

Linear Algebra

Fall 2024, Homework # 2

Due: Oct. 11, 2024

1. (20 pts) For which values of a, b, c is the following matrix invertible? Why?

$$A = \begin{bmatrix} 1 & 1 & a \\ 2 & 3 & b \\ 1 & 2 & c \end{bmatrix}$$

2. (20 pts) Find the determinant of the following matrix. Show your work in sufficient detail.

$$A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 5 & 2 & 6 \\ 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 1 \end{bmatrix}$$

3. (20 pts) Are there constants c_1 and c_2 such that

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Why? Show your work in sufficient detail.

4. (20 pts) Let A and B be two $n \times n$ matrices. Is the equation $(A + B)(A - B) = A^2 - B^2$ always true? Either prove it or find a counter-example.

5. (20 pts) Prove that $\det \begin{pmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{pmatrix} = (a - b)(b - c)(c - a)$