# Linear Algebra
# 線性代數

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# Course Information

- Instructor: 顏嗣鈞; E-mail: hcyen@ntu.edu.tw
- Time: 9:10-12:00, Friday
- Place: BL 103
- Personal Website:
  https://homepage.ntu.edu.tw/~hcyen/index.html
- Course Materials: Textbook + Slides
  - (除了標註※之簡報外，其餘採用李宏毅教授之投影片教材)
- Class web page: the following and NTUCool
  https://homepage.ntu.edu.tw/~hcyen/courses/LA-2024.html
  課程共同網站:
  https://sites.google.com/view/linearalgebra2024fall
- TA: 王璲、 博理館606室 、D07921012@ntu.edu.tw

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# Required Textbook

## Elementary Linear Algebra - A Matrix Approach

2nd Ed., by L. E. Spence, A. J. Insel and S. H. Friedberg

(華泰文化代理)

上課不完全按照教科書內容
但考試範圍都會講到

# Course Outline

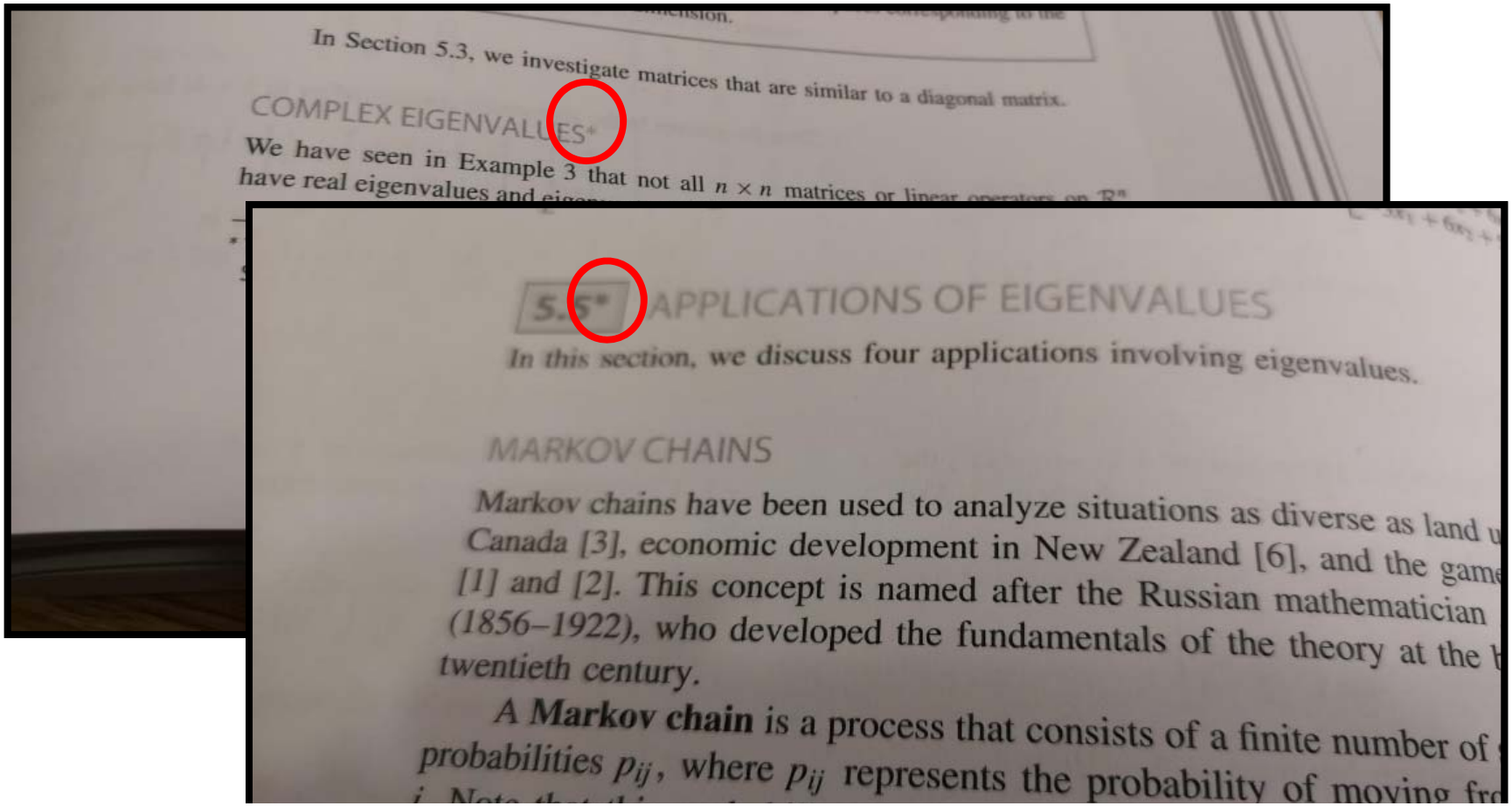
- Chapter 0. Introduction
- Chapter 1. Matrices, Vectors, and Systems of Linear Equations
- Chapter 2. Matrices and Linear Transformations
- Chapter 3. Determinants
- Chapter 4. Subspaces and Their Properties
- Chapter 5. Eigenvalues, Eigenvectors, and Diagonalization
- Chapter 6. Orthogonality
- Chapter 7. Vector Spaces

# Grading

- 期中考(35%)、期末考(35%)、小考/作業 (30%)
- 期中和期末考採各班統一時間舉行及命題
- 作業與考試規則：
  - 作業需準時繳交，遲交者將扣分。嚴禁抄襲，如有違反，將嚴重影響學期成績。
  - 除非生病(需醫師開立證明)，不得請假及要求補考
  - 考試作弊者，除了學期成績不及格外，並將送校議處。

# 考試範圍

- 教科書所有未打星號*章節



In Section 5.3, we investigate matrices that are similar to a diagonal matrix.

COMPLEX EIGENVALUES*

We have seen in Example 3 that not all $n \times n$ matrices

5.5* APPLICATIONS OF EIGENVALUES

*In this section, we discuss four applications involving eigenvalues.*

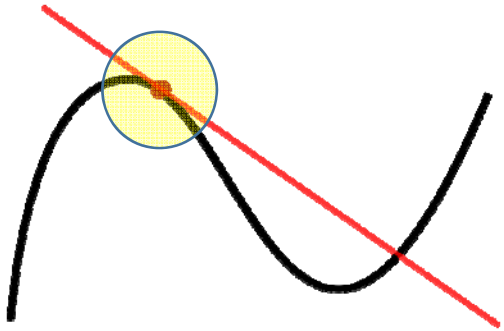
MARKOV CHAINS

*Markov chains have been used to analyze situations as diverse as land u Canada [3], economic development in New Zealand [6], and the game [1] and [2]. This concept is named after the Russian mathematician (1856–1922), who developed the fundamentals of the theory at the twentieth century.*

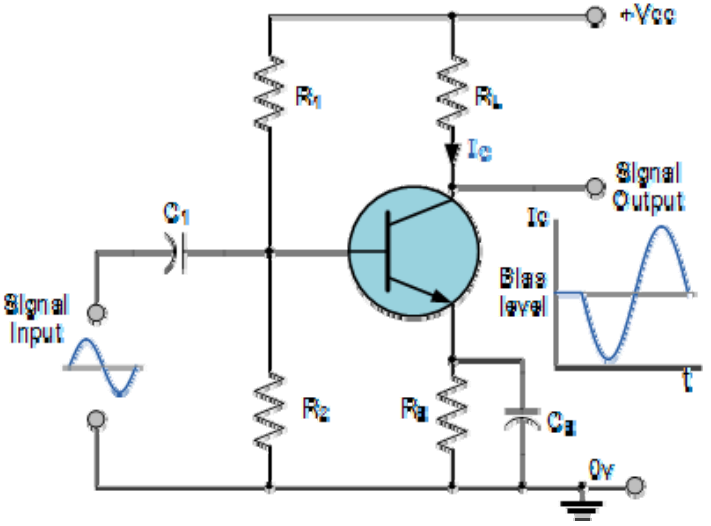
*A* **Markov chain** is a process that consists of a finite number of probabilities $p_{ij}$, where $p_{ij}$ represents the probability of moving fr

# Linear Algebra

- Linear: 線性
  - What is "Linear"? Have to do with line/plane/etc
  - Advantage: Simple → Efficient algorithms
  - Disadvantage: Too simple, (sometimes) not realistic.



"Near" linear if we zoom in

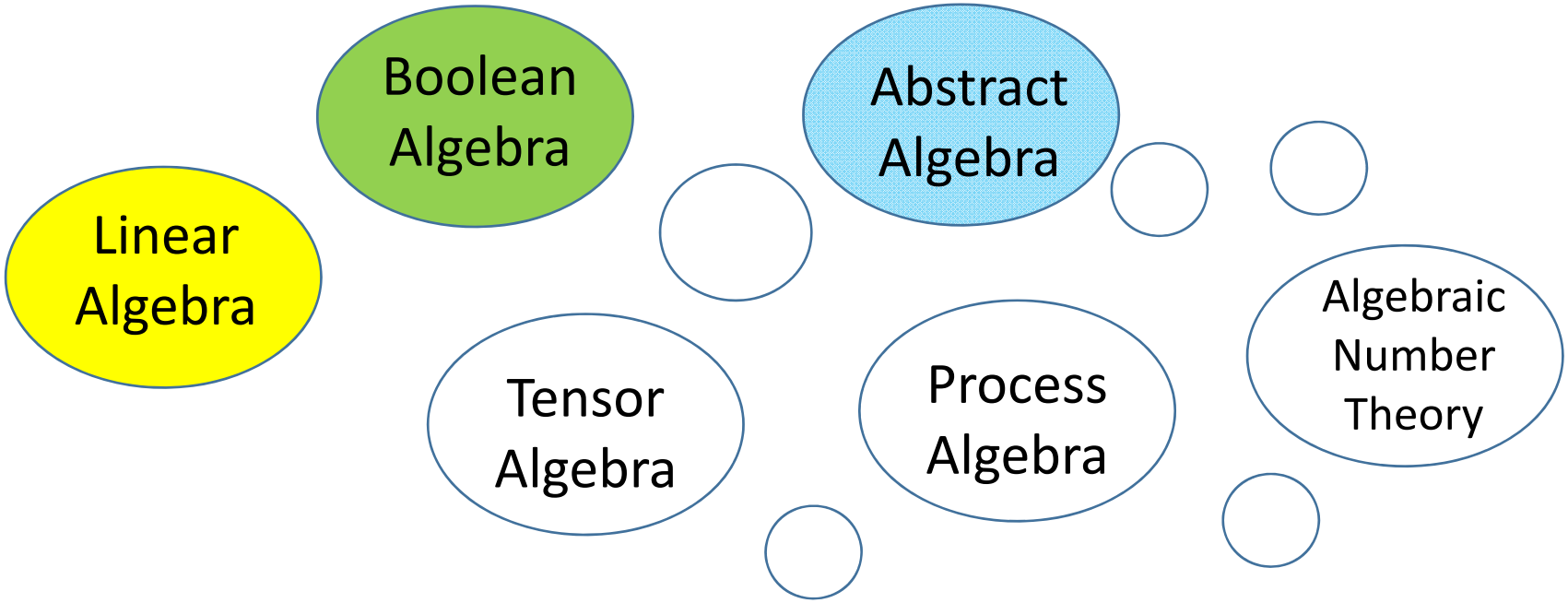



Transistor: a non-linear device
Apply bias to operate in the linear region

圖片資料來源：網站

# Algebra

- **(**Wikipedia) **Algebra** is the study of **variables** and the **rules** for manipulating these variables in formulas; it is a unifying thread of almost all of mathematics.



# Why Study Algebra?

- Algebra provides a unified way of studying many problems in math., science, and engineering. E.g.
  - Linear Algebra（線性代數）: Machine learning, Quantum computing, Coding theory, Circuit theory, Optimization, Control theory, …
  - Boolean algebra（布林代數）: Computer circuits, Computer programming, Formal verification, …
  - Abstract algebra（抽象代數）: Cryptography, Coding theory, Communication, Symmetry, …

# In this Course, Linear Algebra is about …

- Solve the matrix equation $A\mathbf{x} = \mathbf{b}$
  - Many practical problems can be captured by the above matrix equation.
  - Solve system of linear equations using matrices, row reduction, inverses, etc.
- Solve the matrix equation $Av = \lambda v$
  - Solve eigenvalue problem using characteristic polynomial.
  - Understand dynamic of linear transformations via eigenvalues, eigenvectors, diagonalization.

In fact, large classes of engineering problems, no matter how huge, can be reduced to linear algebra.

# What to learn in Linear Algebra?

*Linear System*

# System

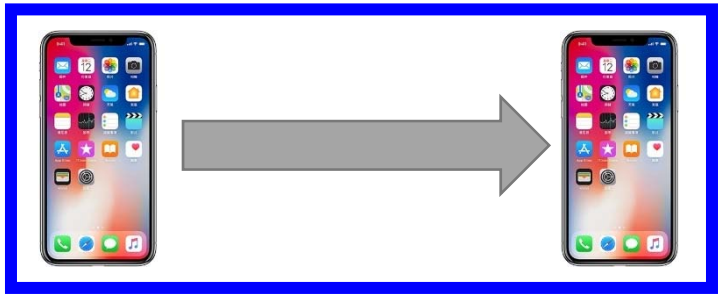
- A system has input and output (function, transformation, operator)

***Speech Recognition System***


***Dialogue System*** (e.g. Siri, Alexa)

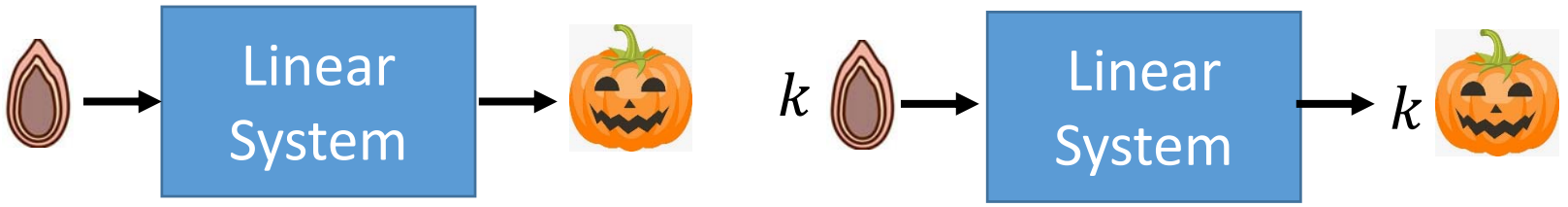
"How are you"  "I am fine"

***Communication System***

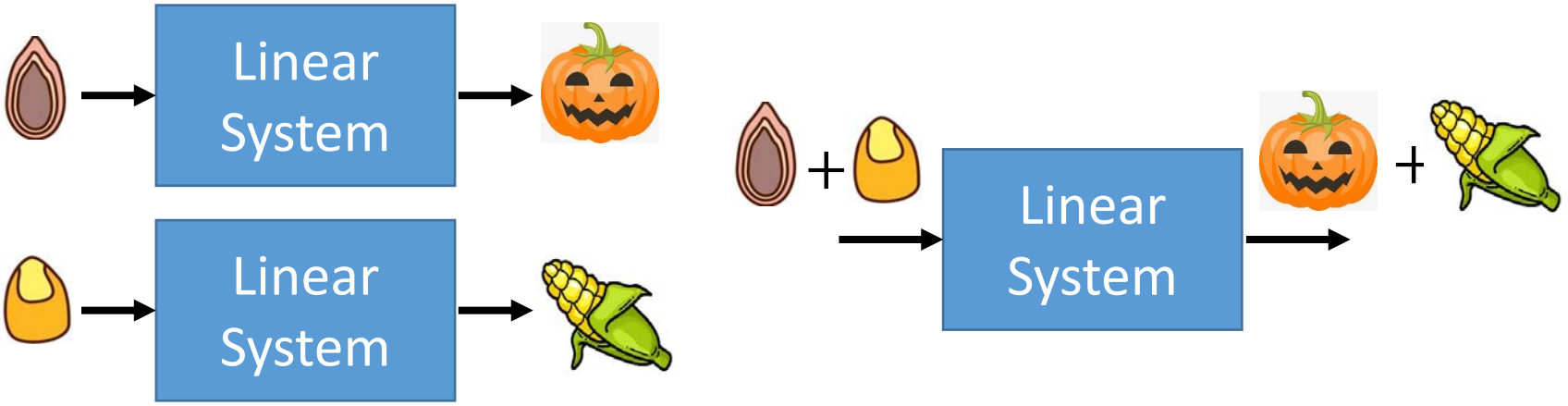
"Hello"  "Hello"

# Linear System

- 1. Persevering Multiplication



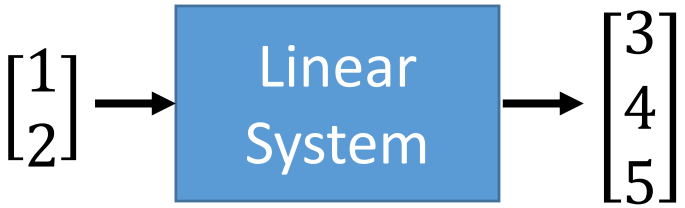

- 2. Persevering Addition

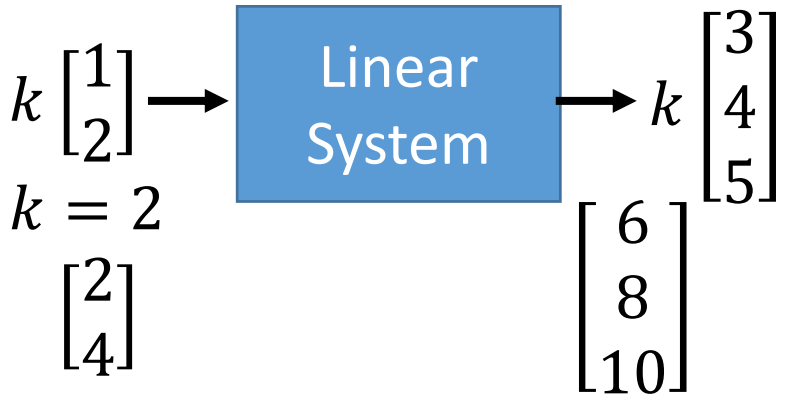


# Linear System

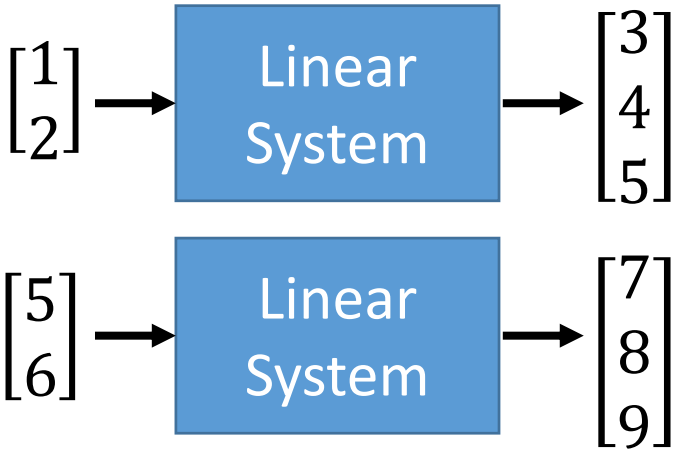
When the input and output are vectors

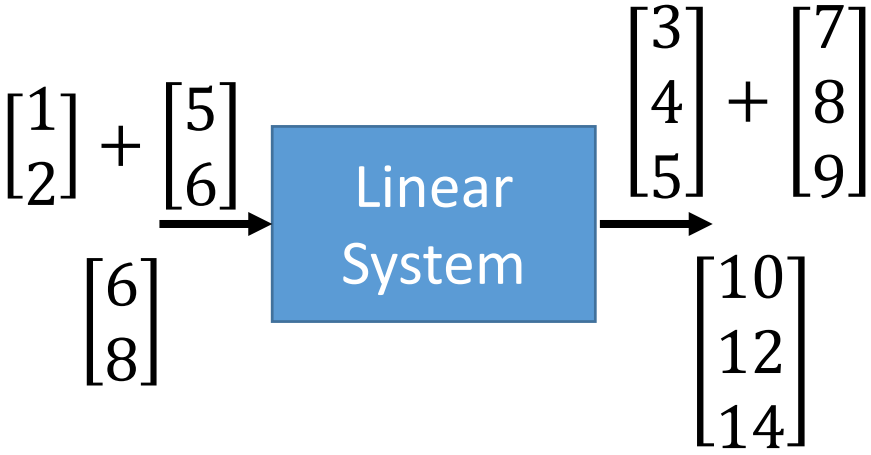
- 1. Persevering Multiplication




- 2. Persevering Addition



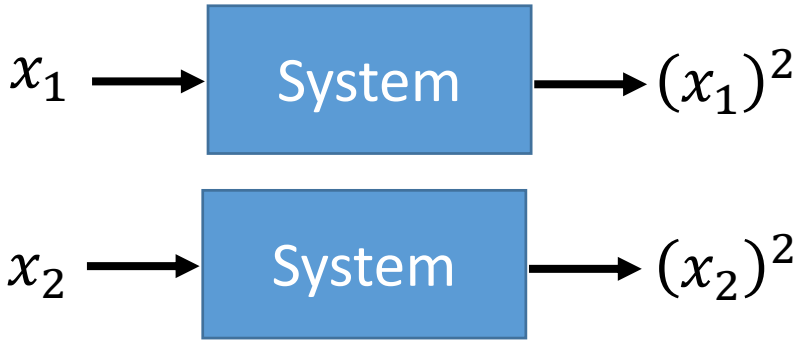
Are they *Linear*?

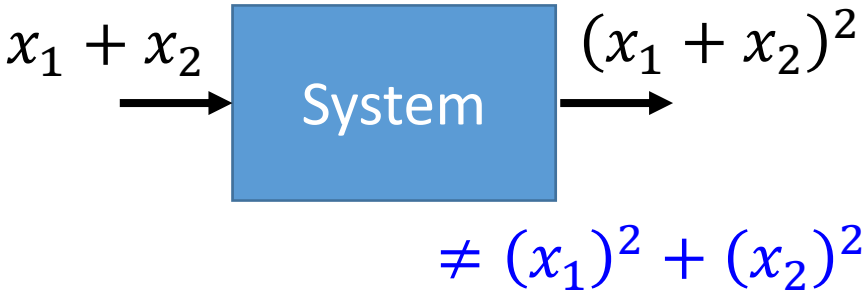
# Linear? **NO**


- 1. Persevering Multiplication



- 2. Persevering Addition



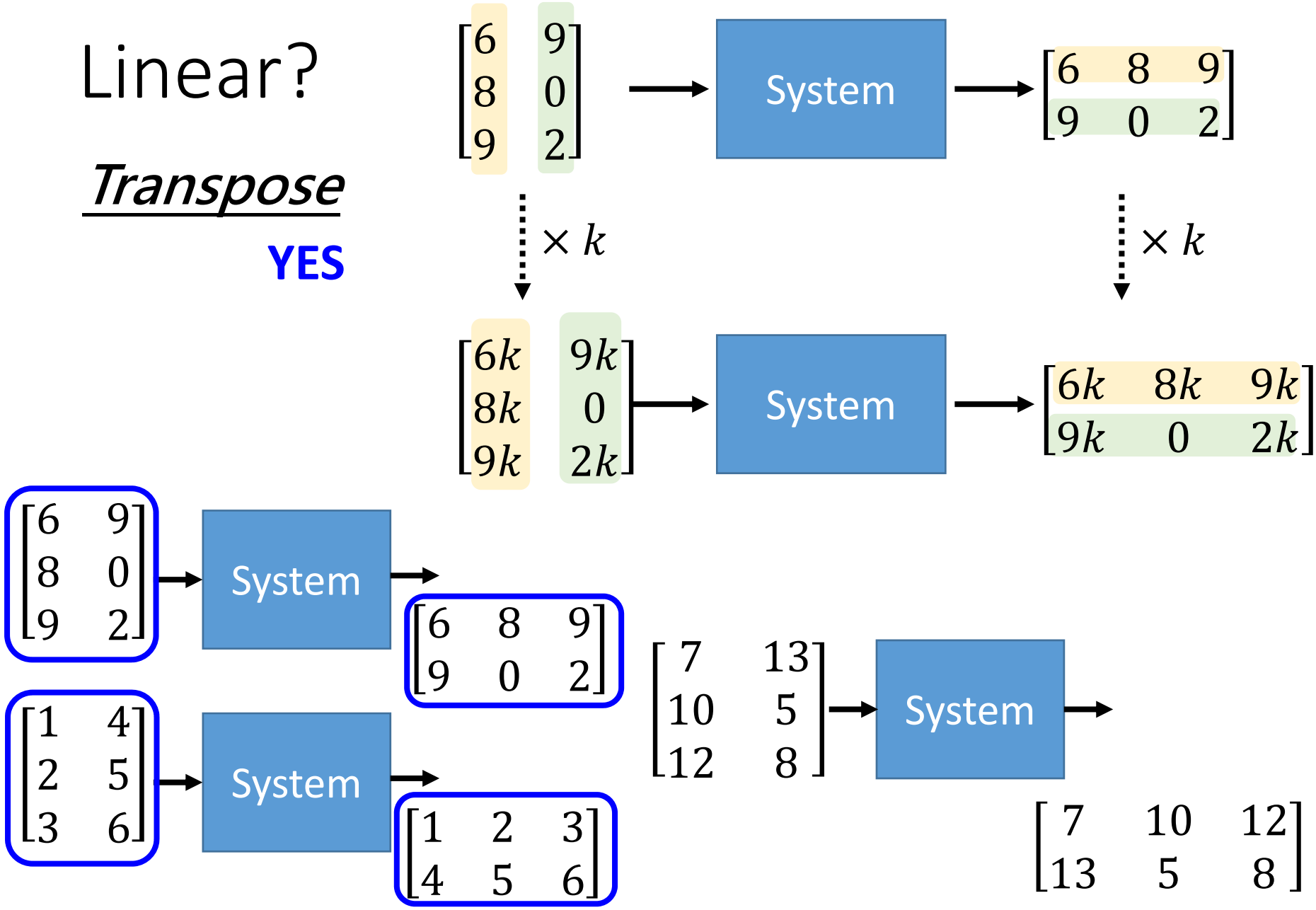



以左上到右下的對角線為軸進行翻轉

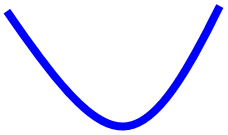
# Linear?

***Transpose***

**YES**



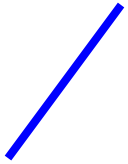
# Linear? **YES**

$x^2$

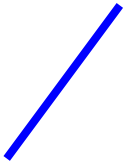
$$f \rightarrow f'$$

$$g \rightarrow g'$$



2x

$$kf \rightarrow kf'$$



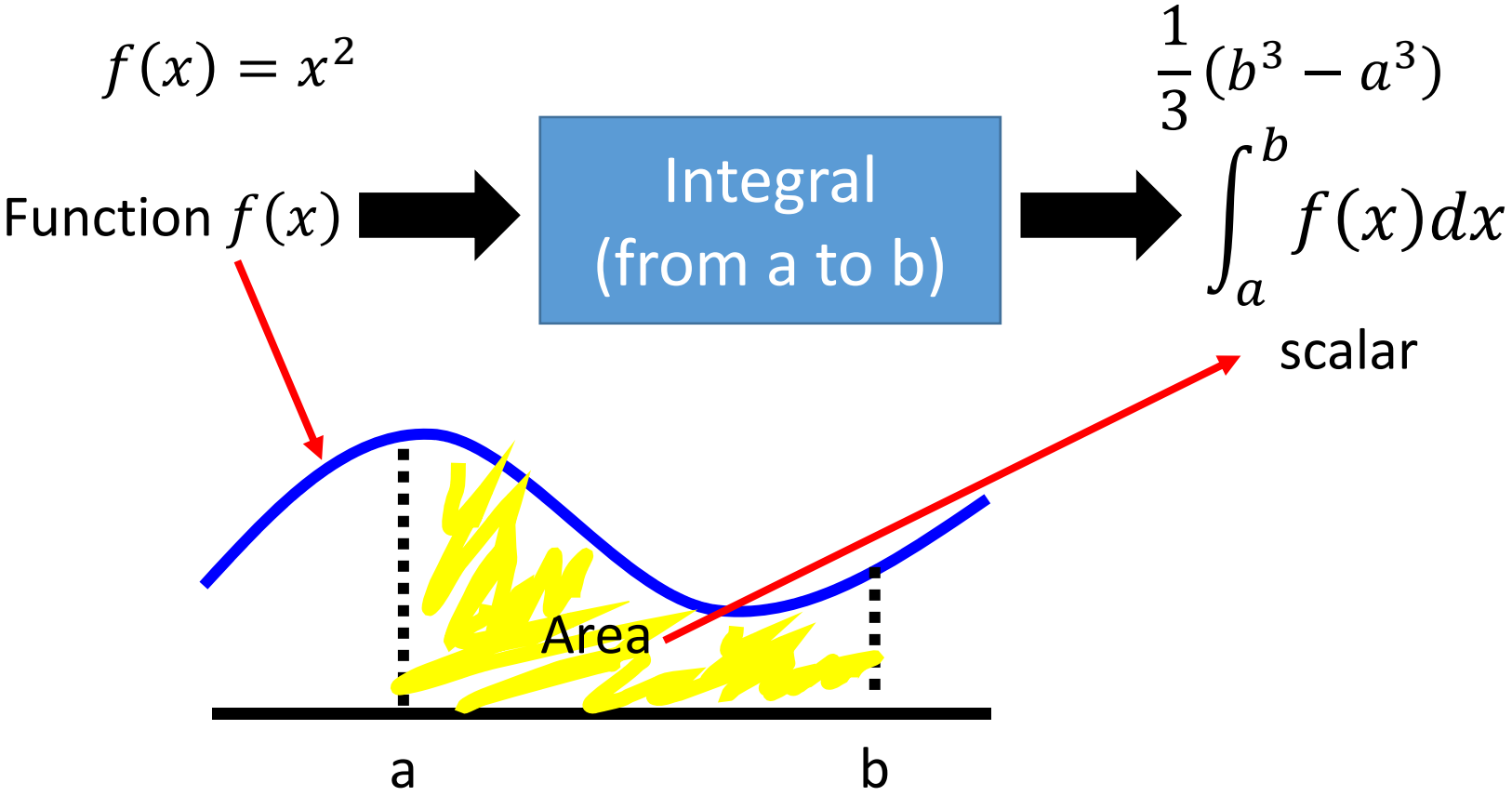
3x

$$f + g \rightarrow f' + g'$$



3

# Linear?


$f(x) = x^2$
$\frac{1}{3}(b^3 - a^3)$
Function $f(x)$
Integral
(from a to b)
$\int_a^b f(x)dx$
scalar
Area
a
b

# Linear? **YES**

$$f(x) \longrightarrow \int_a^b f(x)dx$$

Persevering Multiplication

$$kf(x) \longrightarrow \int_a^b kf(x)dx$$

$$= k\int_a^b f(x)dx$$

---

Persevering Addition

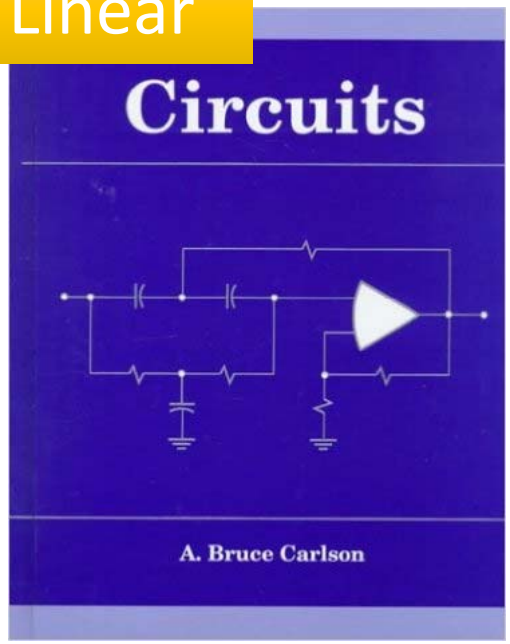
$$f(x) \longrightarrow \int_a^b f(x)dx \qquad g(x) \longrightarrow \int_a^b g(x)dx$$

$$f(x) + g(x) \longrightarrow \int_a^b [f(x) + g(x)]dx$$

$$= \int_a^b f(x)dx + \int_a^b g(x)dx$$

# 電路學

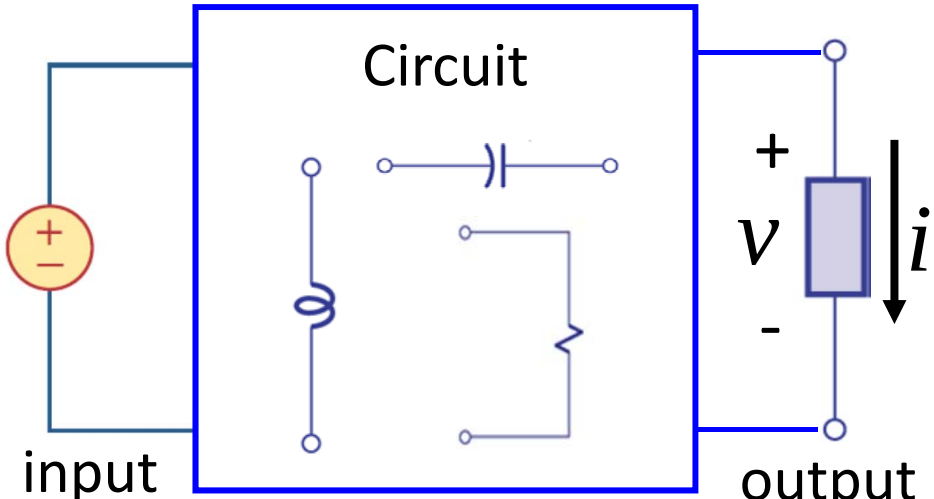
Linear




(大一必修)

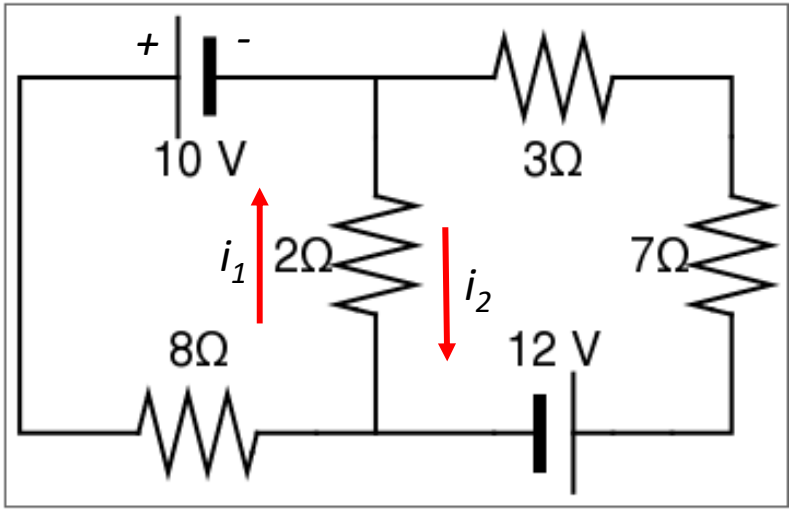
Input: voltage source, current source

output: voltage and current on the load (燈泡、引擎)




Linear System

# An Application




Kirchhoff's Laws

- $10 - 8i_1 - 2(i_1 - i_2) = 0,$
- $12 - 2(i_2 - i_1) - 7i_2 - 3i_2 = 0$

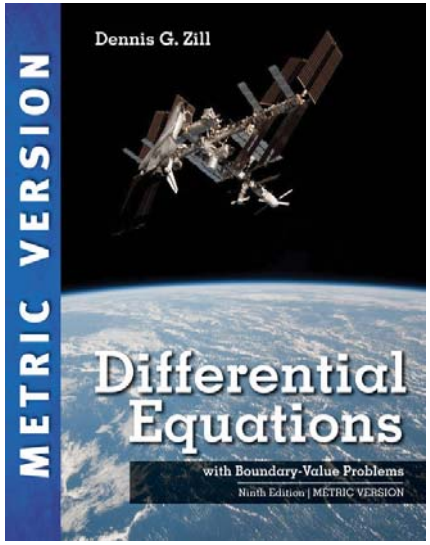
⇩

- $10i_1 - 2i_2 = 10$
- $2i_1 - 12i_2 = -12$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$A = \begin{bmatrix} 10 & -2 \\ 2 & -12 \end{bmatrix} \quad x = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ -12 \end{bmatrix}$$

# 微分方程 (大二上必修)




$$x^{(3)} + a_2x^{(2)} + a_1x^{(1)} + a_0x = 0,$$
$$where\ x^{(1)} = x' \quad x^{(2)} = x'' \quad x^{(3)} = x'''$$

*Let*

$$x_1 = x$$
$$x_2 = x^{(1)}$$
$$x_3 = x^{(2)}$$

$$\mathrm{d/dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

*Then*

$$x'_1 = x_2$$
$$x'_2 = x_3$$
$$x'_3 = x^{(3)} = -a_2x^{(2)} - a_1x^{(1)} - a_0x$$

$$\mathbf{x' = Ax}$$

Linear Algebra meets Differential Equation

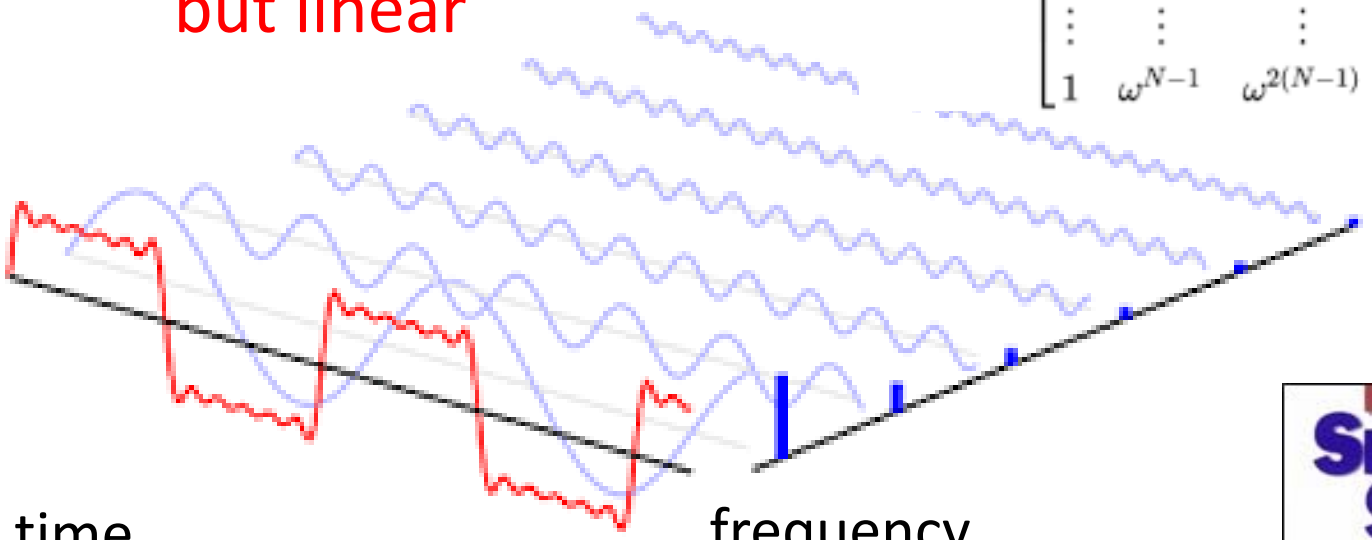
※

# 信號與系統 (大二下必修)

Complex …
but linear

$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

Basically just
change of basis

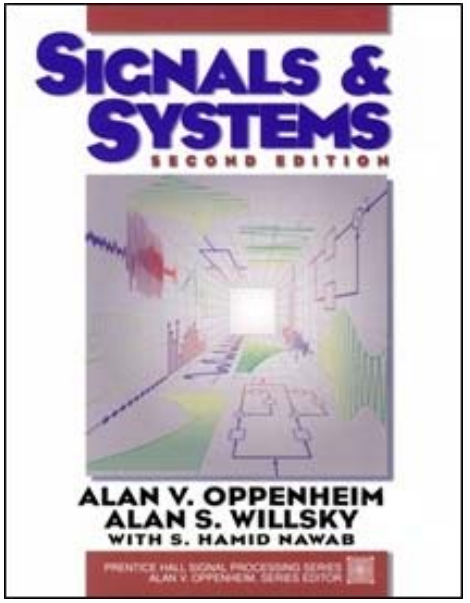


time

frequency

Fourier Transform

Linear System



# Why Taking One Semester?

- Most of you know how to solve systems of linear equations in high school（高斯消去法)
- Why bother spending a whole semester learning Linear Algebra?

  Reason:

  - Often engineers need to solve lots of equations with lots of variables
  - Can we gain useful information about the set of solutions without having to solve the equations?
  - It is also challenging to recognize problems in real-world that can be formulated using linear algebra.

# How to do well in this course?

- Come to the class and pay attention to lectures
- Read the textbook and class notes
- Do homework and exercises carefully
- Ask questions
- Discuss with your classmates (when allowed)
- Do not be afraid of  mathematical proofs
- Practice, practice, practice!