Control System: Homework 07 for Unit 5D, 5E, 5F: Root Locus

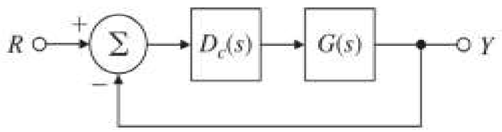
Assigned: Nov 11, 2022

Due: Nov 17, 2022 (23:59)

Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

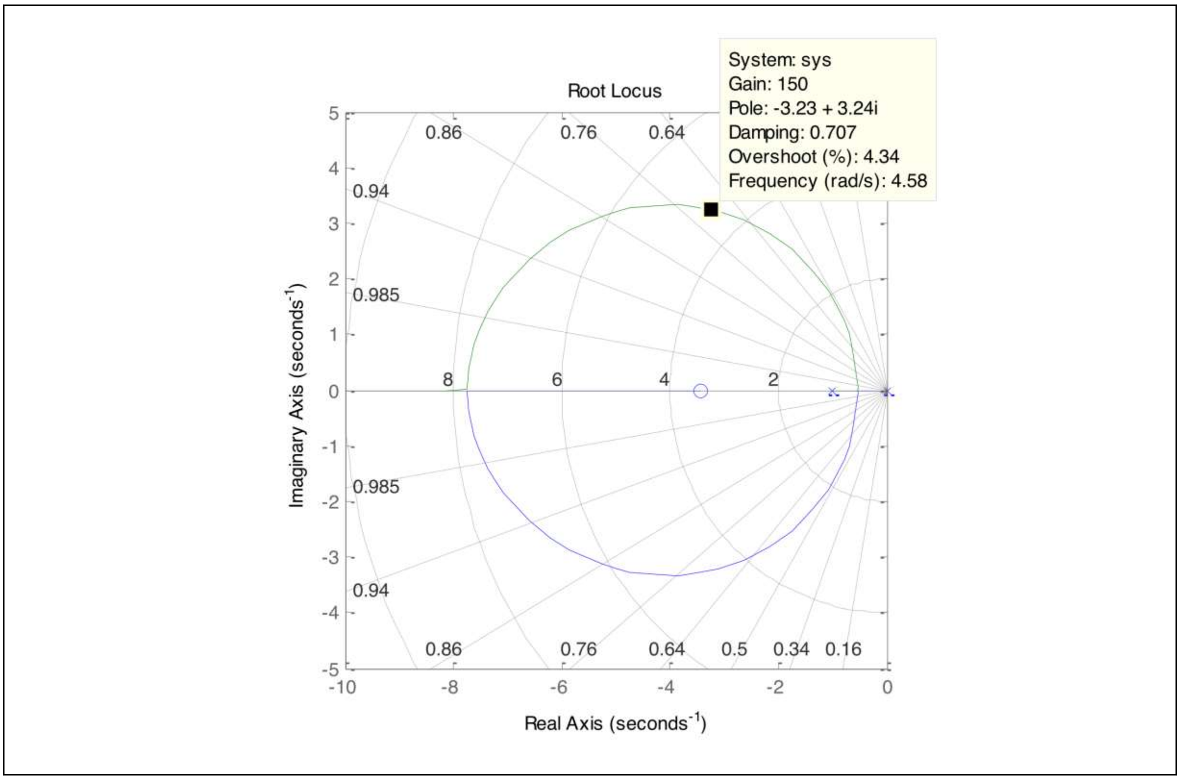
1. (U5D: Lead-Lag Compensator)

23. Suppose the unity feedback system of Fig. 5.59 has an open-loop plant given by $G(s) = \frac{1}{s(s+1)}$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.



Solution:

Setting the pole of the lead to be at $p = -30$, and the zero is at $z = -3$ produces a locus with a circle that goes a bit too high and misses the desired $-3.2 + 3.2j$. So move the zero a bit to the West, i.e. let $z = -3.44$. It does the job, so put your cursor on the spot and find that with a gain of $K = 150$ gives the desired roots. The locus is plotted below.



Root Locus
System: sys
Gain: 150
Pole: -3.23 + 3.24i
Damping: 0.707
Overshoot (%): 4.34
Frequency (rad/s): 4.58
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.86
0.76
0.64
0.94
0.985
0.5
0.34
0.16
8
6
4
2

2. (U5D: Lead-Lag Compensator)

26. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D_c(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.
- The steady-state error to a unit ramp at the reference input must be less than 0.05.

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications, ignoring the error requirement.

(b) What is the velocity constant K_v for your design? Does it meet the error specification?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the Matlab plot of the root locus of your final design.

(e) Give the Matlab response of your final design to a reference step.

Solution:

(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245. With the lead compensator, the overshoot is reduced to 3.64% and the rise time is 0.35 sec.

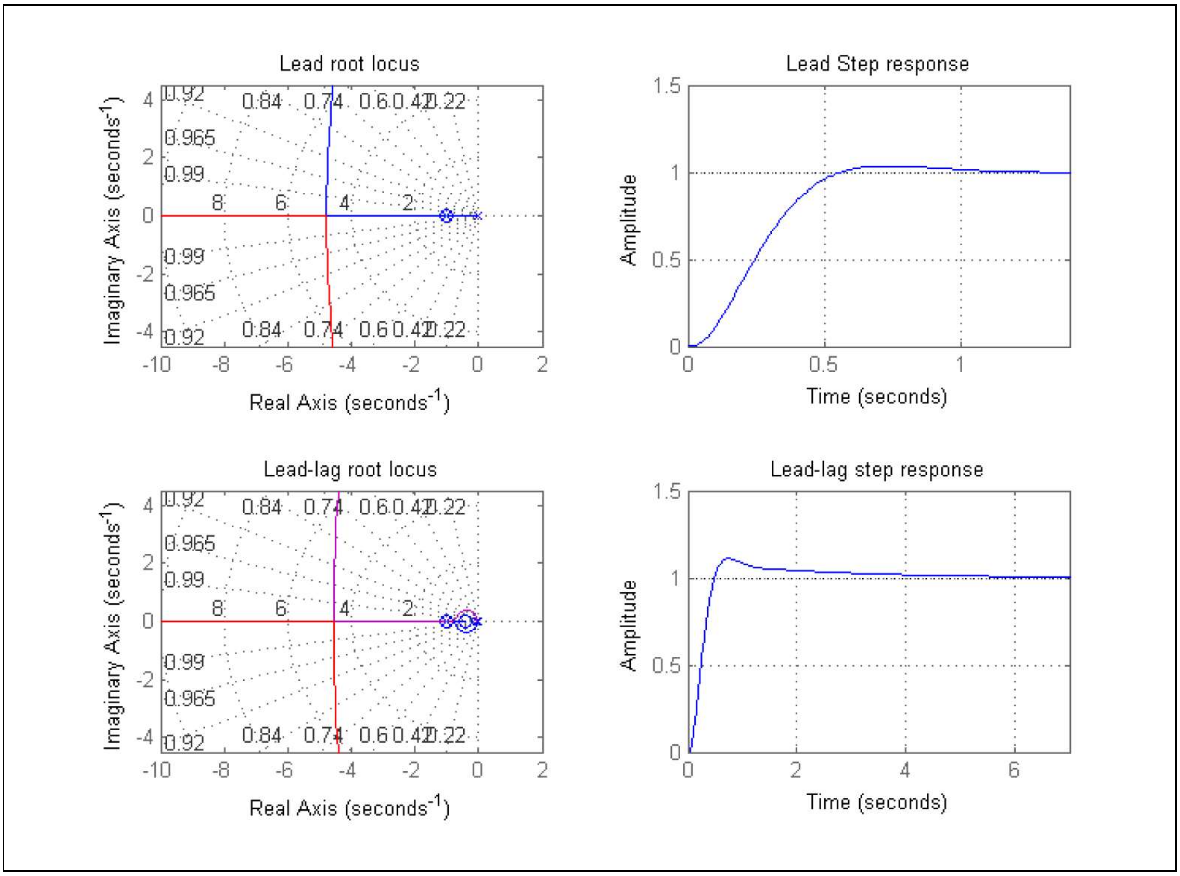
(b)

$$K_v = \lim_{s\to 0} sGD_c = \lim_{s\to 0} s\frac{10}{s(s+1)(s+10)}\frac{245(s+1)}{(s+6)} = 4.083$$

(c) To meet the steady-state requirement, we need a new $K_v = 20$, which is an increase of a factor of 5. If we set the lag zero at $z = -0.4$, the pole needs to be at $p = -0.08$.

(d) The root locus is plotted below.

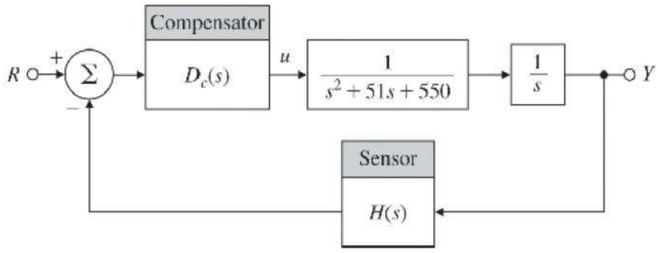
(e) The step response is plotted below.



Lead root locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Lead Step response
Amplitude
Time (seconds)
Lead-lag root locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Lead-lag step response
Amplitude
Time (seconds)

3. (U5E: Design using the Root Locus)

40. Consider the instrument servomechanism with the parameters given in Fig.5.68. For each of the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your final design.



(a) *Lead network* : Let

$$H(s) = 1, \quad D_c(s) = K\frac{s+z}{s+p}, \quad \frac{p}{z} = 6.$$

Select z and K so that the roots nearest the origin (the dominant roots) yield

$$\zeta \geq 0.4, \quad -\sigma \leq -7, \quad K_v \geq 16\frac{2}{3}\text{sec}^{-1}.$$

(b) *Output-velocity (tachometer) feedback*: Let

$$H(s) = 1 + K_T s \quad \text{and} \quad D_c(s) = K.$$

Select K_T and K so that the dominant roots are in the same location as those of part (a). Compute K_v. If you can, give a physical reason explaining the reduction in K_v when output derivative feedback is used.

(c) *Lag network* : Let

$$H(s) = 1 \quad \text{and} \quad D(s) = K\frac{s+1}{s+p}.$$

Using proportional control, is it possible to obtain a $K_v = 12$ at $\zeta = 0.4$? Select K and p so that the dominant roots correspond to the proportional-control case but with $K_v = 100$ rather than $K_v = 12$.

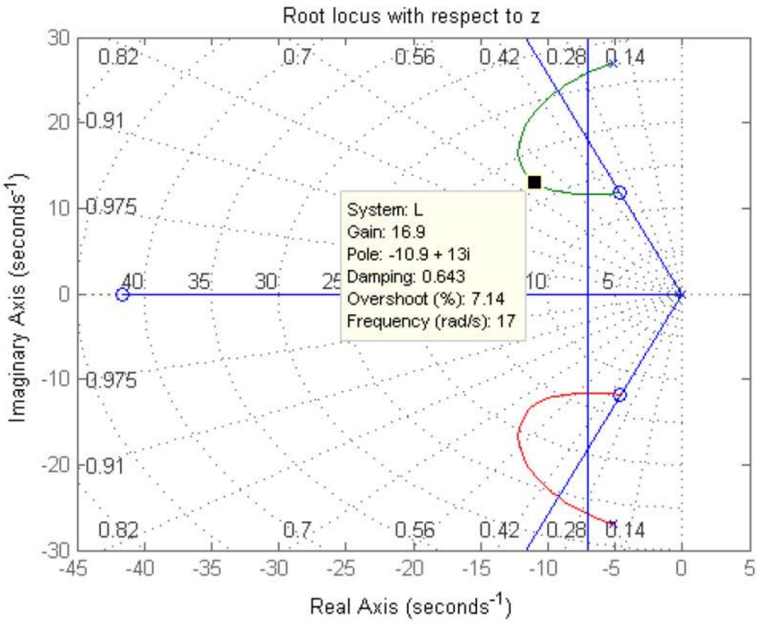
Solution:

(a) Setting $p = 6z$, the velocity constant is

$$K_v = \lim_{s\to 0} s \frac{K(s+z)}{s+6z} \frac{1}{s(s^2+51s+550)} = \frac{K}{3300}$$

Thus the K_v requirement leads to $K \geq 35200$. With $K = 40000$, a root locus can be drawn with respect to z.

$$1 + z\frac{6s(s^2+51s+550)+40000}{s^2(s^2+51s+550)+40000s} = 0$$



Root locus for Problem 5.40(a)

At the point of maximum damping, the values are $z = 16.8$ and the dominant roots are at $s = -11 \pm 13j$. So the compensator is $D_c(s) = 40000\dfrac{s+16.8}{s+100.8}$.

(b) With $H(s) = 1 + K_T s$ and $D_c(s) = K$, the closed-loop transfer function is

$$\frac{Y}{R} = \frac{K}{s^3 + 51s^2 + (550 + KK_T)s + K}$$

For this system to have poles at $s = -11 \pm 13j.$, the characteristic polynomial should be in the form of

$$(s + p)(s^2 + 22s + 290) = s^3 + (p + 22)s^2 + (22p + 290)s + 290p$$

Equating the coefficients leads to $p = 29$, $K = 8410$, and $K_T = 0.045$. With these value, the velocity constant is

$$\frac{1}{K_v} = \lim_{s \to 0} s\left(1 - \frac{Y}{R}\right)\frac{1}{s^2} = \frac{550 + KK_T}{K} \quad \Rightarrow \quad K_v = 9.058$$

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.

(c) Using proportional control ($D_c(s) = K$), the velocity constant is

$$K_v = \lim_{s \to 0} sK\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550}$$

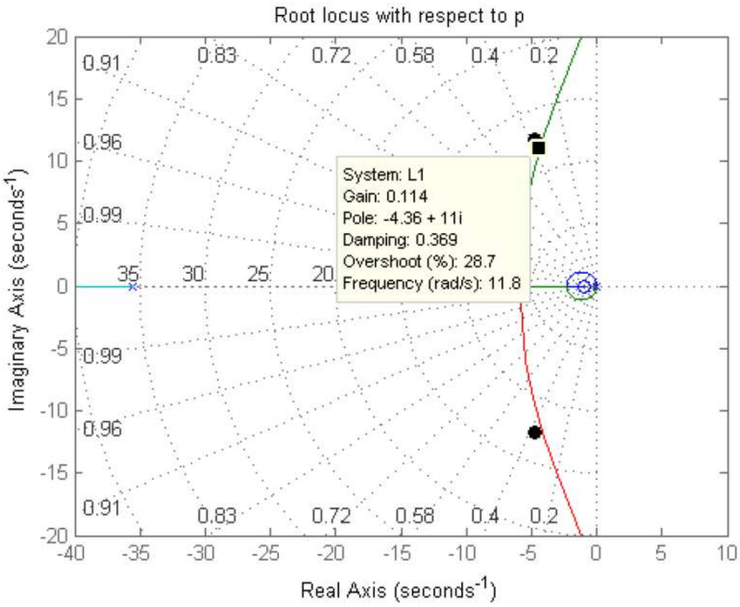
Therefore $K_v = 12$ can be obtained by setting $K = 6600$. With this value, the dominant roots are at $s = -4.7 \pm 11.69j$, and $\zeta = 0.37$.

With $D_c(s) = K\frac{s+1}{s+p}$, the velocity constant is

$$K_v = \lim_{s \to 0} s\frac{K(s+1)}{s+p}\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550p}$$

So $K_v = 100$ can be obtained by setting $\frac{K}{p} = 55000$. Setting $K = 55000p$, a root locus can be drawn with the parameter p

$$1 + p\frac{s(s^2 + 51s + 550) + 55000(s + 1)}{s^2(s^2 + 51s + 550)} = 0$$



Root locus for Problem 5.40(c)

In the plot, the desired pole locations are marked with a dot (•). Thus we can choose $p = 0.11$ to place the poles near the desired locations. Thus the compensator is $D_c(s) = 6050\dfrac{s+1}{s+0.11}$.

參考觀摩的作業

1. (Lead-Lag Compensator)

作者：b08901085 施彥宇

理由：詳細解釋如何計算根軌跡位置

作者：b09901143，李立安

理由：透過題目要求詳細求出給定極點求零點與 K 的方法

作者：b09901166，陳嘉彤

理由：詳細解釋使用 MATLAB 調整 lead compensator 參數達到系統要求

作者：b10202032，卓 然

理由：詳細解釋根軌跡位置與補償器極點、零點、K 值的關係

HW07 – Unit 5, Root Locus

學號：B08901085　　　系級：電機四　　　姓名：施彥宇

- **Question :**

Suppose the unity feedback system of Fig.1 has an open-loop plant given by $G(s) = \frac{1}{s(s+1)}$. Design a lead compensation $D_C(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.

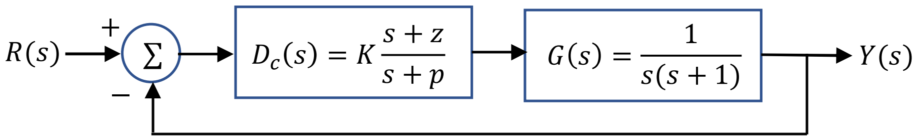


Fig. 1, An unity feedback system

- **Solution :**

Setting the pole of the lead to be at $p = -30$, and the zero is at $z = -3$ produces a locus with a circle that goes a bit too high and misses the desired $(-3.2, -3.2)$. So move the zero a bit to the West, i.e. let $z = -3.44$. It does the job, so put the cursor on the spot and find that with a gain of $K = 150$ gives the desired roots, which means $D_c(s) = 150\frac{s+3.44}{s+30}$. The root locus is plotted below :

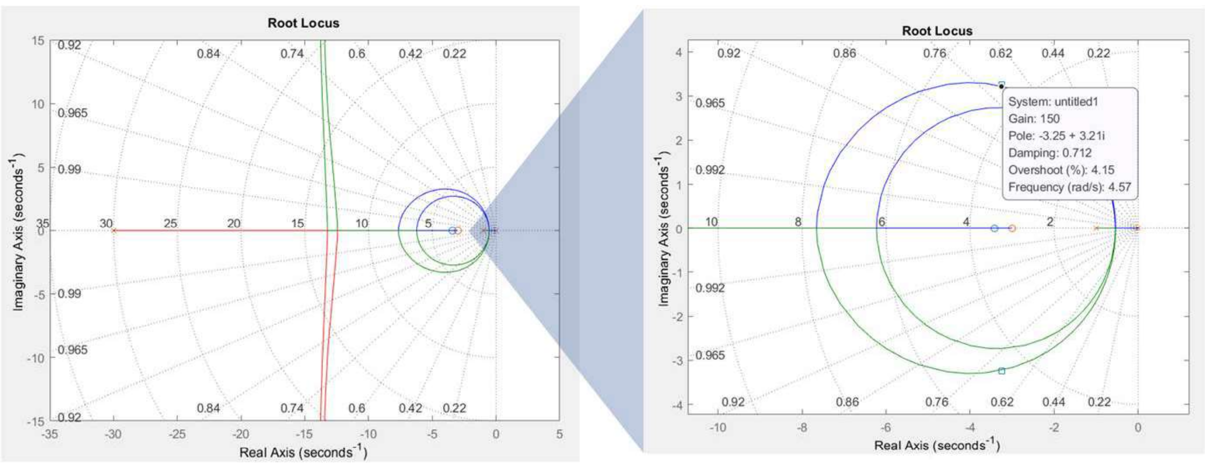


Fig. 2, Root locus of system in Fig,1 with a lead compensator.

- **What I can do more :**

The solution above gets the parameters mainly by trail and error. If one without experience or the sense of designing compensator, it may cause many time for whom to find proper compensator. As a result, I use a method starting from the desired poles and then figure out those parameters, which may help us saving time while designing compensator.

- **What I can do more :**

From the question, the desired poles are $s_d = -3.2 \pm 3.2j$ 、 and the original poles are $p_1 = 0$ 、 $p_2 = -1$. Therefore, the location of these poles are shown in Fig 3 beside :

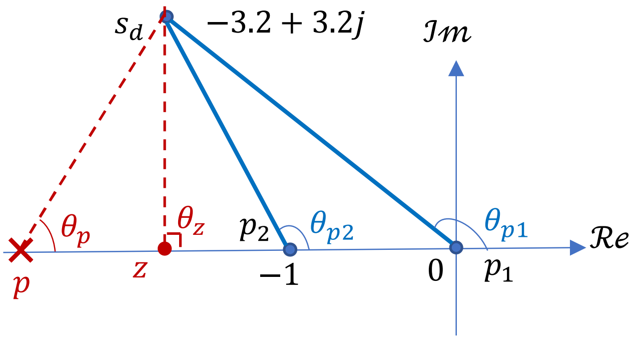


Fig.3, Pole location of system in Fig,1.

Supposing the angle of zero is 90°, which means $z = 3.2$. If s_d is on the root locus, it means that $\angle G(s_d) = 180° \pm 360°n,\ n = 0, \pm1, \pm2 \dots$, so we can get θ_p in :

$$180° = \theta_z - (\theta_{p1} + \theta_{p2} + \theta_p)$$

$$90° = -\left(180° - \tan^{-1}\left|\frac{3.2}{-3.2}\right|\right) - \left(180° - \tan^{-1}\left|\frac{3.2}{-2.2}\right|\right) - \theta_p$$

$$\Rightarrow \theta_p = -349.51° = 10.49° \qquad (1).$$

Then, the position of p can be calculate as :

$$p = z + \frac{\mathcal{Im}\{s_d\}}{tan\ \ _p} = -3.2 - \frac{3.2}{\tan 10.49°} = -20.48 \qquad (2).$$

Thus, the compensator becomes $D_c(s) = k\frac{s+3.2}{s+20.48}$, at this moment, value k can be determined by the product of the distance from s_d to poles and zeros.

$$k = \frac{|s_d - p_1| \times |s_d - p_2| \times |s_d - p|}{|s_d - z|} = \frac{3.2\sqrt{2} \times \sqrt{3.2^2 + 2.2^2} \times \sqrt{3.2^2 + 17.28^2}}{3.2} = 96.5 \qquad (3).$$

Now testing our result on MATLAB, one can observe that the value of gain is 96.8, which is almost the same as our result. This represents our method is reliable and feasible.

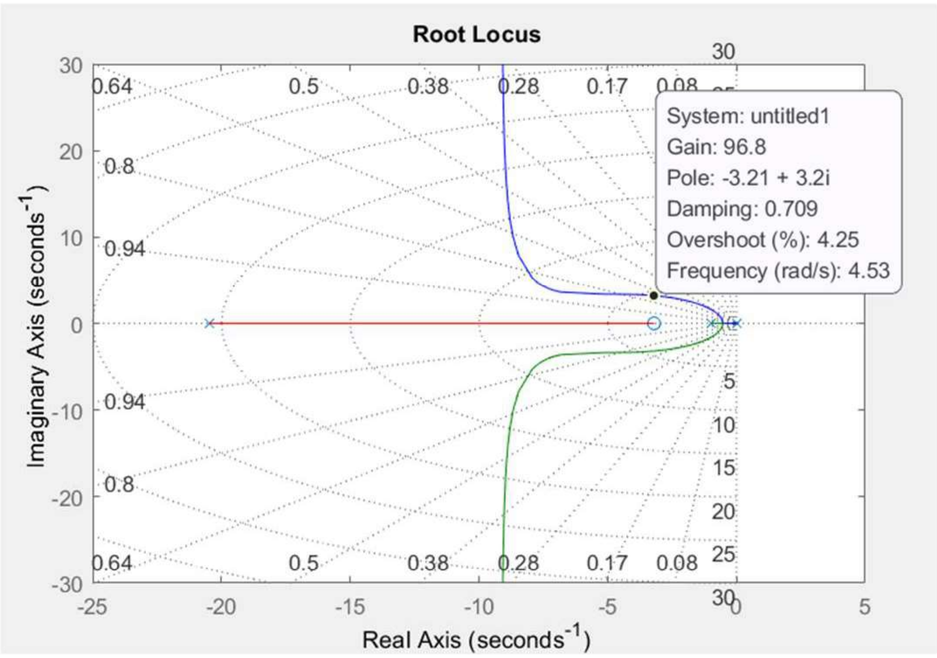


Fig.4, The result of designing compensator using method in page 2

- **What I can do more :**

The method above which supposes $\theta_z = 90°$ is an ideal condition. Because only when frequency goes to infinity has a 90° angle in a phase leading circuit. Also, if one puts z under s_d, p may be too far and is not such feasible. Hence, another method of designing compensator properly can be modified as :

Defining $\theta_z - \theta_p = \theta_m$ is the compensating angle. From the method in page 2, we can get the compensating angle to be :

$$180° = \theta_z - (\theta_{p1} + \theta_{p2} + \theta_p)$$

$$\Rightarrow \theta_z - \theta_p = \theta_m = 180° + (\theta_{p1} + \theta_{p2}) \qquad (4).$$

$$\theta_m = 180° + \left(180° - \tan^{-1}\left|\frac{3.2}{-3.2}\right|\right) + \left(180° - \tan^{-1}\left|\frac{3.2}{-2.2}\right|\right)$$

$$= 439.51° = 79.51° \qquad (5).$$

In order to make $\theta_z < 90°$, we set θ_s is the angle between s_d to origin and horizontal line, which is shown in Fig,5.

$$\theta_s = 180° - \tan^{-1}\left|\frac{3.2}{3.2}\right| = 135° \quad (6).$$

And we can set $\angle z s_d w = \angle p s_d w = \frac{\theta_m}{2}$ to get p and z. The condition of $\theta_z < 90°$ is $\theta_{s'} + \frac{\theta_m}{2} < 90°$. As a result, we set $\theta_{s'} = 45°$.

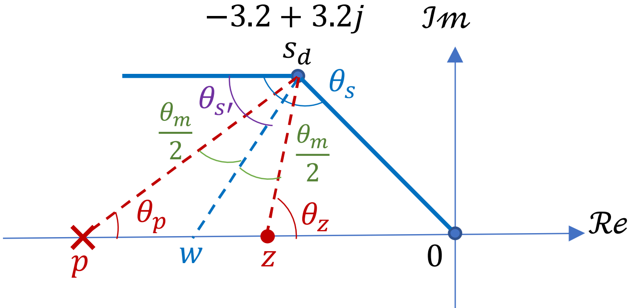


Fig.5, θ_s and w in s-domain

As a result, p and z can be calculated by the following equations :

$$\theta_z = \theta_{s'} + \frac{\theta_m}{2} = 84.755° \Rightarrow z = \mathcal{Re}\{s_d\} - \frac{\mathcal{Im}\{s_d\}}{tan\theta_z}$$

$$= -3.2 - \frac{3.2}{\tan 84.755°} = -3.49 \qquad (7).$$

$$\theta_p = \theta_{s'} - \frac{\theta_m}{2} = 5.245° \Rightarrow p = \mathcal{Re}\{s_d\} - \frac{\mathcal{Im}\{s_d\}}{tan\theta_p}$$

$$= -3.2 - \frac{3.2}{\tan 5.245°} = -38.06 \qquad (8).$$

$$k = \frac{|s_d - p_1| \times |s_d - p_2| \times |s_d - p|}{|s_d - z|} = \frac{3.2\sqrt{2} \times \sqrt{3.2^2 + 2.2^2} \times \sqrt{3.2^2 + 34.86^2}}{\sqrt{3.2^2 + 0.29^2}} = 191.46 \qquad (9).$$

Thus, the compensator becomes $D_c(s) = k\frac{s+3.49}{s+38.06}$, at this moment, value k can be determined in the same way in equation (3). And the testing graph of this method is shown below in Fig,6.

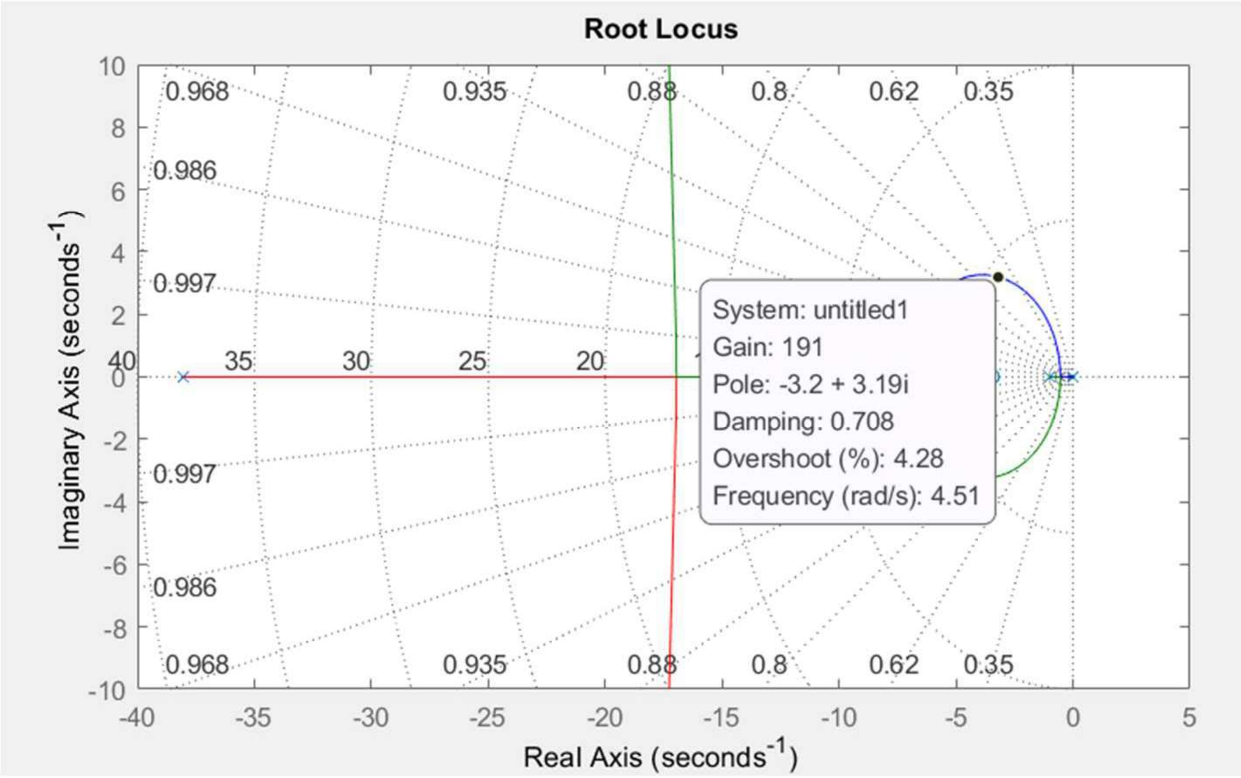


Fig.6, The root locus of designing compensator using method in page 3.

In this case, the compensating angle is larger than $60°$, and the distance of p is still far away from z. Therefore, one can redesign this compensator by separating the original one into two stages for feasibility.

HW 07: Root Locus	Control Systems, Fall 2022, NTU-EE
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Problem 1

$$1+D_c(s)G(s)=0$$

$$\Rightarrow 1+K\frac{s+z}{s(s+1)(s+p)}=0$$

$$\Rightarrow s(s+1)(s+p)+K(s+z)=0$$

$$\Rightarrow s^3+(p+1)s^2+(p+K)s+Kz=0$$

poles at $-3.2\pm3.2j \Rightarrow (s+3.2)^2=-(3.2^2)$
$=-10.24$

$$\Rightarrow s^2+6.4s+20.48=0$$

$\Rightarrow s^2+6.4s+20.48$ 為 $s^3+(p+1)s^2+(p+K)s+Kz$ 的因式

$$\Rightarrow (s^2+6.4s+20.48)(s+a)=s^3+(p+1)s^2+(p+K)s+Kz$$

$\Rightarrow \begin{cases} 6.4+a=p+1 \\ 20.48+6.4a=p+K \\ 20.48a=Kz \end{cases}$ assume we know p $\Rightarrow \begin{cases} a=p-5.4 \\ K=5.4p-14.08 \\ z=\dfrac{20.48p-110.592}{5.4p-14.08} \end{cases}$

lead compensation

$$\Rightarrow z=\frac{20.48p-110.592}{5.4p-14.08}<p$$

$$\Leftrightarrow 20.48p-110.592<5.4p^2-14.08p$$

$$\Leftrightarrow 5.4p^2-34.56p+110.592>0$$

$$\Leftrightarrow p^2-6.4p+20.48>0$$

$$\Leftrightarrow (p-3.2)^2+10.24>0$$

恆成立 if p is on the real axis.

set $p=30 \Rightarrow \begin{cases} z=\dfrac{20.48\times30-110.592}{5.4\times30-14.08} \doteqdot 3.41 \\ \text{gain } K=5.4\times30-14.08=147.92 \end{cases}$

解答給的是 $p=-30$ (應為+30)

$z=-3.44$ (: +3.44) 相差不大

$K=150$

poles at $-3.23\pm j3.24$

P	Z	K
25	3.32	120.92
30	3.41	147.92
35	3.47	174.92

$\dfrac{dz}{dp}=\dfrac{10.5912}{(x-2.60741)^2}$

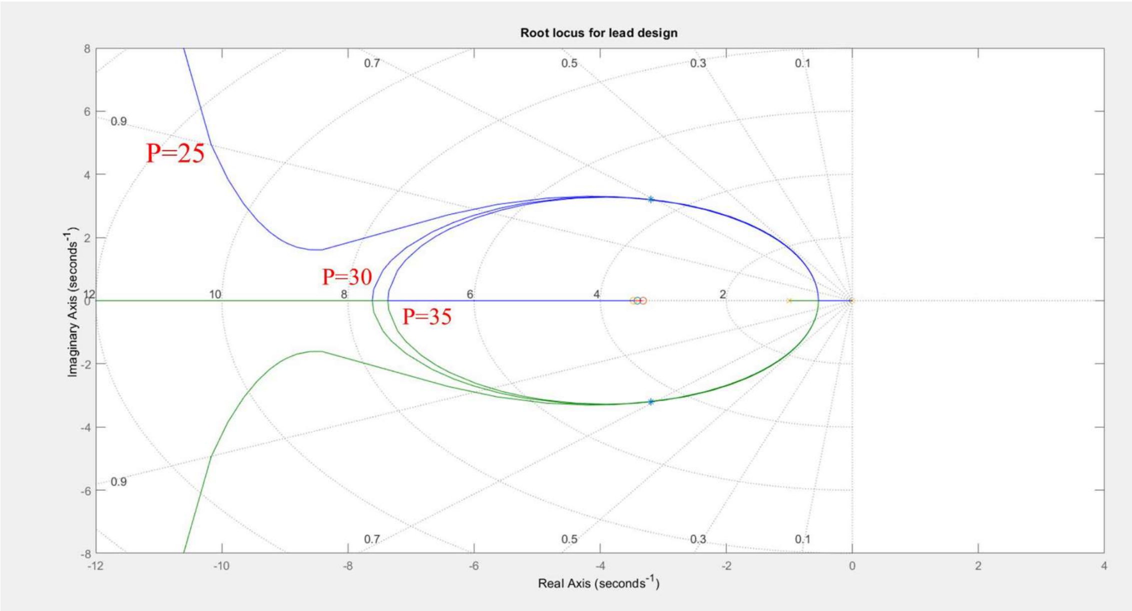
assume $K>0$

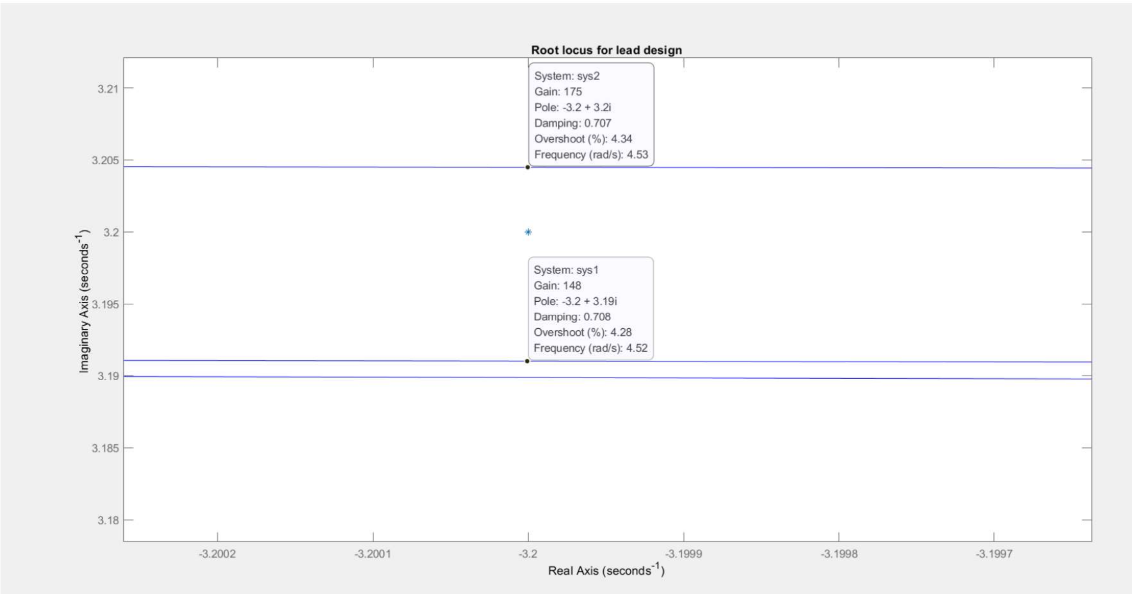
⇒ P↑則z↑且K↑

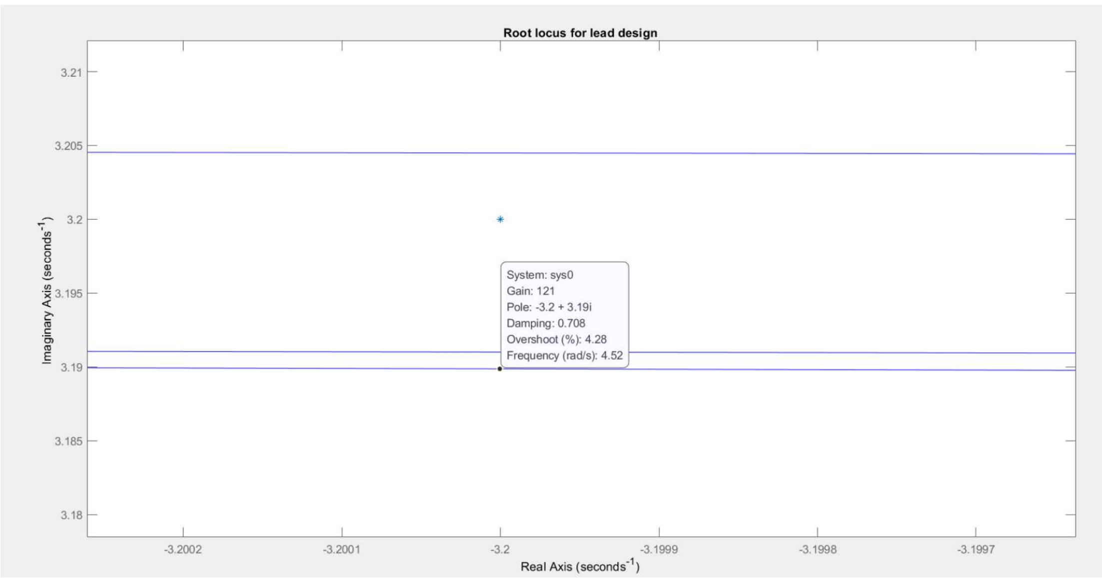
Matlab code:

```
clf;
s=tf('s');
sys1=(s+3.41)/((s*(s+1))*(s+30));
sys0=(s+3.32)/((s*(s+1))*(s+25));
sys2=(s+3.47)/((s*(s+1))*(s+35));
rlocus(sys1)
axis([-12 4 -8 8])
title('Root locus for lead design')
hold on
rlocus(sys0)
hold on
rlocus(sys2)
r1=roots([1 31 177.92 503.808]);
plot(r1,'*')
%roots are at -3.2+j3.2 and -3.2-j3.2
z=0.1:.2:.9;
wn=2:2:35;
sgrid(z, wn)
hold off
```

HW 07: Root Locus	Control Systems, Fall 2022, NTU-EE
Name: 李立安 B09901143	Date: 11/17, 2022







References

[1: HW07_Unit5_ BSOL.PDF]

HW07_Unit5B_SOL.PDF

1. Problem 1 (Lead-Lag Compensator)

First, we guess the pole at $p = -30$ and zero at $z = -3$. Then run the *rlocus* function in Matlab. It is a little bit too high and thus misses the desired poles. As shown in Fig.1, poles are located at $s = -3 \pm 2.7j$.

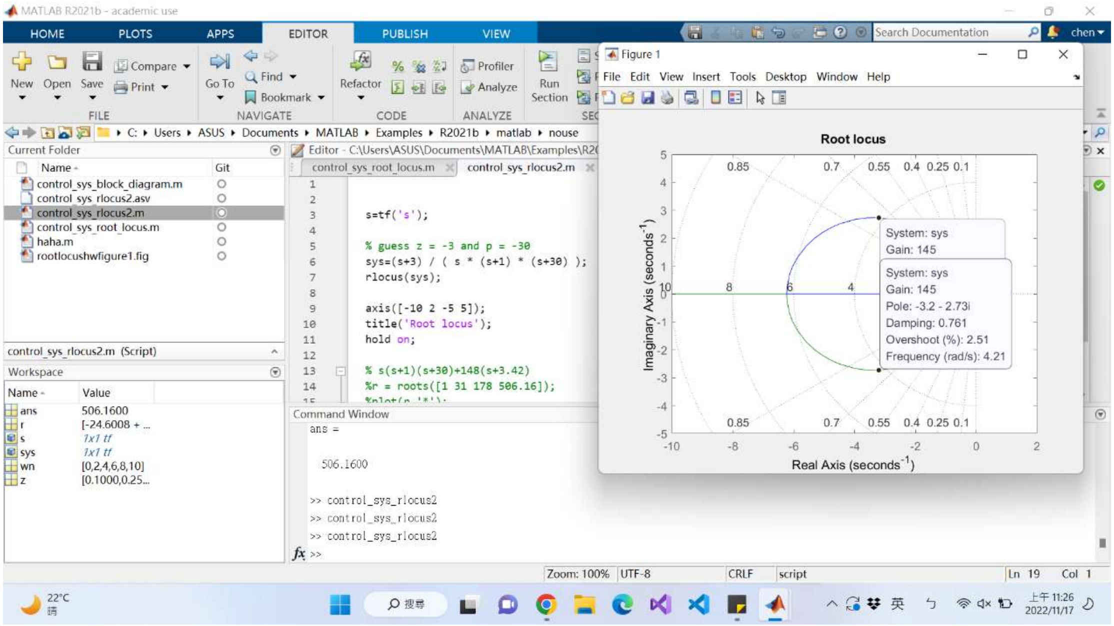


Fig.1

So we move the zero a little to the leftside. When zero goes to $z = -3.42$. It does let the dominant poles located at $s = -30 \pm 3.2j$. As shown in Fig.2, ploes are located on the rootlocus.

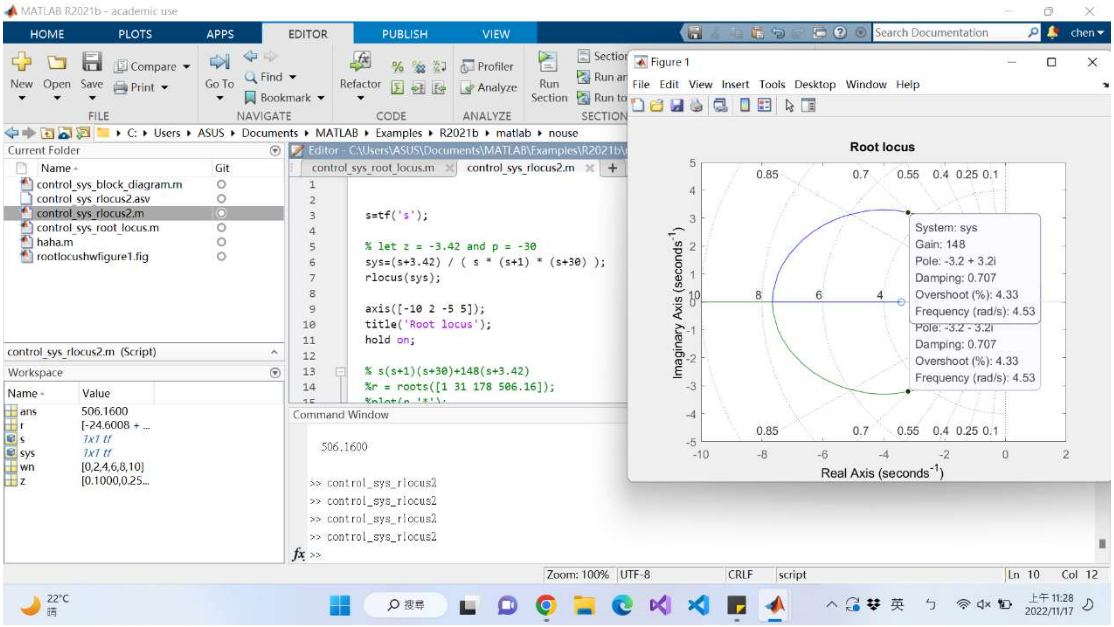


Fig.2

Given by the *rlocus* function, we know the gain $K = 148$. So the characteristic equation should be modified as s*(s+1)*(s−30)+148*(s−3.42)=0. To find the roots of this equation, we use the *roots* function in Matlab and plot the location of roots by the '*' symbols. As shown in Fig.3, we can find the roots is located on the root locus.

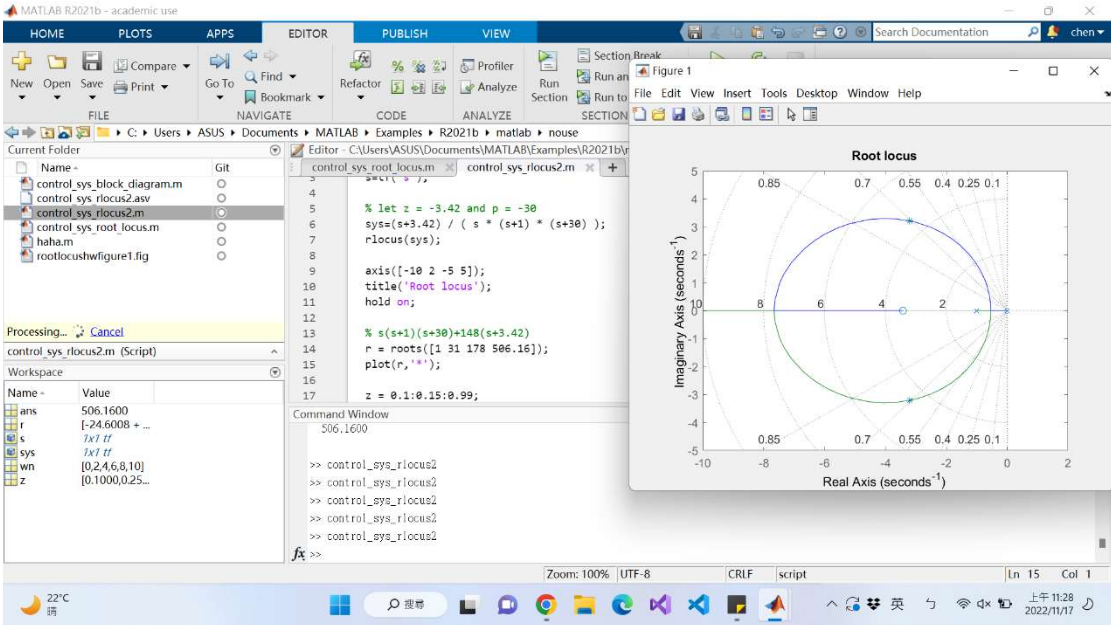


Fig.3

Thus the desired system may be determined by the lead compensation

$$D_C(s) = 148\frac{s+3.42}{s+30}$$

The Matlab code :

```
s=tf('s');

% let z = -3.42 and p = -30
sys=(s+3.42) / ( s * (s+1) * (s+30) );
rlocus(sys);

axis([-10 2 -5 5]);
title('Root locus');
hold on;

% s(s+1)(s+30)+148(s+3.42)
r = roots([1 31 178 506.16]);
plot(r,'*');

z = 0.1:0.15:0.99;
wn = 0:2:10;
sgrid(z, wn);
```

Control System HW7

B10202032 物理二 卓然

1. (U5D: Lead-Lag Compensator)

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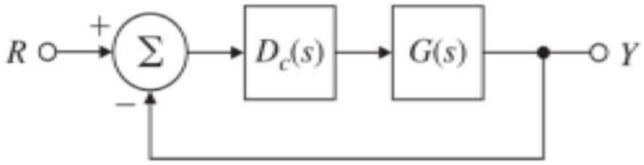
23. Suppose the unity feedback system of Fig. 5.59 has an open-loop plant given by $G(s) = \frac{1}{s(s+1)}$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.



The reference answer provided a purely experimental method of solving this problem. However, that answer is entirely based on trial and error and the result provided is not totally exact (with $(p, K, z) = (30, 150, 3.44)$, the pole on the plot is $s = -3.23 + j3.24$, not exactly $-3.2 + j3.2$). Therefore, I'll provide another mainly-analytical alternative to complement the reference answer.

The transfer function of the closed-loop system is

$T(s) = \frac{Y(s)}{R(s)} = \frac{D_c(s)G(s)}{1+D_c(s)G(s)}$, and therefore the characteristic equation is $1 + D_c(s)G(s) = 0$. Using $D_c(s) = K\frac{s+z}{s+p}$ and $G(s) = \frac{1}{s(s+1)}$, the equation becomes $1 + K\frac{s+z}{s+p}\frac{1}{s(s+1)} = 0$

Note that there is an additional pole at $s=-p$, and an additional zero at $s=-z$. For $s \neq 0, -1, -p$, the equation becomes $s(s+1)(s+p) + K(s+z) = 0$

$\Rightarrow \quad s^3 + (p+1)s^2 + (p+K)s + Kz = 0. \quad \cdots (1)$

From the problem statement, it is required that

$s = -3.2 \pm j3.2 \Rightarrow s + 3.2 = \pm j3.2$

$\Rightarrow (s+3.2)^2 = (\pm j3.2)^2 \Rightarrow s^2 + 6.4s + 10.24 = -10.24$

$\Rightarrow \quad s^2 + 6.4s + 20.48 = 0$

Therefore, $s = -3.2 \pm j3.2$ are roots of (1)

$\Rightarrow$ (1) is divisible by $s^2 + 6.4s + 20.48$.

Calculate $[s^3 + (p+1)s^2 + (p+K)s + Kz] \div [s^2 + 6.4s + 20.48]$:

$$
\begin{array}{r|llll}
 & 1 & p-5.4 & & \\
\hline
1 \quad 6.4 \quad 20.48 & 1 & p+1 & p+K & Kz \\
-) & 1 & 6.4 & 20.48 & \\
\hline
 & & p-5.4 & p+K-20.48 & Kz \\
-) & & p-5.4 & 6.4(p-5.4) & 20.48(p-5.4) \\
\hline
 & & & (-5.4p+K+14.08) & (Kz-20.48(p-5.4))
\end{array}
$$

Therefore, to make $s=-3.2 \pm j3.2$ roots of (1), we must have

$$
\begin{cases}
-5.4p + K + 14.08 = 0 & \cdots (2) \\
Kz - 20.48(p-5.4) = 0 & \cdots (3)
\end{cases}
$$

with the third pole located at $s = p-5.4$.

Note that the equations (2) and (3) have 3 unknowns with 2 constraints, and therefore there are infinitely many sets of solution (p, K, z) that satisfies these conditions, giving us freedom when designing the system.

Let $p = t \Rightarrow (2): -5.4t + K + 14.08 = 0$

$\Rightarrow K = 5.4t - 14.08$

$\Rightarrow (3): (5.4t - 14.08)z - 20.48(t - 5.4) = 0$

$$\Rightarrow \quad z = \frac{20.48(t-5.4)}{(5.4t-14.08)}.$$

$5.4t - 14.08 \geq 0$
$5.4t > 14.08$
$t >$

Therefore, the solutions are $\begin{cases} p = t \\ K = 5.4t - 14.08 \\ z = \dfrac{20.48(t-5.4)}{(5.4t-14.08)} \end{cases}, \ t \in (2.61, \infty)$

where $t \in (2.61, \infty)$ makes the pole be located at the left half s-plane and ensures stability, and also makes sure that $K > 0$, which is the basic setup of Chapter 5.

Now, let us consider the case $t = 30$ (an additional pole at $s = -30$) adopted in the reference answer and compare the results. In this case,

$$\begin{cases} K = 5.4(30) - 14.08 \\ z = \dfrac{20.48(30-5.4)}{5.4(30) - 14.08} \end{cases} \Rightarrow \begin{cases} K = 147.92 \\ z = 3.406 \end{cases}$$

Note that this set of solution is close to the one provided in the reference solution, $(p, K, z) = (30, 150, 3.44)$. However the solution derived above is exact, which is definitely better than the one in the reference solution. The errors caused by the approximation provided in reference solution are (setting $p=30$):

$$\begin{cases} K: \text{error} = \dfrac{K_{ref} - K_{exact}}{K_{exact}} = \dfrac{150 - 147.92}{147.92} = 1.41\% \\ z: \text{error} = \dfrac{z_{ref} - z_{exact}}{z_{exact}} = \dfrac{3.44 - 3.406}{3.406} = 1.00\% \end{cases}$$

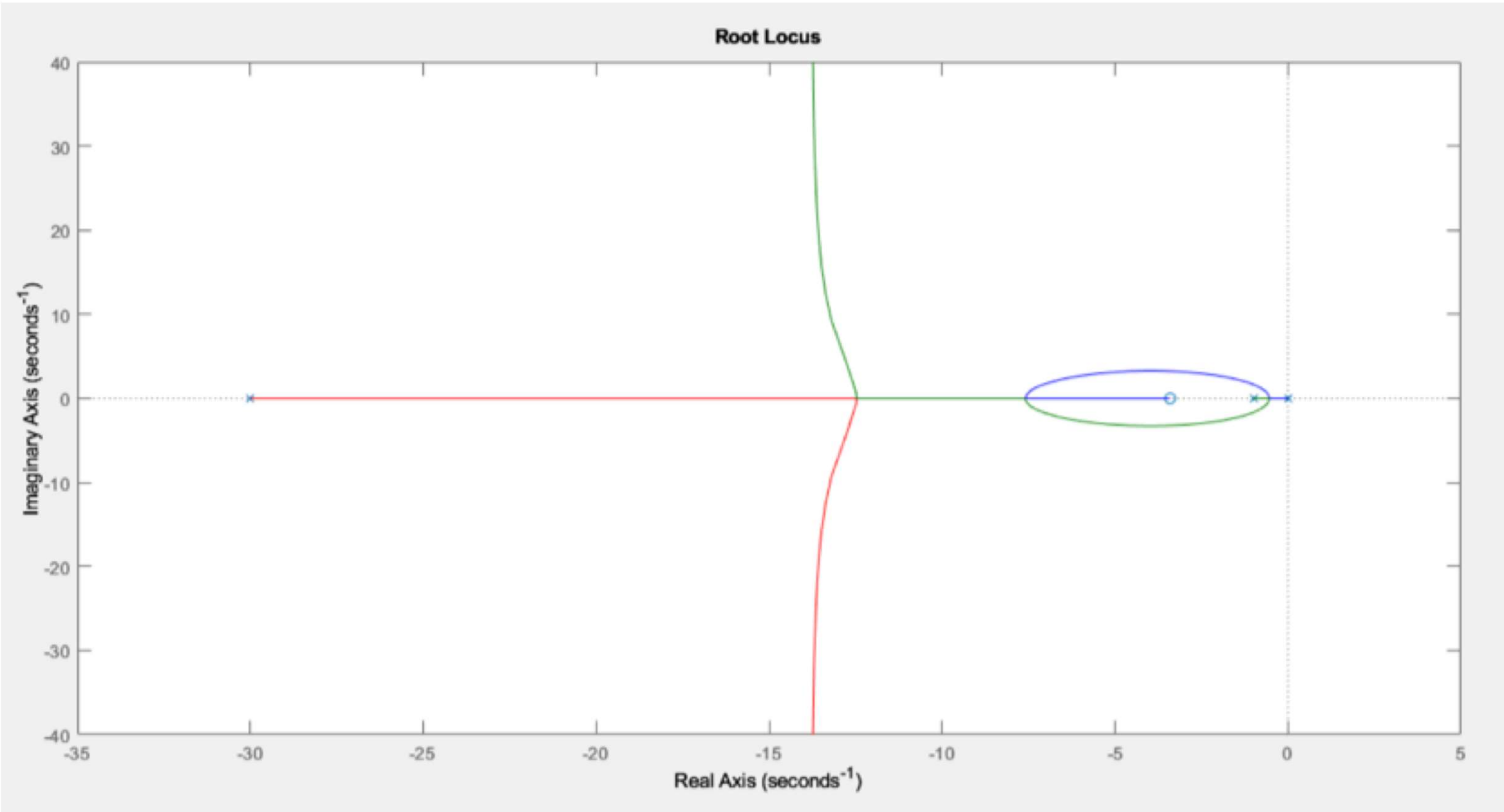
A MatLab plot of the root locus with my solution $(p, z) = (30, 3.406)$ is shown in the next page, and the point $s = -3.2 + j3.19$ is marked (it should be $s = -3.2 + j3.2$, but since Matlab only accepts the first 5 digits of z, this imprecision is inevitable).

* Since $L(s) = \frac{s+z}{(s+p)s(s+1)} = \frac{s+z}{s^3+(p+1)s^2+ps+0}$,

the MatLab code is:

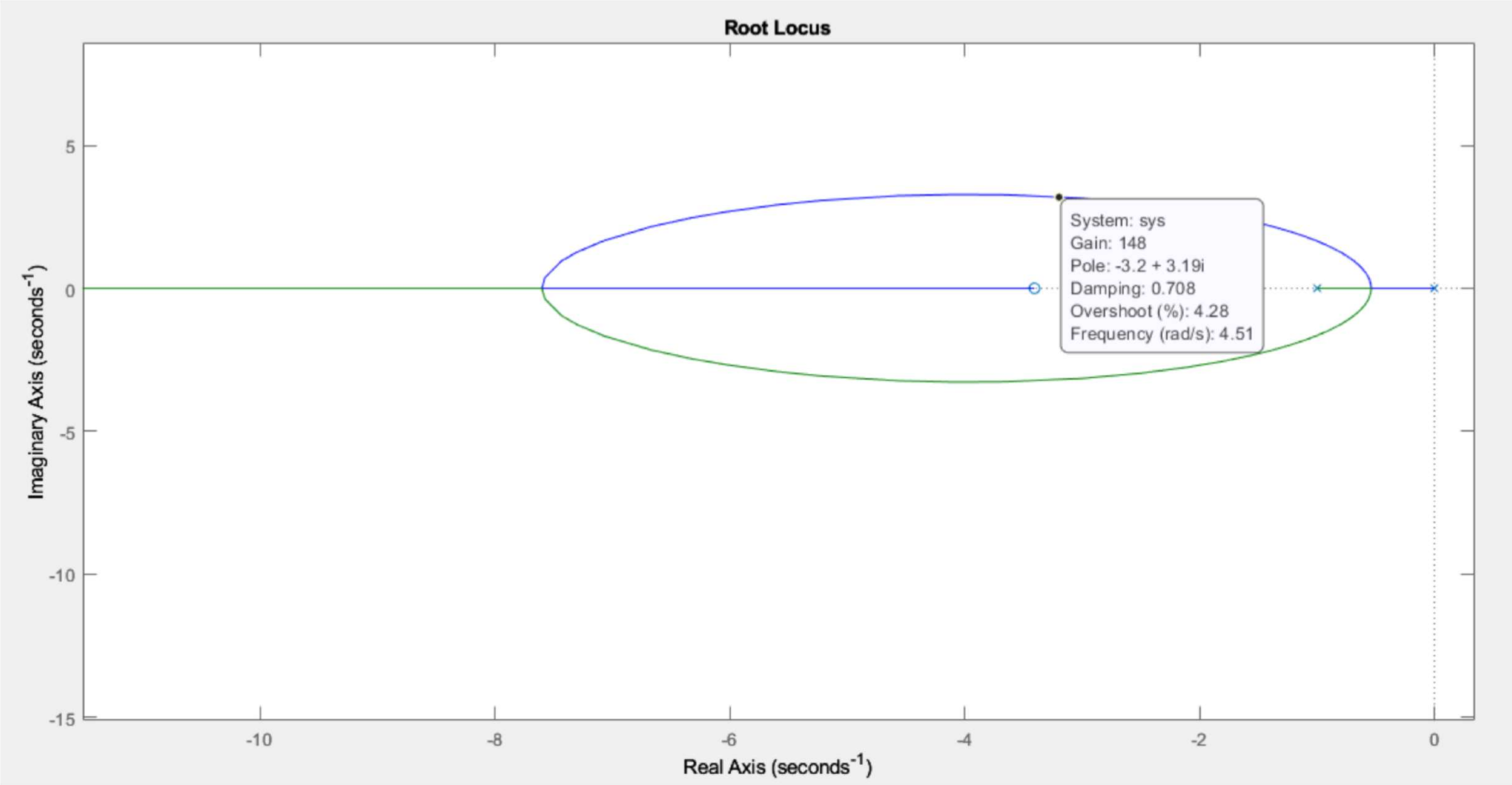
```
p=30;
z=20.48*(p-5.4)/(5.4*p-14.08);%the exact value of z
sys=tf([1 z],[1 p+1 p 0]);
rlocus(sys)
```

The full plot of the root locus is:



(Note the locations of the poles and the zero.)

Looking closely near $s=-3.2$:



(Note that the gain $K=148$, which is close to the exact value 147.92.)

參考觀摩的作業

2. (Lead-Lag Compensator)

作者：	b08901176，陳育楷
理由：	討論用不同方式設計 lead compensator 讓系統達到目標

1&2 (U5D: Lead-leg Compensation)

Problem:

In original problems, problem1 solves the required dominant pole by **adding one zero and adding one pole**, but problem2 (a) solves it by delete **adding one pole and deleting one pole.** Compare the two method, and discuss how they influence the pole of zero of $D_c(s)$.

Solution:

令 adding zero and adding pole 為解法一，adding pole and deleting pole 為解法二。

在 **problem1** 中，題目要求的 dominant pole 是$-3.2 \pm 3.2j$。

用解法一解 problem1 時，

characteristic equation 的因式要有$(s+3.2)^2+3.2^2=\boxed{s^2+6.4s+20.48}$。

Characteristic equation 是$s(s+1)(s+p)+K(s+z)=\boxed{s^3+(1-p)s^2+(K-p)s-zK}$。

如果令 non-dominant frequency 是 25 Hz，

則$s^3+(1-p)s^2+(K-p)s-zK=(s^2+6.4s+20.48)(s+25)$

$=s^3+31.4s^2+180.48s+512 \Longrightarrow \boxed{p=-30.4, K=150.08, z=\frac{-512}{150.08}\approx -3.412}$。

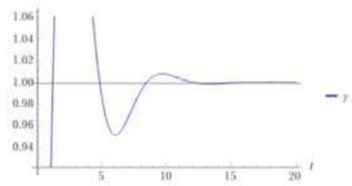
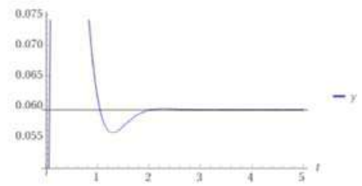
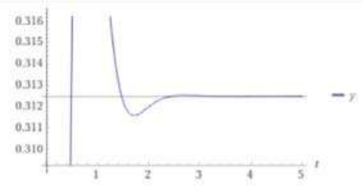
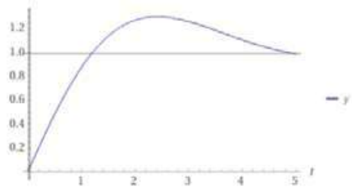
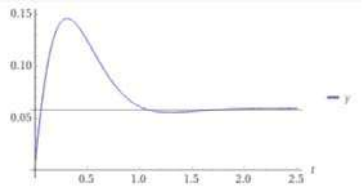
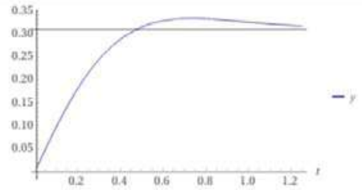
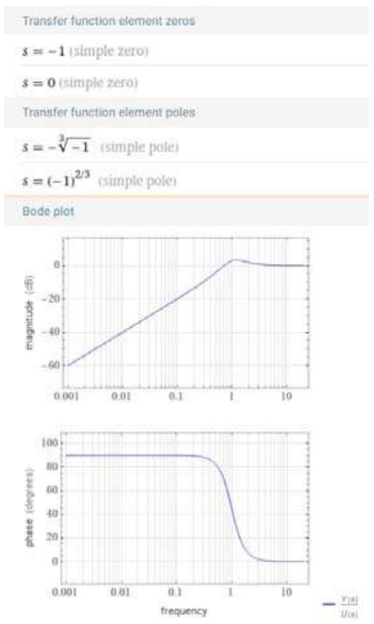
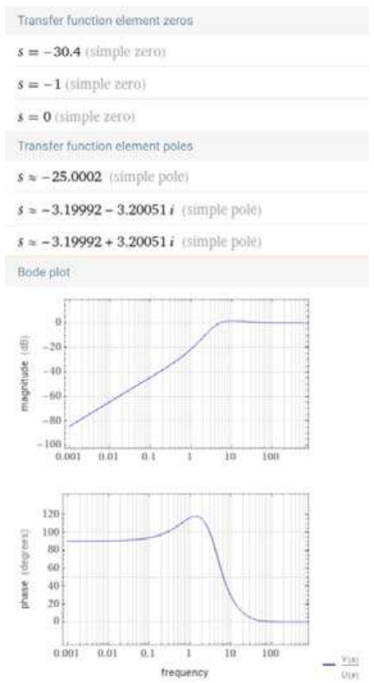
令 non-dominant frequency 是 25 Hz，會和原本題目算出的結果相似。

用解法二解 problem1 時，

令$D_c(s)$的$\boxed{z=-1}$用來消掉 G(s) 在分母的$(s+1)$，

Characteristic equation 是$s(s-p)+K=s^2+6.4s+20.48 \Longrightarrow \boxed{p=-6.4, K=20.48}$。

Plot:

	Original	解法一	解法二
$D_c(s)$	1	$150.08\frac{s+3.412}{s+30.4}$	$20.48\frac{s+1}{s+6.4}$
Unit-ramp response (rise time)			
Unit-ramp response (overshoot)			
Blot plot			

Observation:

	解法一	解法二
方法說明	$D_c(s)$的 zero 和 pole 會拿來修正系統的 dominant pole。	$D_c(s)$的 zero 會和G(s)的 pole 抵消，簡化題目要求的未知數。
相同的部分	兩者都是 lead compensation，rise time 有很大的改善。	
$D_c(s)$的 p	Unity feedback system 的其中一個 zero 是 p，根據 characteristic equation 的平方項係數，為了要和 non-dominant pole 抵消效果，會設得比 dominant pole 大，因此 p 會令得比 z 大很多。	按簡化後的題目湊出所要求的 dominant pole 算出 p。
$D_c(s)$的 z	dominant pole 相比於 non-dominant pole 會很大程度影響到 z 的大小（non-dominant pole 從 25 升到 65，z 只 3.4 升到 3.6）。	$D_c(s)$的 zero 為了和G(s)的 pole 抵消，會設成G(s)的其中一個 pole。

參考觀摩的作業 3. (Design using the Root Locus)
無