Control System: Homework 06 for Unit 5A, 5B, 5C: Root Locus

Assigned: Oct 28, 2022

Due: Nov 10, 2022 (23:59)

Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

1. (U5B: Root Locus)

3. For the characteristic equation

$$1 + \frac{K}{s^2(s+1)(s+5)} = 0,$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) Sketch the locus.

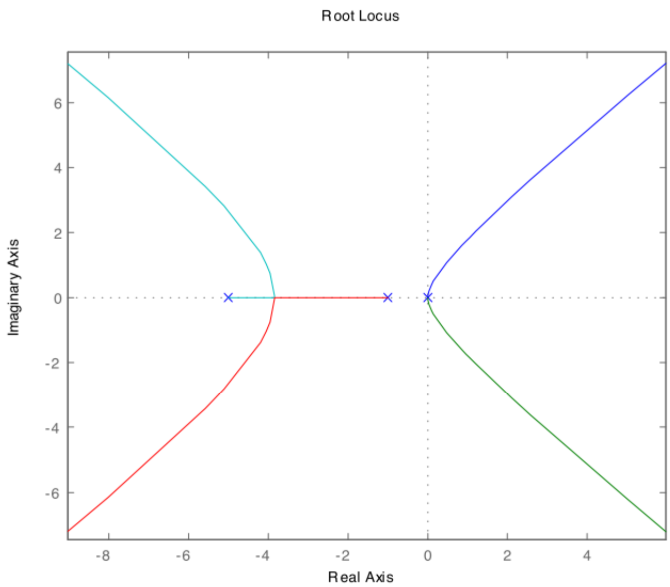
(d) Verify your sketch with a Matlab plot.

Solution:

(a) The real axis segment is $-1 > \sigma > -5$.

(b) $\alpha = -6/4 = -1.5$; $\phi_i = \pm 45°, \pm 135°$

(c) The plot is shown below.



2. (U5B: Root Locus)

4. *Real poles and zeros.* Sketch the root locus with respect to K for the equation $1+KL(s)=0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

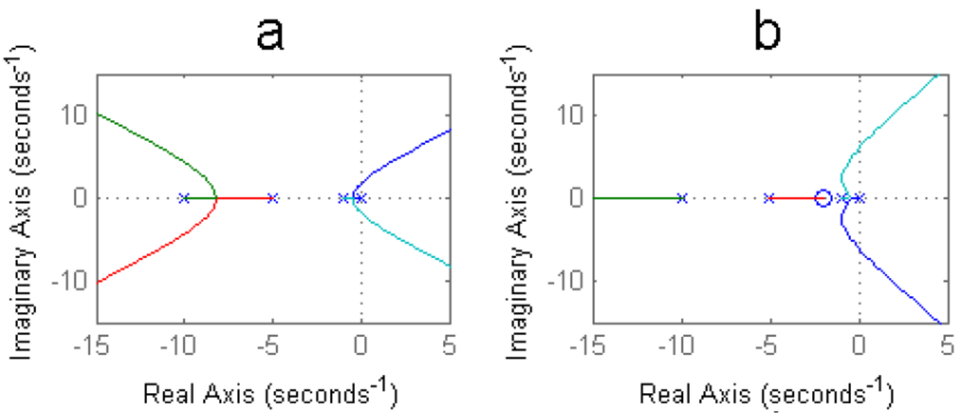
   (a) $L(s) = \dfrac{2}{s(s+1)(s+5)(s+10)}$

   (b) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

Solution:
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -4$; $\phi_i = \pm 45°, \pm 135°$

(b) $\alpha = -4.67$; $\phi_i = \pm 60°, 180°$



3. (U5B: Timing Property and Root Locus)

13. For the system in Fig. 5.53,

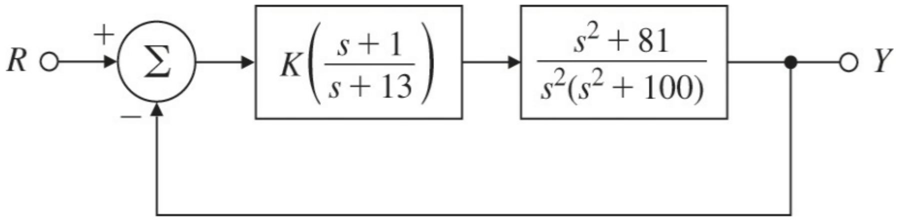


Fig. 5.53 Feedback system for Problem 5.13

(a) Find the locus of closed-loop roots with respect to K.

(b) Is there a value of K that will cause all roots to have a damping ratio greater than 0.5?

(c) Find the values of K that yield closed-loop poles with the damping ratio $\zeta = 0.707$.

(d) Use Matlab to plot the response of the resulting design to a reference step.

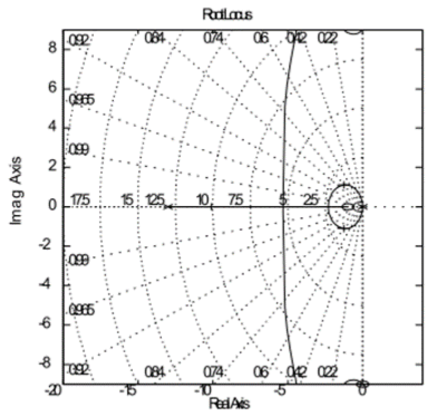
Solution:

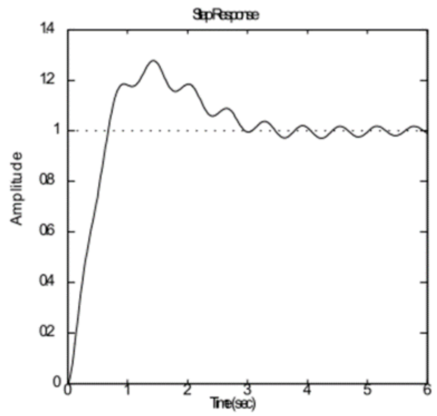
(a) The locus is plotted below

(b) There is a K which will make the 'dominant' poles have damping 0.5 but none that will make the poles from the resonance have that much damping.

(c) Using rlocfind, the gain is about 35.

(d) The step response shows the basic form of a well damped response with the vibration of the response element added.



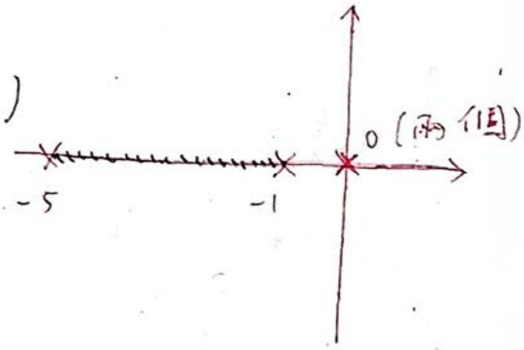


參考觀摩的作業 1. (Root Locus)
作者：b08901115，范博淵 理由：詳細推導根軌跡繪製以及計算根軌跡分離的位置
作者：b09901029，楊 珩 理由：討論 repeated poles with multiplicity k 在 s=0 的根軌跡圖對不同 k 的影響
作者：b09901143，李立安 理由：詳細求出根軌跡圖離開實軸的點，討論四階系統不同 pole 的位置對根軌跡圖的影響
作者：b10202032，卓 然 理由：詳細推導根軌跡繪製以及計算根軌跡分離的位置

1.

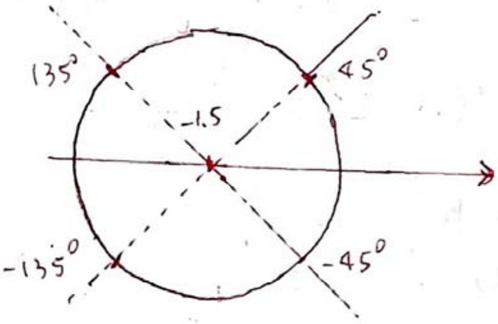
(a) left of an odd number of poles and zeros

$-5 < \sigma < -1$ （在 poles : $\{0,0,-1\}$ 之左）



(b) $$d = \frac{\sum P_i - \sum Z_i}{n-m} = \frac{(-1)+(-5)-0}{4-0} = -1.5$$

$$\phi_l = \frac{180^\circ + 360^\circ \cdot (l-1)}{n-m} = \frac{180^\circ + 360^\circ \cdot (l-1)}{4} = 45^\circ + 90^\circ \cdot (l-1), \quad l = 0,1,2,3$$



$$= \begin{cases} 45^\circ \\ 135^\circ \\ 225^\circ \\ 315^\circ \end{cases}$$

(c) Rule 1 : 從 0, 0, -1, -5 射出 4 個 branch.

Rule 2 : 實軸上 $-1 < \sigma < -5$ 區域是 locus 之一

Rule 3 : $k \to \infty$，4 個 branch 會以 (b) 求出的 4 條漸近線衝到無窮遠。

Rule 4 : 從 pole 0 射出的角度為 $\frac{-0^\circ - 0^\circ - 180^\circ - 360^\circ \cdot (l-1)}{2}$

$$\begin{cases} -90^\circ \\ +90^\circ \end{cases} \leftarrow = -90^\circ - 180^\circ \cdot (l-1), \quad l = 1,2$$

從 pole -1 射出的角度為 $\frac{-(-180^\circ)\times 2 - 0^\circ - 180^\circ - 360^\circ \cdot (l-1)}{1}$

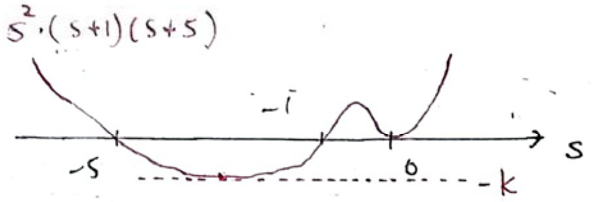
$$= -180^\circ \quad l = 1$$

從 pole -5 的角度為

$$-(-180^\circ)\times 2 - (-180^\circ) - 180^\circ = 0^\circ$$

Rule 5 : characteristic equation :

$s^2(s+1)(s+5)+K=0$，在某一 K 有重根，



（從 -5 向右，從 -1 向左，會撞上）

$s^2(s+1)(s+5)$，s 帶入實數，在 $-5 \le s \le -1$，會和 $(-K)$ 有解，

重根發生在斜率為零，

$$\frac{d}{ds}\left[s^2(s+1)(s+5)\right]=\frac{d}{ds}\left(s^4+6s^3+5s^2\right)=4s^3+18s^2+10s=0$$

$$s(2s^2+9s+5)=0$$

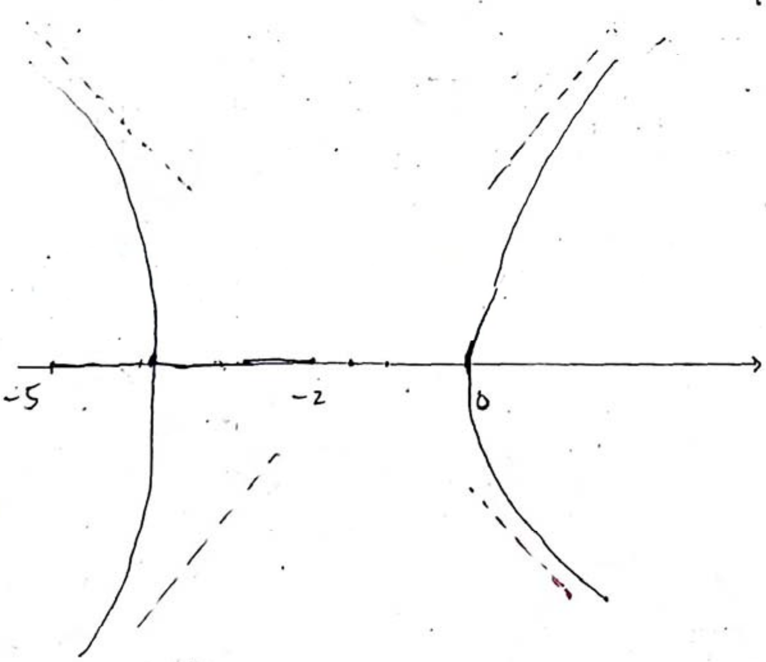
$s=0, -0.65, -3.85$，即 -3.85（介於 -5 到 -1）

-3.85 代回 $s^2(s+1)(s+5)=-48.58$ (01)

當 K = 48.58，有重根（重覆兩次）在 -3.85

相撞後，離開角度為 $\frac{180^\circ+360^\circ\cdot(l-1)}{2}=90^\circ+180^\circ\cdot(l-1)$, $l=1,2$

即 $\begin{cases}90^\circ\\-90^\circ\end{cases}$



HW 06: Root Locus	Control Systems, Fall 2022, NTU-EE
Name: 楊珩 B09901029	Date: 11/10, 2022

1. (U5B: Root Locus)

3. For the characteristic equation

$$1 + \frac{K}{s^2(s+1)(s+5)} = 0,$$

(a) Draw the real-axis segments of the corresponding root locus.

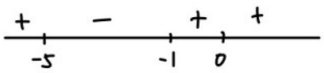
(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) Sketch the locus.

(d) Verify your sketch with a Matlab plot.

(a) $1 + \frac{K}{S^2(S+1)(S+5)} = 0$

$\Rightarrow S^2(S+1)(S+5) < 0$



$\Rightarrow -5 < S < -1$

The real axis segment is $-5 < \sigma < -1$

(b) $\alpha = \frac{\sum p_i - \sum z_i}{n-m}$

$= \frac{(-1)+(-5)}{4}$

$= -1.5$

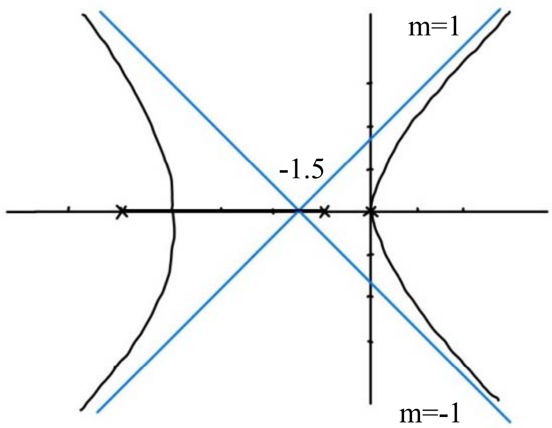
$\phi_l = \frac{180^\circ + 360^\circ(l-1)}{n-m} \quad (l = 1, 2, \ldots n-m)$

$= \pm 45^\circ, \ \pm 135^\circ$

The asymptotes of the locus are shown in (c).

The blue line are the asymptotes of the locus.

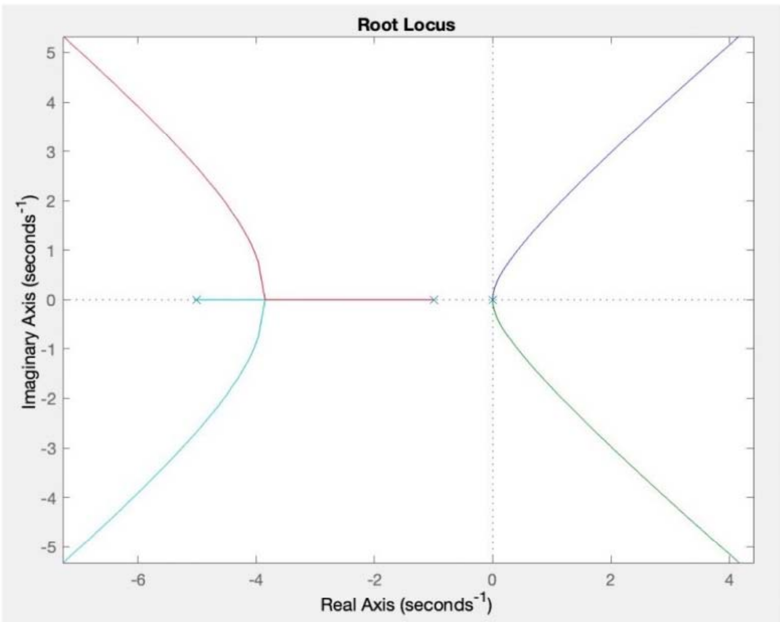
(c)



(d)

```
s=tf('s');
sys = tf([1 6 5 0 10000],[1 6 5 0 0]);

rlocus(sys)
```



now if we let $L(s) = \frac{1}{s^k (s+1)(s+5)}$

$\Rightarrow$ the real axis segment is still $-5 < \sigma < -1$ if k is even.

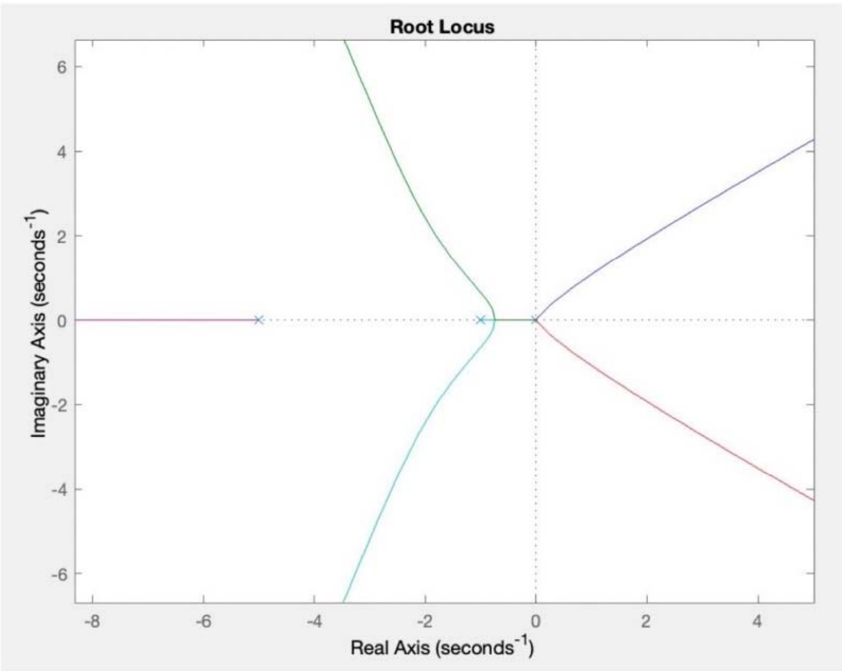
the real axis segment change to $\sigma < -5$, $-1 < \sigma < 0$ if k is odd.

however $\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{(-1)+(-5)}{k+2} = \frac{-6}{k+2}$

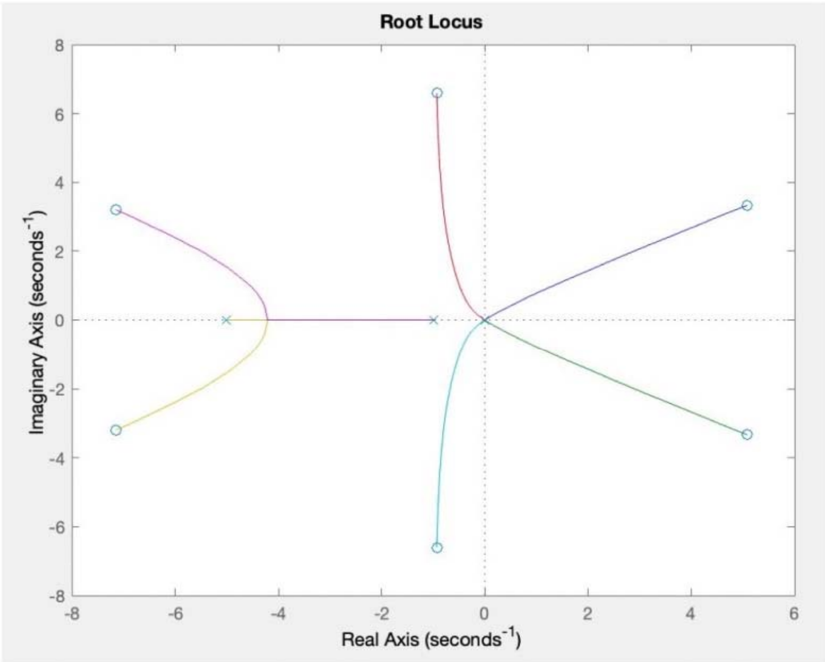
$\phi_l = \frac{180^\circ + 360^\circ (l-1)}{n-m} = \frac{180^\circ + 360^\circ (l-1)}{k+2}$ $(l = 1, 2, \ldots, n-m)$

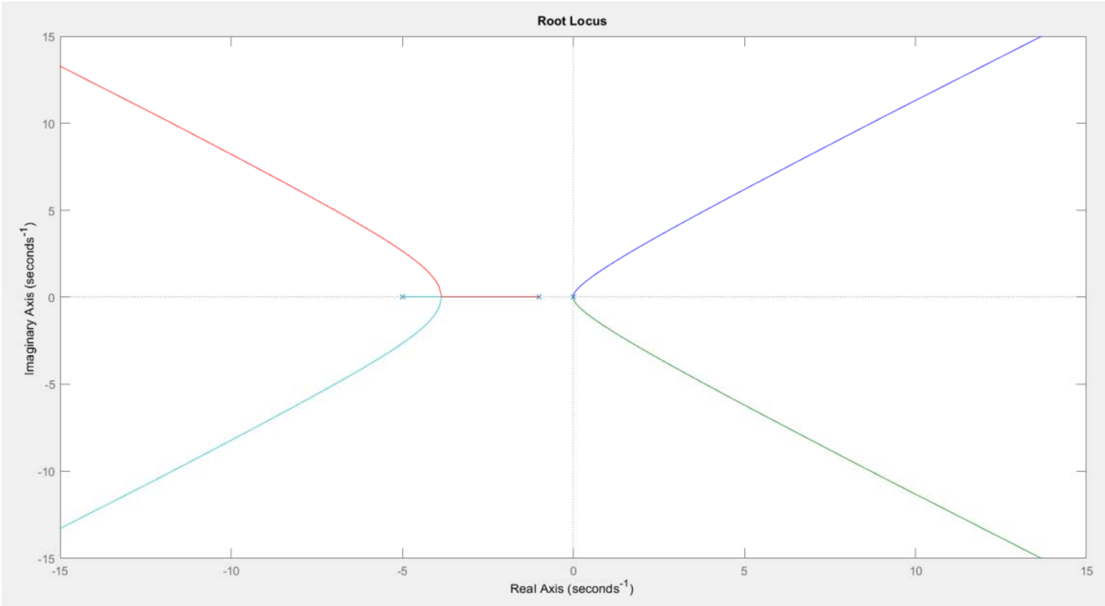
當k不斷上升，α會越接近原點，且 root locus 的圖形會從 k=2 的朝四角發散一直變成朝多邊發散

ex: k=3 (朝正五邊形的五個角度發散)，$\alpha = -1.2$



ex: $k=4$ ($\phi_i = \pm 30^\circ, \pm 90^\circ, \pm 150^\circ$), $\alpha = -1$




Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
15
10
5
0
-5
-10
-15
-15
-10
-5
0
5
10
15

HW 06: Root Locus	Control Systems, Fall 2022, NTU-EE
Name: 李立安 B09901143	Date: 11/10, 2022

Problem 1

1. $1+\frac{K}{s^2(s+1)(s+5)}=0$

$\Rightarrow s^2(s+1)(s+5)+K=0 \Rightarrow K=-s^2(s+1)(s+5) \Rightarrow$

$\Rightarrow \frac{dK}{dS}=-(4s^3+18s^2+10s)$

$\frac{dK}{dS}=0$ 時 $S=0,\ \frac{-9\pm\sqrt{41}}{4}$ (正不合, 沒在 -1 和 -5 之間)

$\Rightarrow$ breakpoint: $0,\ \frac{-9-\sqrt{41}}{2}\approx -3.85$

poles: $0, -1, -5$ (0為重根)

zeros: none

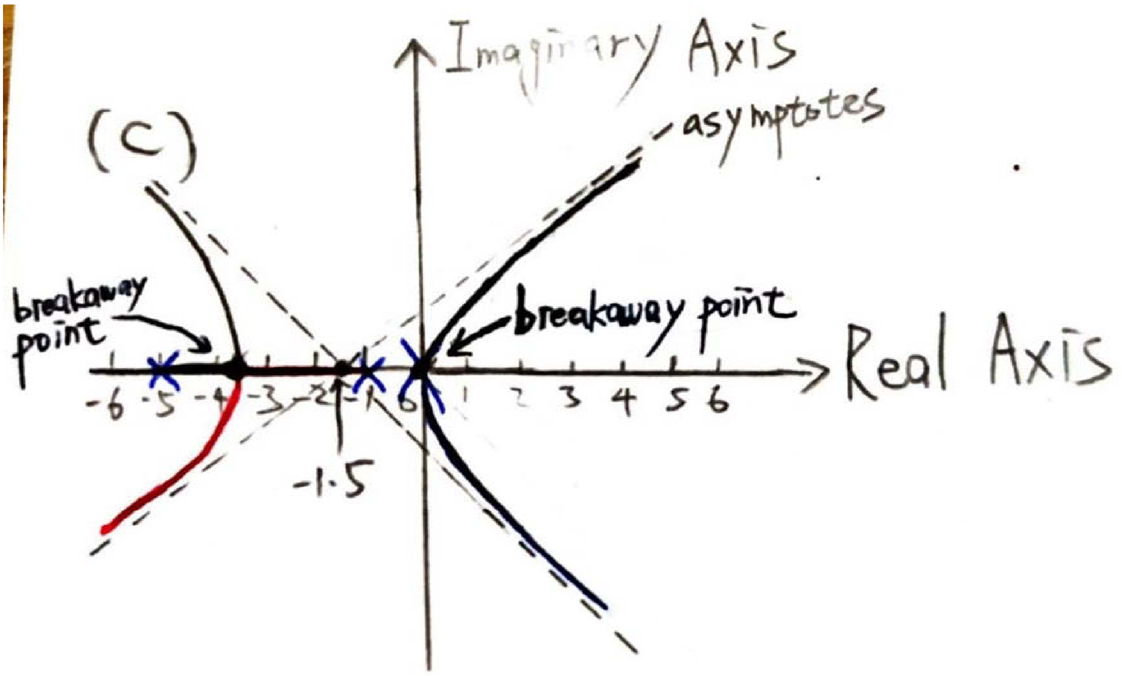
(a) real axis segment: $-5<\sigma<-1$

(b) $\alpha=\frac{0+(-1)+(-5)}{4-0}=-\frac{3}{2}$

$\phi_i=\frac{180^\circ+360^\circ(l-1)}{4}=45^\circ+90^\circ(l-1)$

$=90^\circ\cdot l-45^\circ,\ l=1,2,3,4$

$\Rightarrow \phi_i=\pm45^\circ, \pm135^\circ\ (225^\circ\rightarrow-135^\circ,\ 315^\circ\rightarrow-45^\circ)$



(c)
Imaginary Axis
asymptotes
breakaway point
breakaway point
Real Axis
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
-1.5

(d)

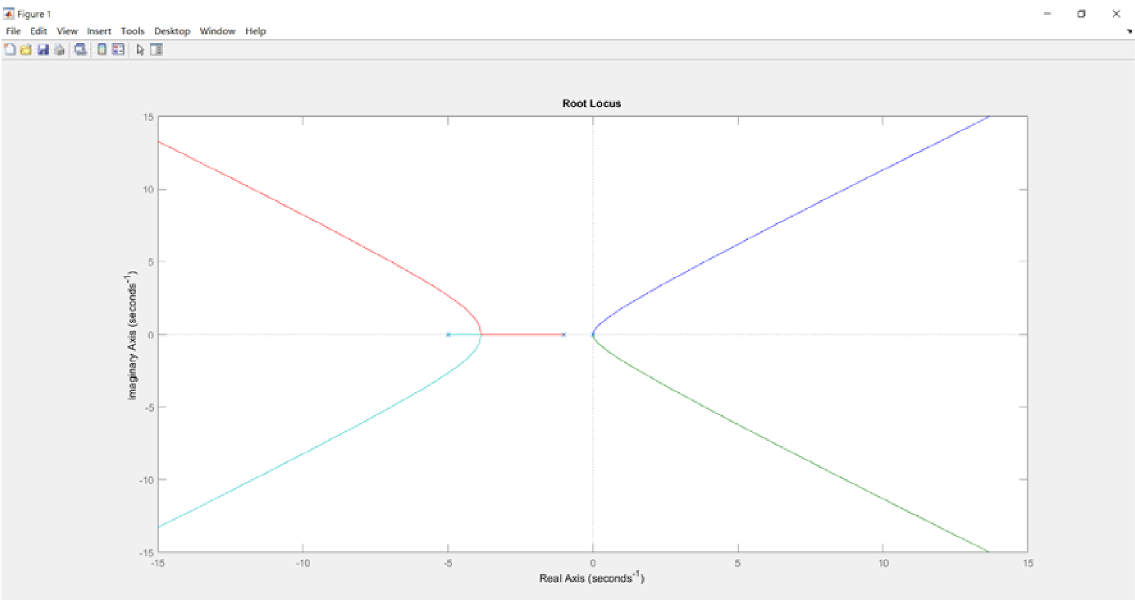


Figure 1
File Edit View Insert Tools Desktop Window Help
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
15
10
5
0
-5
-10
-15
-15 -10 -5 0 5 10 15

HW 06: Root Locus	Control Systems, Fall 2022, NTU-EE
Name: 李立安 B09901143	Date: 11/10, 2022

general 4階 no zeros, $a(s)=s^2(s+a)(s+b)$, $b>a>0$

$\Rightarrow s^2(s+a)(s+b)=0 \Rightarrow K=-s^2(s+a)(s+b) \Rightarrow \frac{dK}{ds}=-(4s^3+3(a+b)s^2+2abs)$

poles: $0,-a,-b$ (0為重根)　(a) $-b<\sigma<-a$　$\frac{dK}{ds}=0$ 時 $s=0,\ \frac{-3(a+b)\pm\sqrt{9a^2+9b^2-14ab}}{8}$ (正不合)

zeros: none

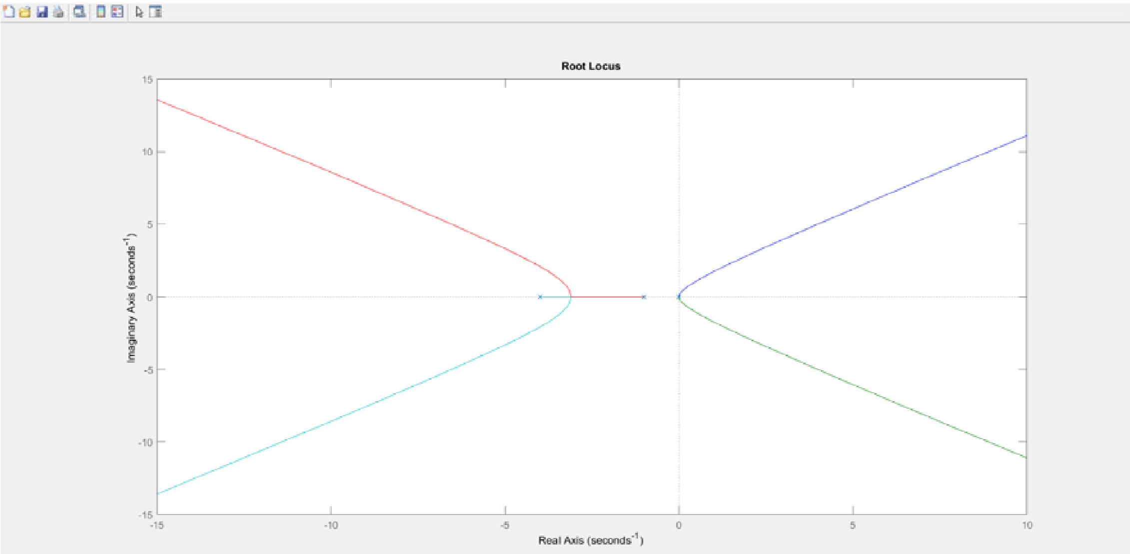
$\Rightarrow$ breakpoint: $0,\ -\frac{3}{8}(a+b)-\frac{\sqrt{9a^2+9b^2-14ab}}{8}$

(b) $\alpha=\frac{0+(-a)+(-b)}{4-0}=-\frac{a+b}{4}$

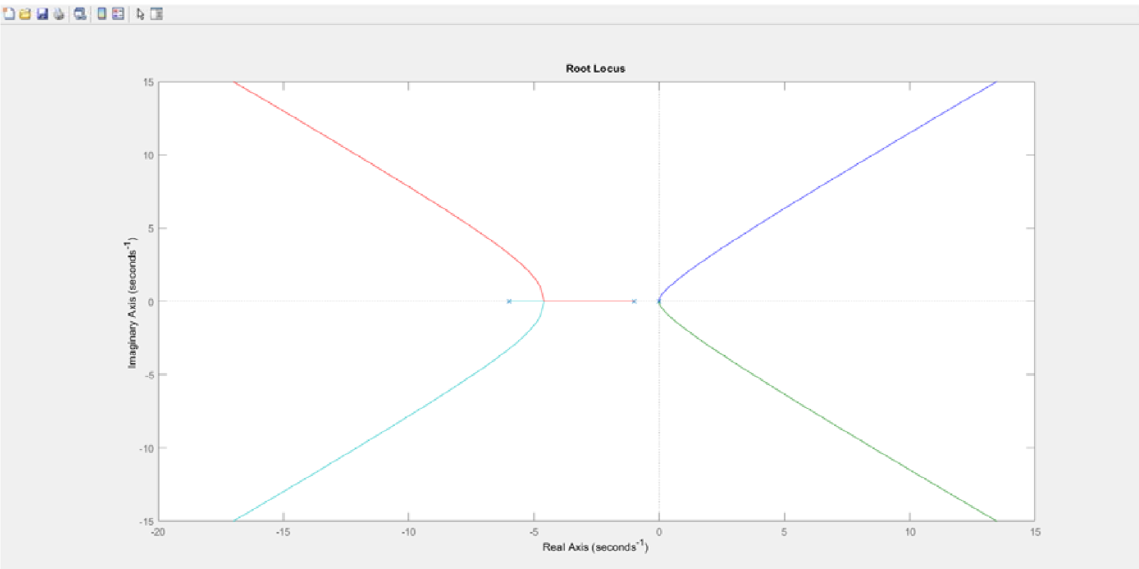
ϕ_i 同上

(0) a=1, b=5 如上(d)

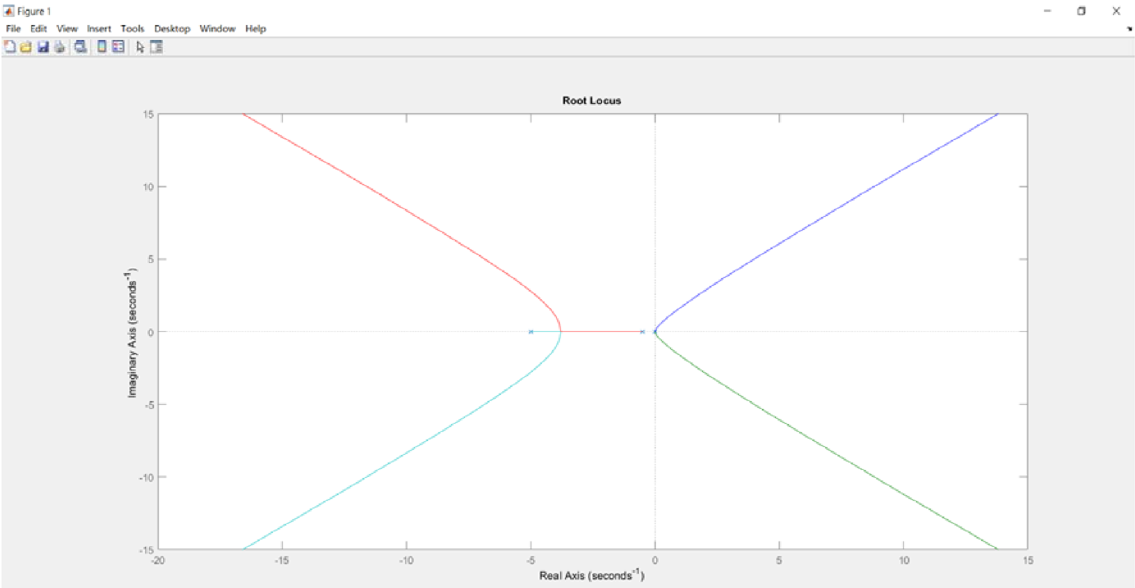
(1) a=1, b=4



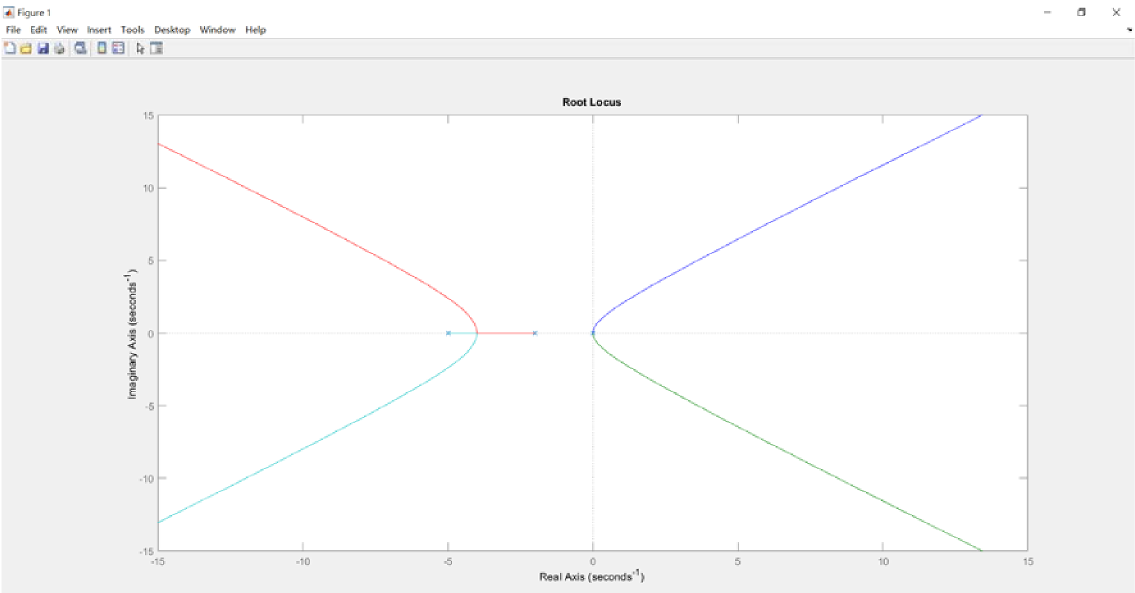
(2) a=1, b=6



(3) a=0.5, b=5

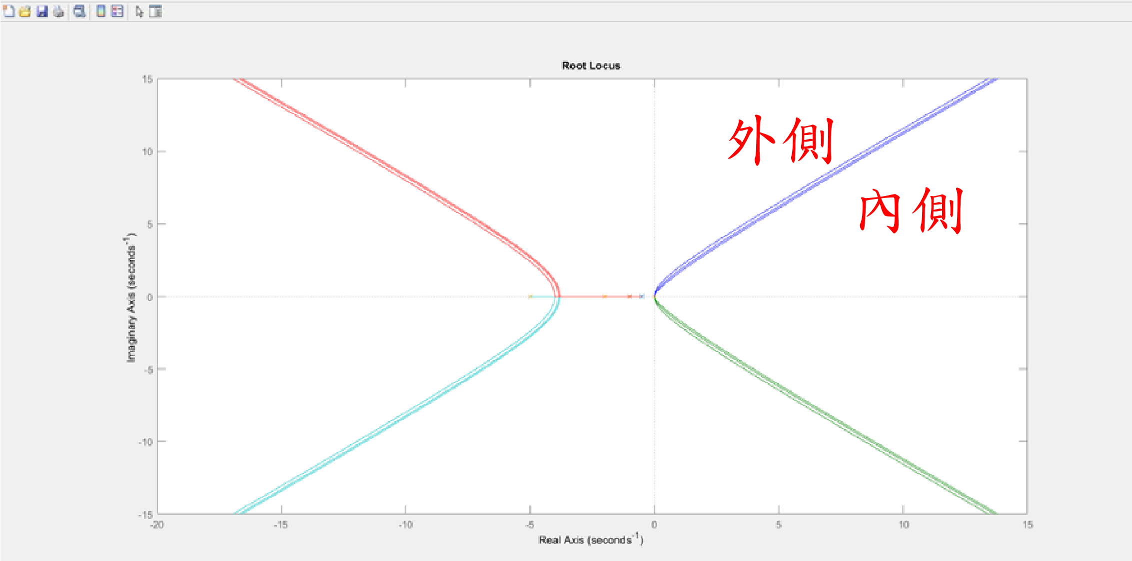


(4) a=2, b=5



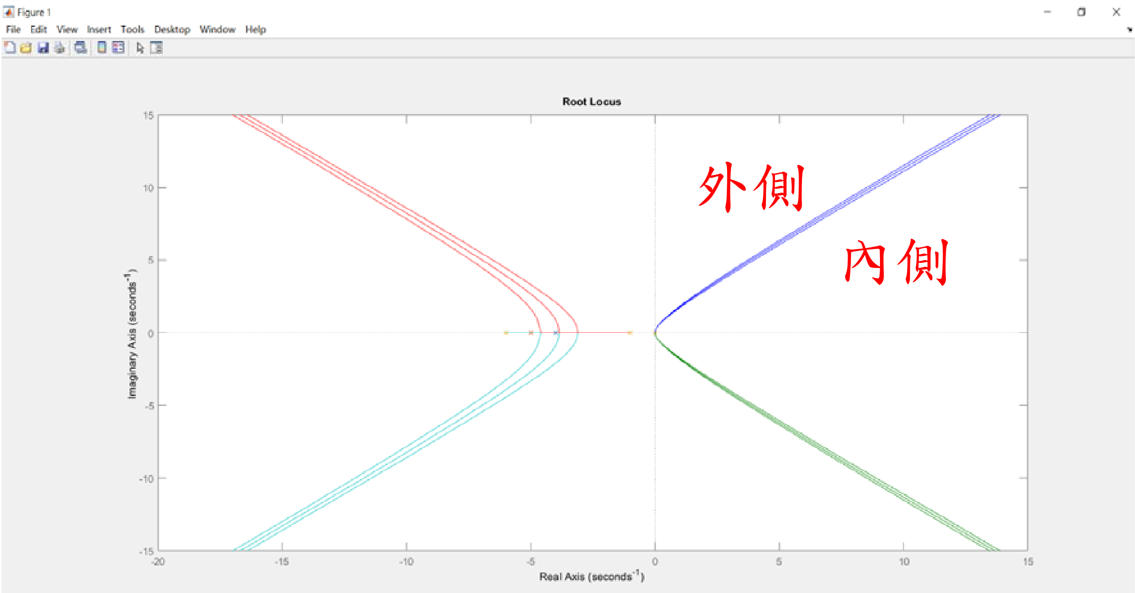
(5) 改變 a 畫在一起的圖，a 較小時，除了 0 之外的另一個 breakaway point 的值較大(負較少)

```
s=tf('s')
sysL=1/(s*s*(s+0.5)*(s+5))
sysL2=1/(s*s*(s+1)*(s+5))
sysL3=1/(s*s*(s+2)*(s+5))
rlocus(sysL)
hold on
rlocus(sysL2)
hold on
rlocus(sysL3)
```



(6) 改變 b 畫在一起的圖，b 較小時，除了 0 之外的另一個 breakaway point 的值較大(負較少)，且在 real axis 軸上大於 0 的軸線在較內側

```
s=tf('s')
sysL=1/(s*s*(s+1)*(s+4))
sysL2=1/(s*s*(s+1)*(s+5))
sysL3=1/(s*s*(s+1)*(s+6))
rlocus(sysL)
hold on
rlocus(sysL2)
hold on
rlocus(sysL3)
```



References

[1: HW06_Unit5A_SOL.PDF]

HW06_Unit5A_SOL.PDF

Control System HW6
B10202032 物理二 卓然

Some interesting questions are discussed in page ⑥ to ⑧.

3. For the characteristic equation

$$1+\frac{K}{s^2(s+1)(s+5)}=0,$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) Sketch the locus.

(d) Verify your sketch with a Matlab plot.

A. First, I will solve this problem with details.

(a) The characteristic equation can also be written as $s^2(s+1)(s+5)+K=0$. Assuming that $K\in(0,\infty)$, to make the equation holds, we must have $s^2(s+1)(s+5)=-K<0$. Now consider only the real part of s, σ, since the problem only asks for the real segment of the root locus. $\sigma\in\mathbb{R} \Rightarrow$

$$\sigma^2(\sigma+1)(\sigma+5)<0 \Longleftrightarrow (\sigma+1)(\sigma+5)<0 \Longleftrightarrow -5<\sigma<-1.$$

Therefore, the real segment is exactly $\boxed{-5<\sigma<-1}$.

(b) According to the Rule 3 discussed in lecture, we first calculate the value

$\alpha\equiv\frac{\sum_i p_i-\sum_i z_i}{n-m}$, where n, m is the degree of the denominator and the numerator, respectively;

$\sum_i p_i$, $\sum_i z_i$ denotes the sum of poles and zeros.

Comparing to the standard form $1+KL=0$, we get

$L(s) \equiv \frac{1}{s^2(s+1)(s+5)}$. Hence, $(n,m) = (4,0)$,

$\sum_i p_i = 0+0+(-1)+(-5) = -6$ and $\sum_i z_i = 0$

since it has no zero. Therefore, $\alpha = \frac{-6-0}{4-0} = \boxed{-1.5}$

Next, calculate $\phi_l \equiv \frac{\pi + 2\pi(l-1)}{(n-m)}$, where

$l = 1, 2, \ldots, n-m$. In this case, $n-m=4$. Therefore,

$$\phi_1 = \frac{\pi+2\pi(1-1)}{4-0} = \frac{\pi}{4}$$

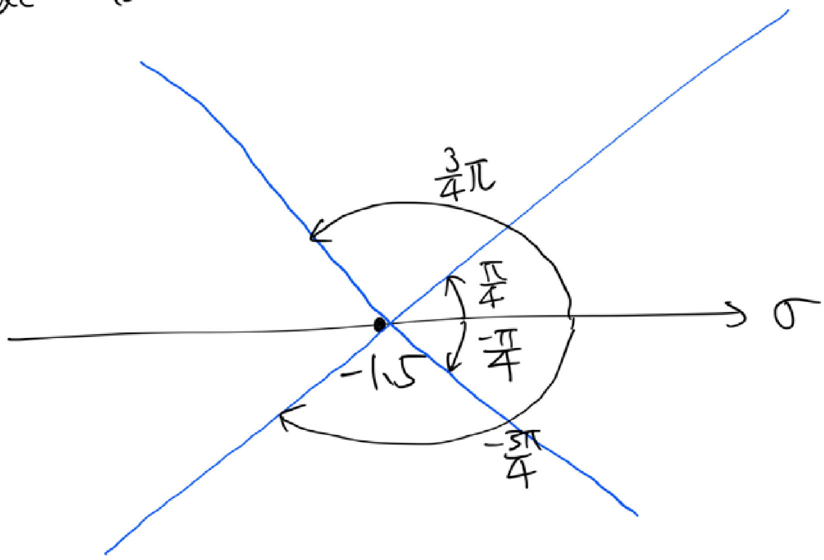
$$\phi_2 = \frac{\pi+2\pi(2-1)}{4-0} = \frac{3\pi}{4}$$

$$\phi_3 = \frac{\pi+2\pi(3-1)}{4-0} = \frac{5\pi}{4} \xrightarrow[\text{angle}]{\text{homogeneous}} \phi_3 = \frac{-3\pi}{4}$$

$$\phi_4 = \frac{\pi+2\pi(4-1)}{4-0} = \frac{7\pi}{4} \xrightarrow[\text{angle}]{\text{homogeneous}} \phi_4 = \frac{-\pi}{4}.$$

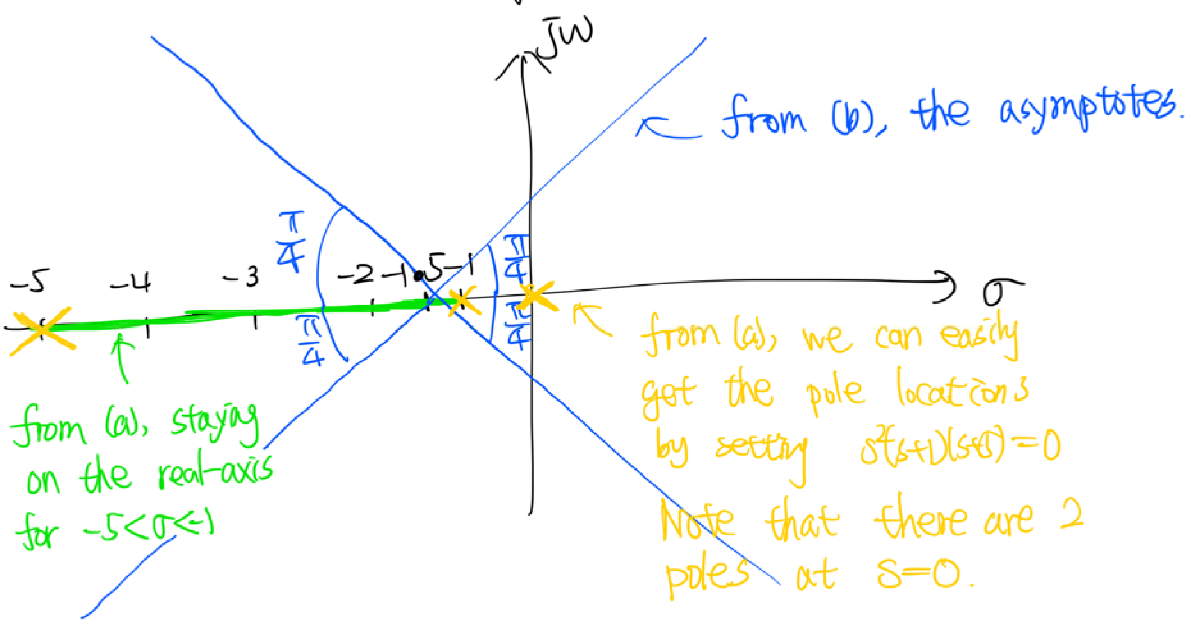
According to Rule 3, there are $n-m=4$ asymptotes for the root locus intersecting at $s=\alpha=-1.5$ and going to the directions with angles ϕ_i, $i=1\sim4$ to the real-axis.

That is:



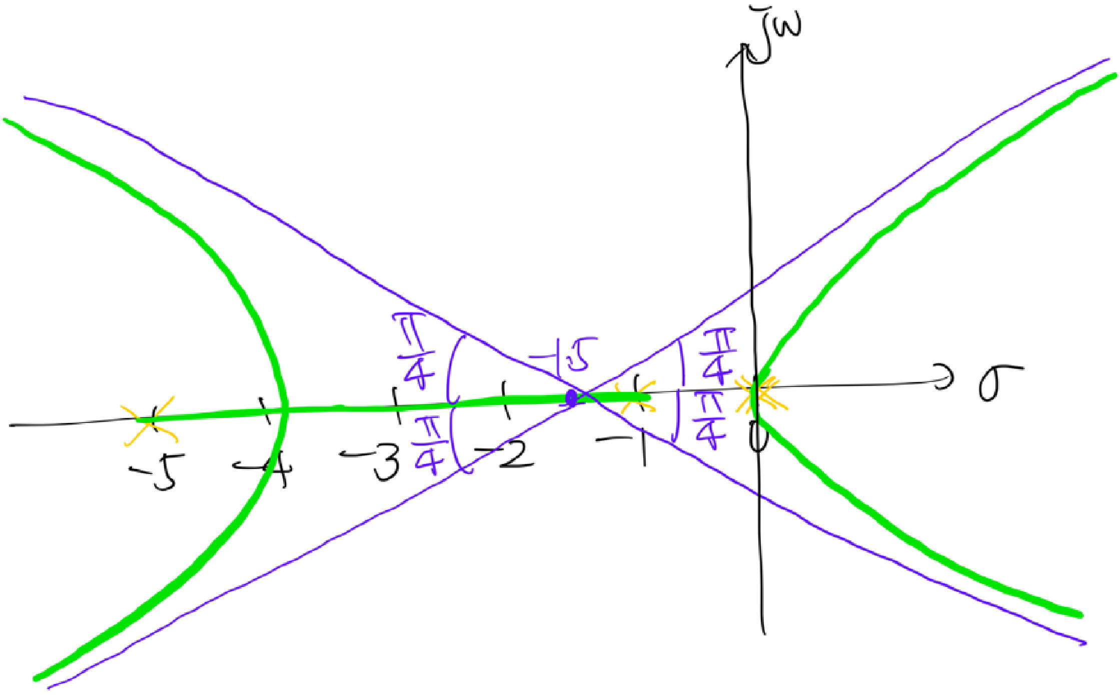
where the blue lines are the asymptotes of the root locus.

(c) According to the information derived from (a) and (b), we can plot several traits of the root locus:



Note that this agrees with the Rule 2 since there are no locus in the segment $\sigma \in (-\infty,-5)$ and $\sigma \in (-1,\infty)$ since #(poles in the right) + #(zeros in the right) = even number for these segments. (Note that $s=0$ counts as two poles due to the s^2 term.)

Furthermore, from Rule I, all the loci start with a corresponding pole. Therefore, we can see that the root locus must look similar to this:



the green curves are the root locus.

(d) Using MatLab

$(s+1)(s+5)$ s^2+6s+5

Multiplying out, we get $L(s) = \dfrac{1}{s^4+6s^3+5s^2+0s+0}$.

Thus, by simply typing in

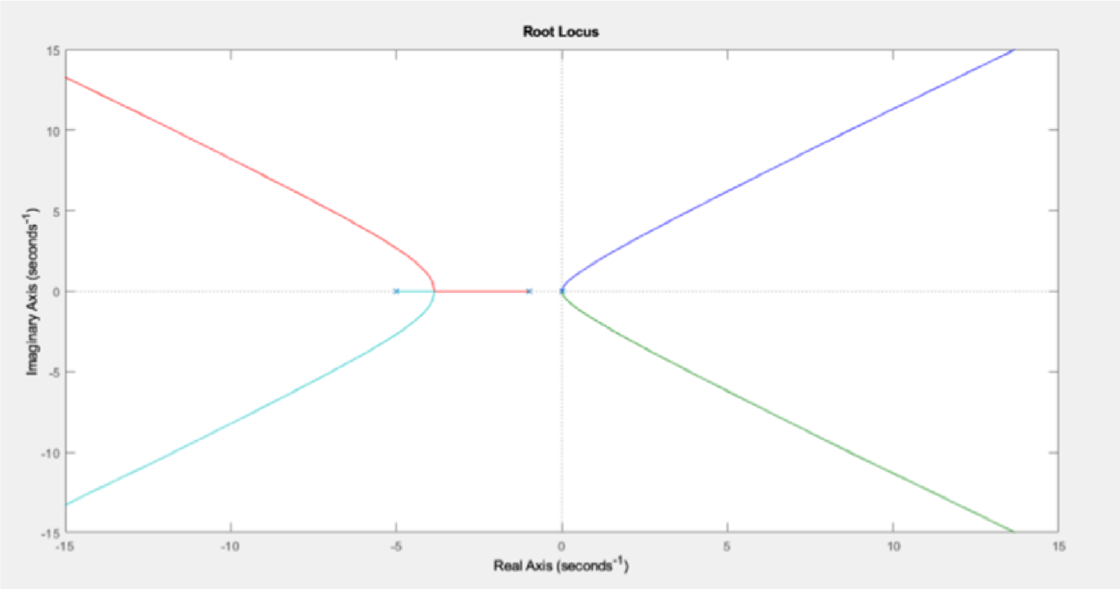


Figure 1. The root locus for (d).

```
sys = tf([1],[1 6 5 0 0]);
rlocus(sys)
```

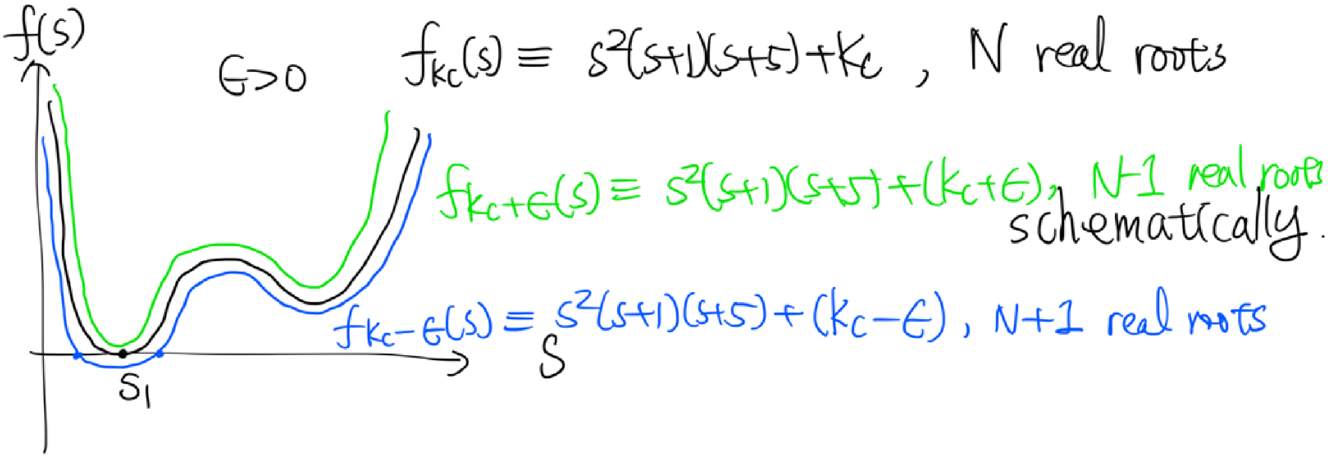
We can get the root locus plotted on the s-plane, as shown in Figure 1. This agrees with all the predices made in (a), (b), and (c).

B. Some interesting questions from the characteristic equation.

1. I believe that it is natural to ask where the intersection of the red and the blue breaches is. That point marks the critical point such that the # of real poles of $s^2(s+1)(s+5)+K=0$ suddenly decrease by 2.

i.e. the breakaway point

This is a Calculus problem. Suppose that the intersection occurs when $K=K_c$, then



schematically.

To allow this, a local minimum of

$f_{K_c}(s) \equiv s^2(s+1)(s+5)+K_c$ must be zero.

$$\frac{d}{ds} f_{K_c}(s) = \frac{d}{ds}\left(s^4 + 6s^3 + 5s^2 + K_c\right)$$

$$= 4s^3 + 18s^2 + 10s = 0$$

$$\Rightarrow \quad s(2s^2 + 9s + 5) = 0$$

$$\Rightarrow \quad s = 0, \quad s = \frac{-9 \pm \sqrt{81-40}}{4} = \frac{-9 \pm \sqrt{41}}{4}$$

(not the case wanted)

$$\Rightarrow \begin{cases} s_1 \equiv 0 \\ s_2 \equiv \frac{-9+\sqrt{41}}{4} \approx -0.649 \\ s_3 \equiv \frac{-9-\sqrt{41}}{4} \approx -3.851 \end{cases}$$

Since s_1, s_2 don't lie on the segment $-5 < Re\{s\} < -1$, they are not considered to be the breakaway point. Therefore, it should be $s_3 \equiv \frac{-9-\sqrt{41}}{4} \approx -3.851$

To find K_c, substitute s_3 in and get

$$(-3.851)^2(-3.851+5)(-3.851+1) + K_c = 0$$

$$\Rightarrow K_c = 48.58$$

This matches the result seen on the result of MatLab.

Through these, we have analyzed a crucial characteristic of the equation.

2. Another problem that seems pretty amazing is the exact equations of the root locus curves. Since they have slant asymptotes at "reasonable" directions, it is natural to guess that they are parts of a hyperbola. However, now I cannot find a possible way to explicitly discuss this issue. Perhaps some classmates can discuss this with me? That will be wonderful.

參考觀摩的作業 . (Root Locus)
作者：b089011 ， 理由： 求根的 根軌跡圖的實軸

HW6

B08901176
陳育楷

1&2. (U5B: Root Locus)

Use $L(s)=\frac{b(s)}{a(s)}=\frac{-1}{K}$ to explain the meaning of the real-axis segments of the corresponding root locus and why the root locus has $\text{degree}\,(a(s))$ branchs.

Solution:

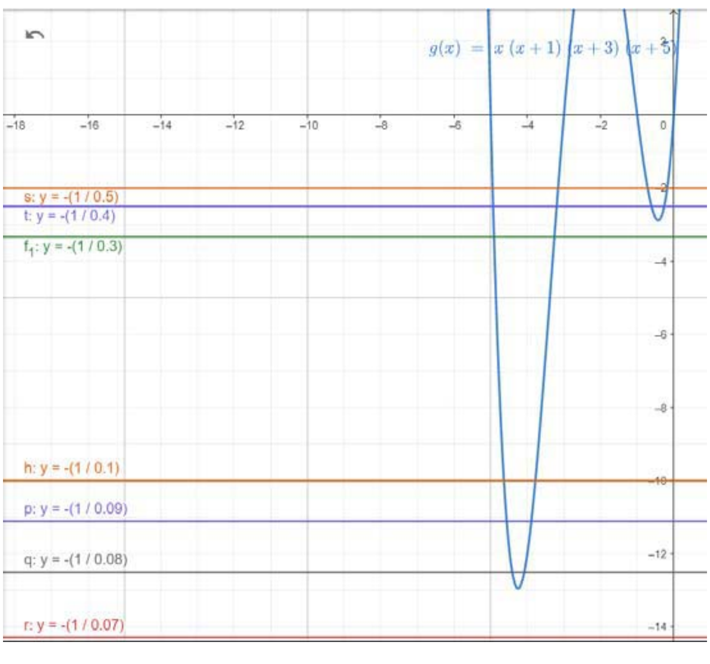
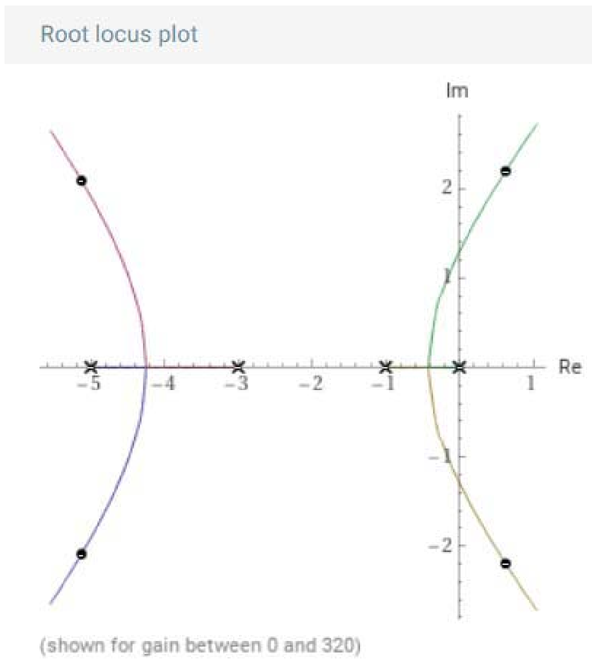
通常$a(s)$的 degree 比$b(s)$來的大，因此我們在分析根軌跡實數線段的部分時，不妨將它視為

$$\begin{cases} y=b(x) \\ y=\frac{-1}{K}a(x) \end{cases}$$

兩組方程式的解。

接下來舉個例子，用兩組方程式解來說明根軌跡實數線段的意義

$$\text{Let } L(s)=\frac{1}{s(s+1)(s+3)(s+5)}=\frac{-1}{K}$$

$\begin{cases} y=x(x+1)(x+3)(x+5) \\ y=\frac{-1}{K} \end{cases}$	Root Locus
	

B08901176
陳育楷

- 當K從無限降到$K = 0.3 \sim 0.4$時，Root Locus 右半的 branch 從$s = 0$和$s = -1$往中間匯合到約$s = -0.426$，兩組方程式的解從**四相異實根**變成**兩相異實根和二重根**，再降下去兩組方程式的解變成**兩相異實根和一組共軛虛根**，為 Root Locus 貢獻 2 個 branch。
- 當從$K = 0.3 \sim 0.4$降到時$K = 0.07 \sim 0.08$時，Root Locus 左半的 branch 從$s = -3$和$s = -5$往中間匯合到約$s = -4.256$，兩組方程式的解從**兩相異實根和兩共軛虛根**變成**二重根和兩共軛虛根**，再降下去兩組方程式的解變成**兩組兩共軛虛根**，為 Root Locus 貢獻再 2 個 branch。
- 當K從無限下降，$b(s)$會**一次跑出一至多組共軛虛根**為 Root Locus 貢獻偶數個 branch，$b(s)$最高次是奇數次，最後會再多貢獻**一個實數射線** branch。That is why the root locus has $\text{degree}\left(a(s)\right)$ branchs.

參考觀摩的作業

. (o t Root Locus)

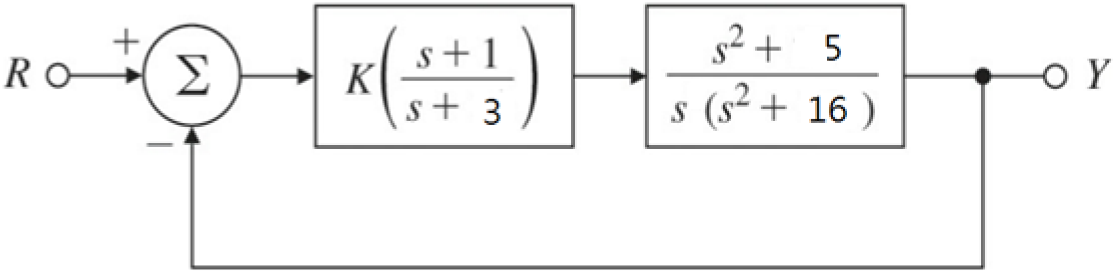
作者：b09502033，

理由： 系統 點 點位置 dampi ratio 以及 系統 階響 ， 不同 階響 的

作者：b10901125，

理由：討論 階系統 階響 的 根軌跡圖對 的

Problem 3 (改了幾個參數)



(a) Find the locus of closed-loop roots with respect to K.

(b) Is there a value of K that will cause all roots to have a damping ratio greater than 0.5?

(c) Find the values of K that yield closed-loop poles with the damping ratio $\zeta = 0.15$.

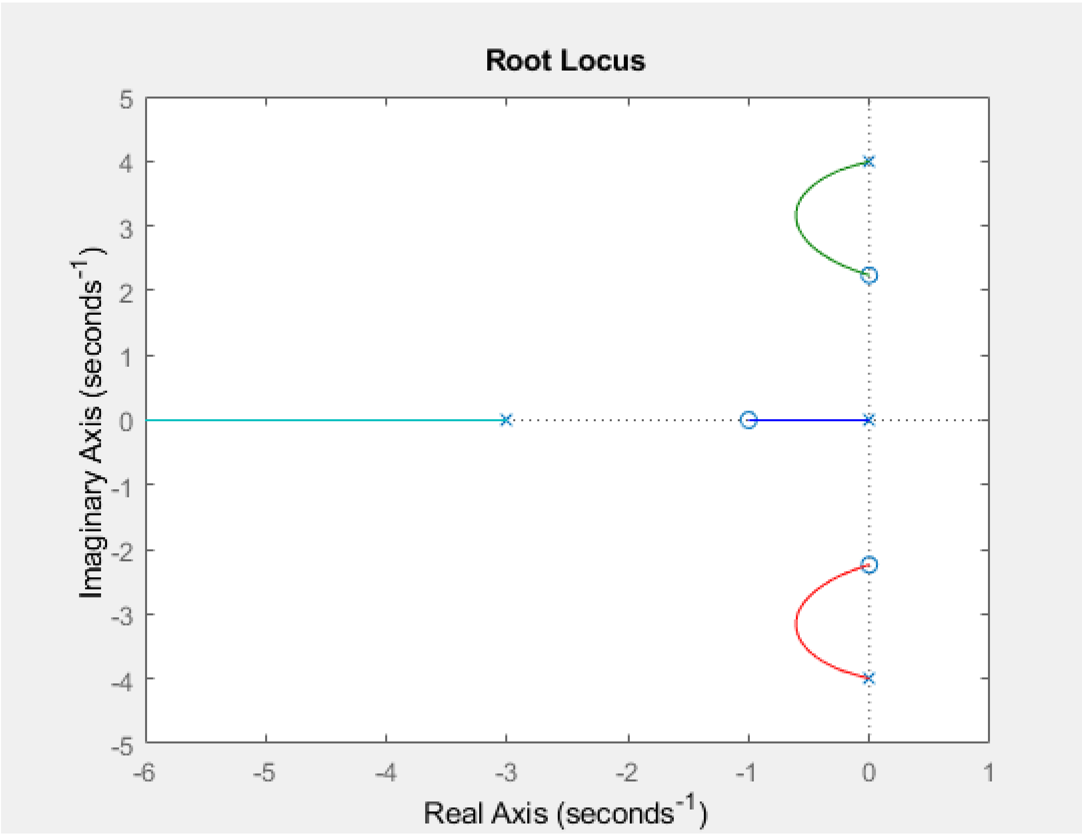
(d) Use Matlab to plot the response of the resulting design to a reference step.

Answer

(a)

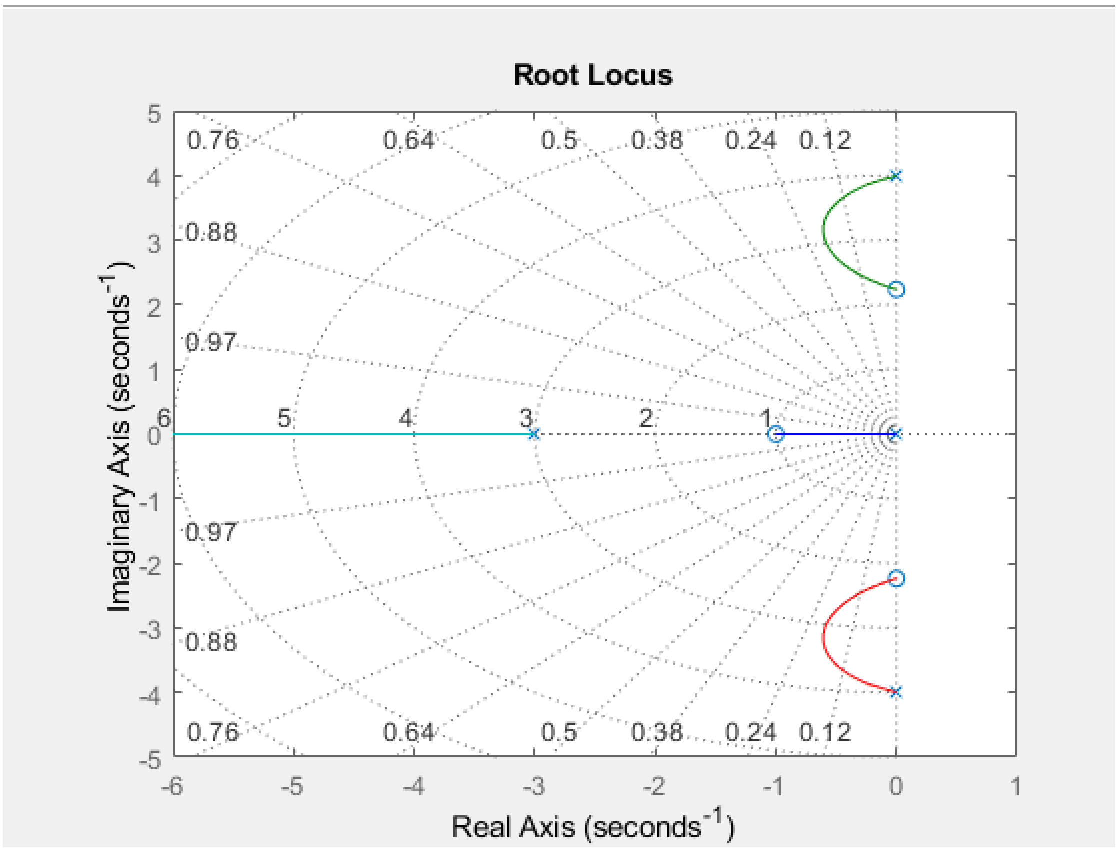
$$1 \quad K \quad (s) = 1 \quad K\frac{1}{\quad} \quad \frac{25}{(\qquad 16)}$$

出來的　　　　　圖　下

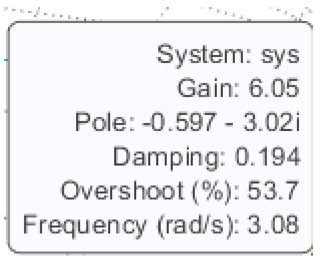


(b)

利用 matlab 中的 sgrid 函數，可以更明顯的看出這個系統在 s-domain 上的 damping ratio 以及 natural frequency。



此系統的 damping 最大約為 0.194，沒有對應的 K 值能使 damping ratio 達到 0.5。下圖為以紅色線段 damping ratio 最大的座標點為例。

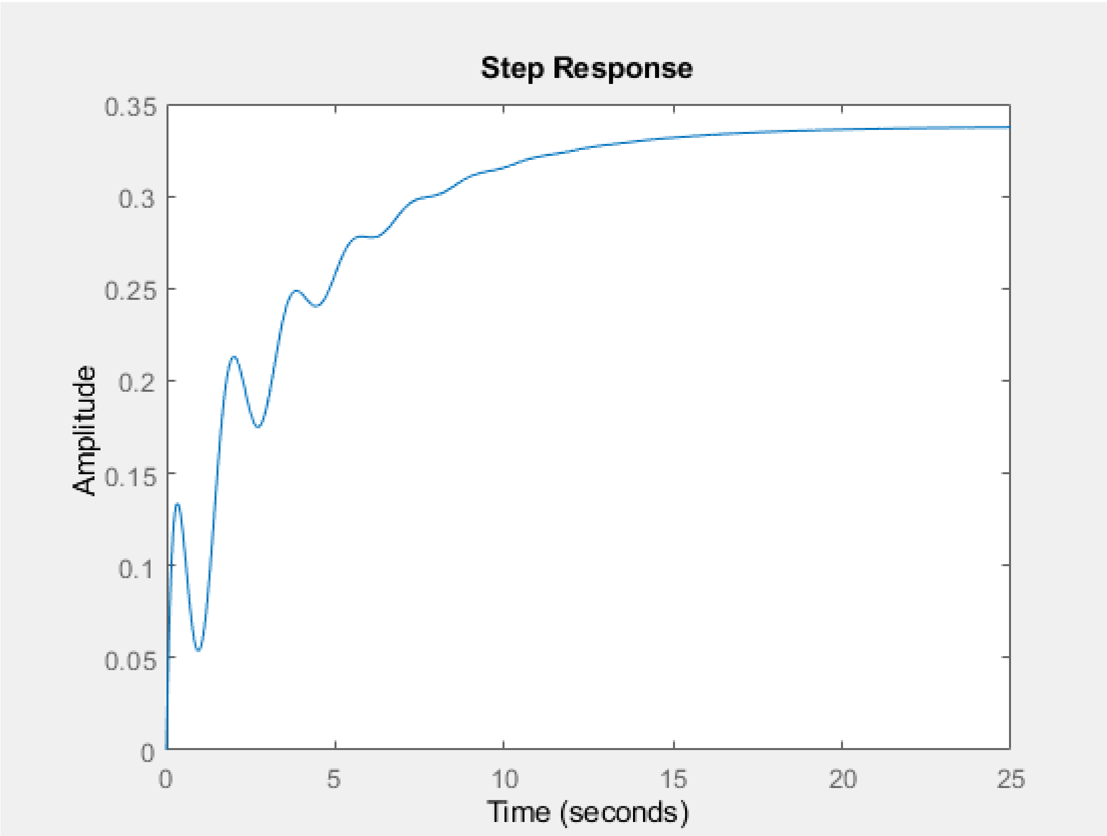


(c)

利用 rlocfind 函數，找到 damping ratio 為 0.15 的時候，Gain 約為

2.96。

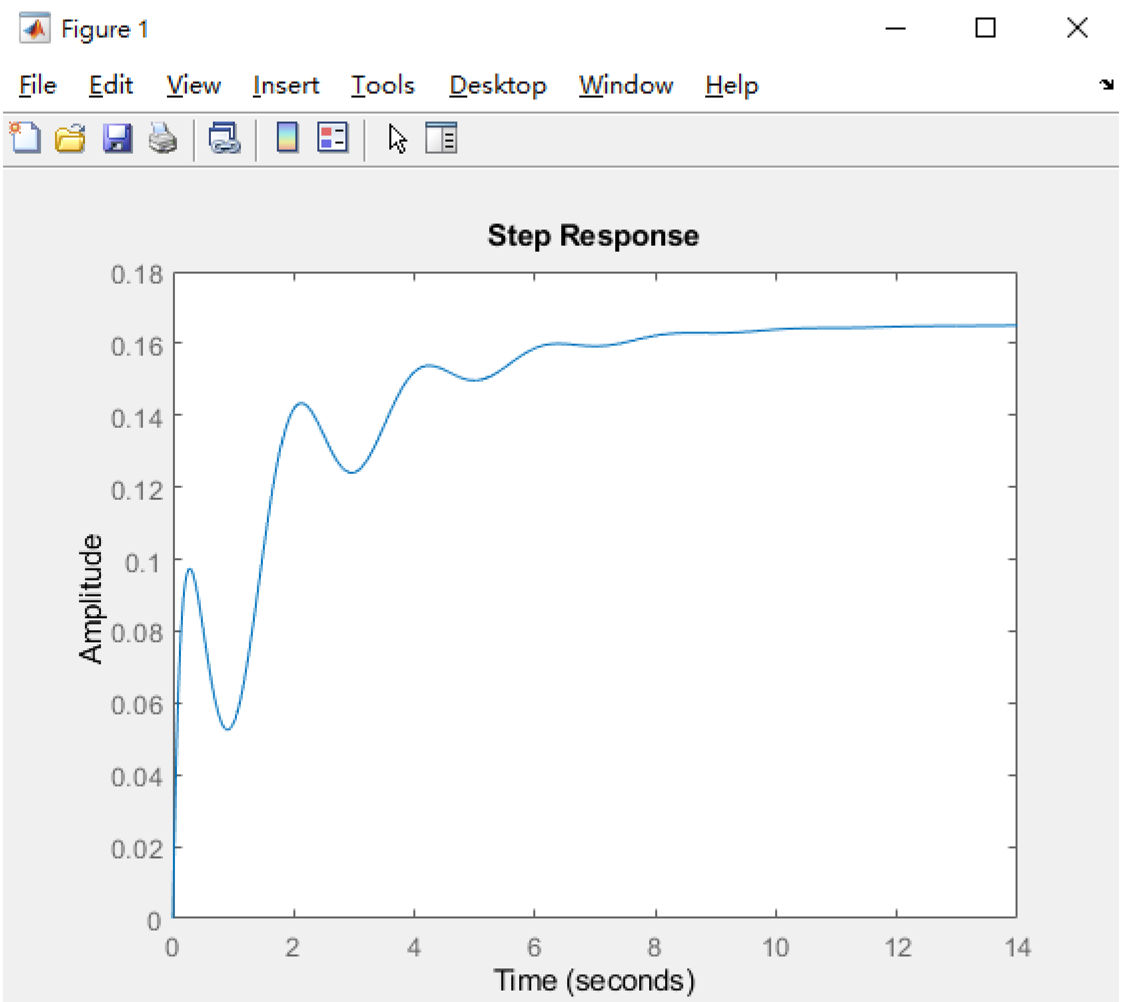
(d)



(

K=2.96，Step Response 的振盪不是很明顯。

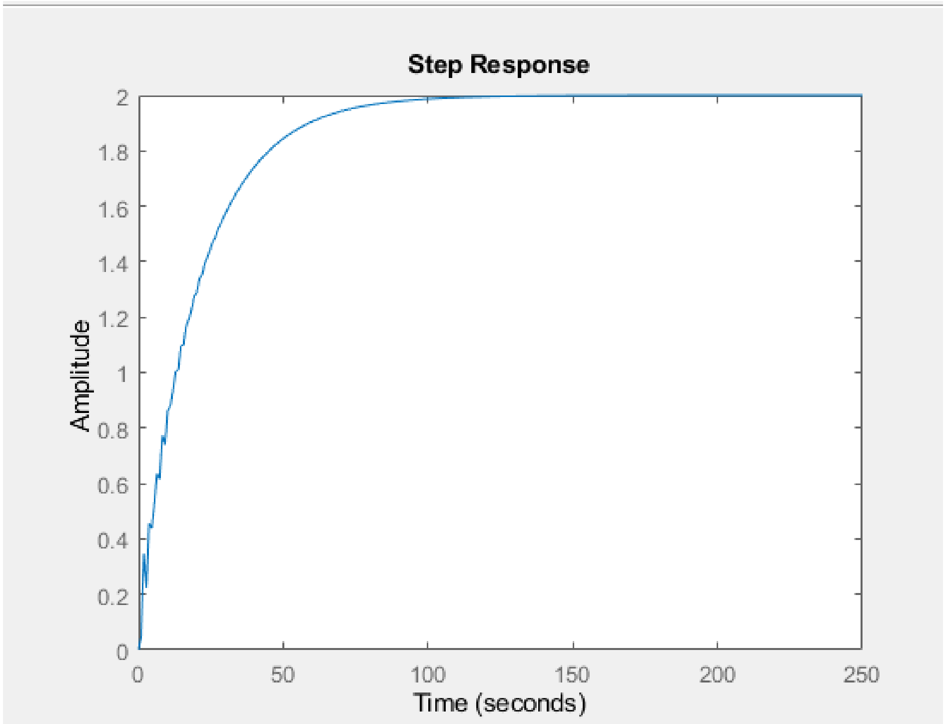
若是和(b)中得到的參數進行比較:K=6.05



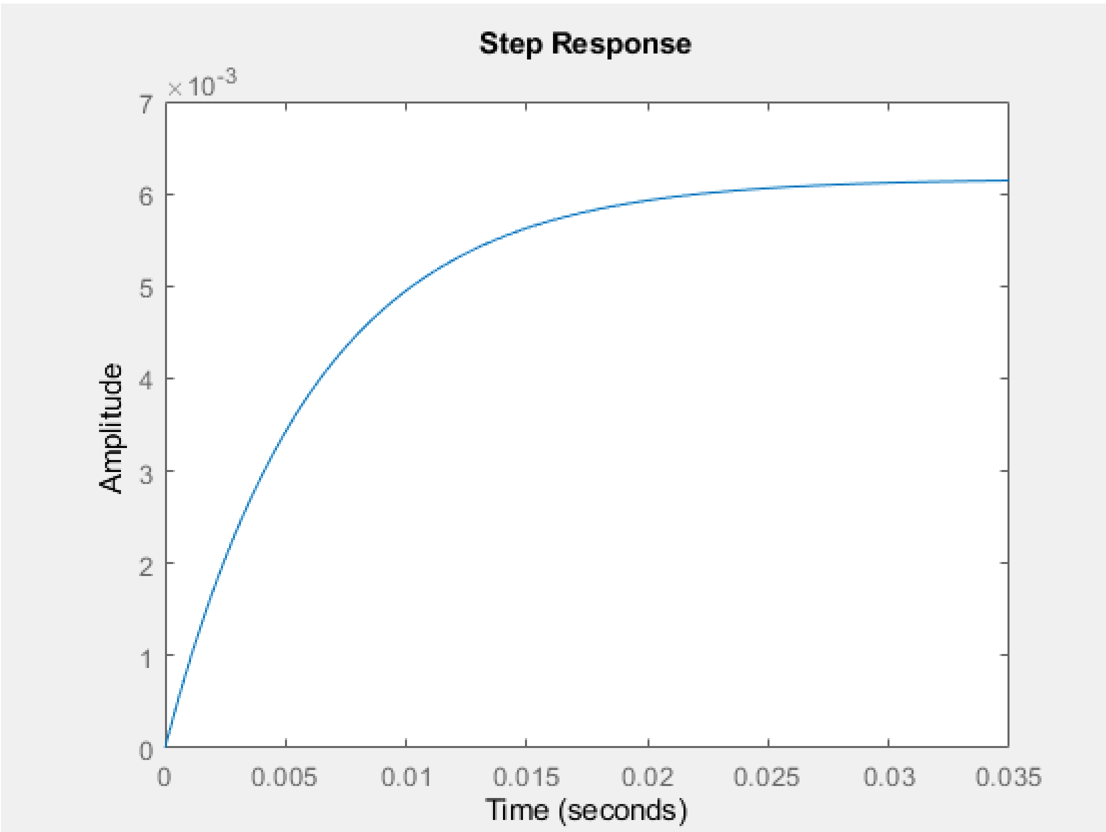
振盪依舊不是很明顯，可能是 Damping Ratio 太小了。

但若是和其他太大或太小的 K 值比起來，就可以明顯感覺到差異。以下以兩個不同的 K 值為例，可以看出振盪幅度小很多。

K=0.5，damping 的幅度小，頻率快

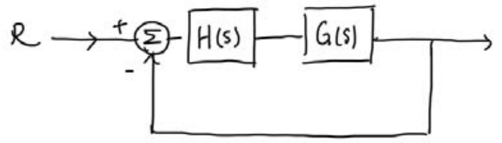


K=160，已經看不出來有 damping 了



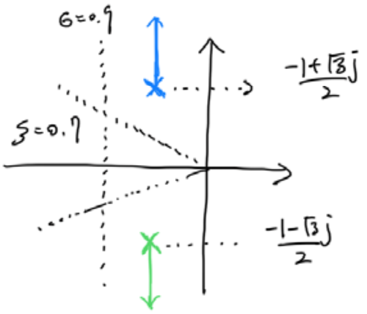
Problem 3

We have a simple damped spring-mass system that follows the equation of motion $\ddot{x}+\dot{x}+x = F(t)$. However, we want its unit step response to having a settling time of fewer than 5 seconds and an overshoot smaller than 5%. In order to meet these requirements, we try to use P control and PD control to modify the roots. From these requirements, we obtain the restrictions for roots: $\sigma \leq 0.9$; $\zeta \leq 0.7$.



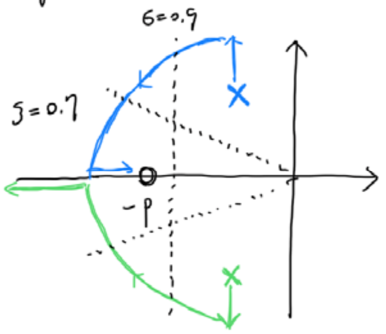
$G(s) = \frac{1}{s^2+s+1}$

• P control: $H(s) = k \Rightarrow 1 + k \cdot \frac{1}{s^2+s+1} = 0$



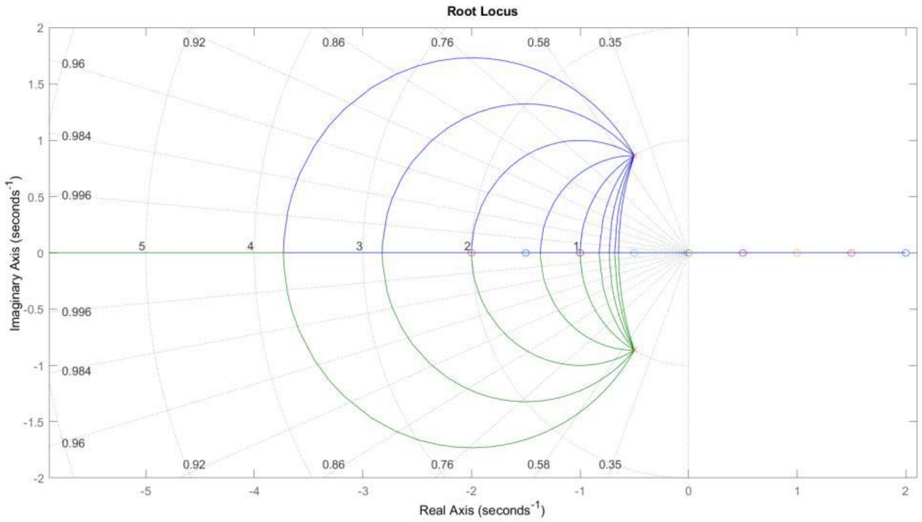
⇒ unable to reach our requirement.

• PD control: $H(s) = k(s+p) \Rightarrow 1 + k \cdot \frac{s+p}{s^2+s+1} = 0$



⇒ able to fulfill our goal!

In addition, we can use the MATLAB to plot the root loci of different p's varying from -2 ~ 2 to observe how it influence the system.

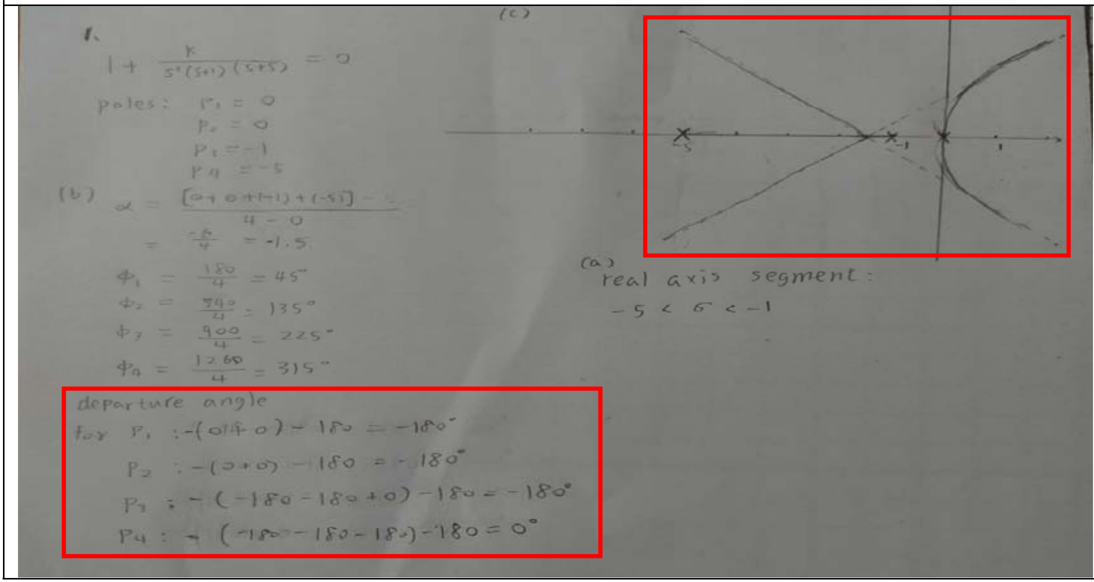


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.92
0.86
0.76
0.58
0.35
0.96
0.984
0.996
5
4
3
2
1

參考觀摩的錯誤

1. (根軌跡圖的漸進線角度和 departure angle of poles 搞混)

Prob. 1



1. 錯誤的 departure angle。

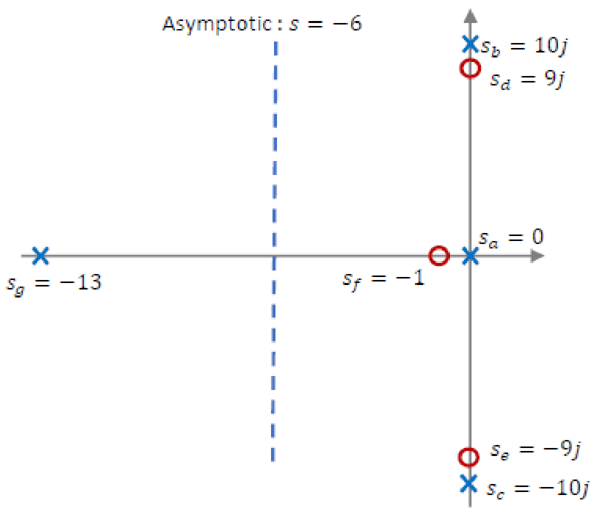
2. 根軌跡圖左半邊和漸進線畫在一起搞錯了。

參考觀摩的作業

2. (根軌跡圖的 departure angle of poles 求錯)

Prob. 3

Now we can have the root locus plot looks like :



✓ **Rule 4** :

For departure angles :

$$2\phi_a = (-90° + 0° + 90°) - (-90° + 90° + 0°) - 180° - 360° \Rightarrow \phi_a = 90°$$

$$\phi_b = (90° + 90° + \tan^{-1} 10) - \left(90° + 90°| + \tan^{-1}\frac{10}{13}\right) - 180° = -179.2°$$

$$\phi_c = (-90° - 90° - 9\tan^{-1} 10) - \left(-90° - 90° - \tan^{-1}\frac{10}{13}\right) - 180° = -180.8°$$

$s = 0$: repeated pole with multiplicity 2 → 2 departure angle

phi_a 少了 repeated pole 的-90 度分離角度，

好像對，只是少了另一個 -90

phi_b 少算 repeated pole 給的 90 度，

好像是：對於 poles 的部分， 90x2 + 90 + atan(10/13)

phi_c：應該會跟 phi_b 對稱！

同理，

因為求出來的角度明顯與正解給出來的根軌跡圖差異很大，
例如 s=10j 的極點應該會由上而下逆時針繞一圈回到 s=9j 的零點，
而非以分離角度 -179.2 度接近水平離開 s=10j。

參考觀摩的作業

3. (根軌跡離開實軸的點求錯)

Prob. 2

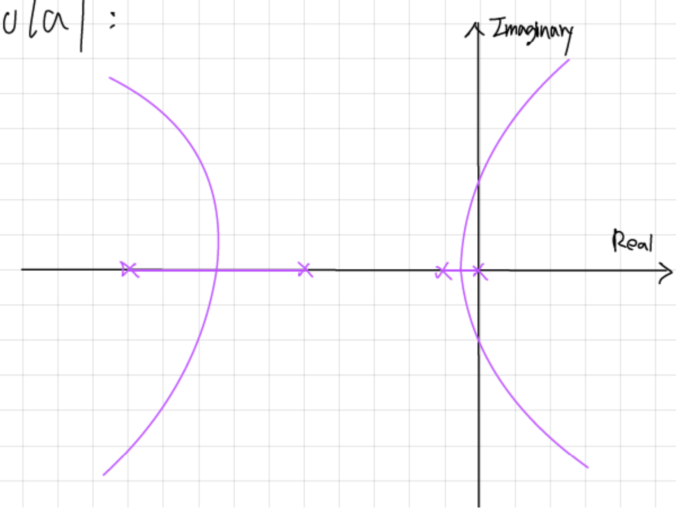
Rule 5:

- The locus can have multiple roots at points on the locus and the branches will approach a point of q roots at angles separated by $\frac{180^o - 360^o(l-1)}{q}$
- And will depart at angles with same separation.

for 0, -1 ⇒ separated by $\frac{180^o}{2} = 90^o$ at $\frac{0+(-1)}{2} = -0.5$

for -5, -10 ⇒ separated by $\frac{180^o}{2} = 90^o$ at $\frac{-5+(-10)}{2} = -7.5$

Total:



求 s(s+1)(s+5)(s+10)極值點對應 s 實數解

主要是求錯實數軸上軌跡分離的位置。
正解應該要比照 continuation locus 的作法，
求出 s(s+1)(s+5)(s+10) 有極值且落在 (0, -1) 和 (-5, -10) 區間的根軌跡，
也就是上式的一次導數 2s^3+24s^2+65s+25 的根。
不過求三次或四次多項式可能需要用電腦計算，
手算比較困難。