

Control System: Homework 05 for Unit 4C, 4D, 4E: Dynamic Response

Assigned: Oct 21, 2022

Due: Oct 27, 2022 (23:59)

Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

1. (U4C: Three terms controller)

4.34 Consider the satellite-attitude control problem shown in Fig. 4.45 where the normalized parameters are

$J = 10$ spacecraft inertia, $\text{N}\cdot\text{m}\cdot\text{sec}^2/\text{rad}$

θ_r = reference satellite attitude, rad.

θ = actual satellite attitude, rad.

$H_y = 1$ sensor scale, factor V/rad .

$H_r = 1$ reference sensor scale factor, V/rad .

w = disturbance torque, $\text{N}\cdot\text{m}$.

- (a) Use proportional control, P, with $D_c(s) = k_P$, and give the range of values for k_P for which the system will be stable.
- (b) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to reference inputs.
- (c) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to disturbance inputs.
- (d) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to reference inputs.
- (e) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to disturbance inputs.
- (f) Use PID control, let $D_c(s) = (k_P + \frac{k_I}{s} + k_D s)$, and determine the system type and error constant with respect to reference inputs.

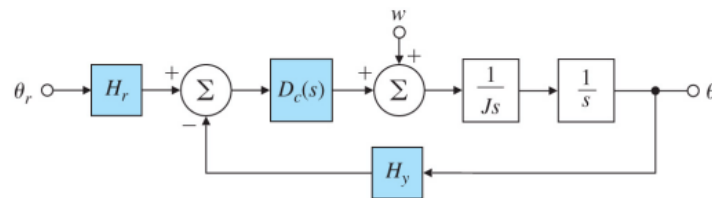


Figure 4.45: Satellite attitude control

Solution:

- (a) $D_c(s) = k_P$; The characteristic equation is

$$1 + H_y D_c(s) \frac{1}{Js^2} = 0$$

$$Js^2 + H_y k_P = 0$$

or $s = \pm j\sqrt{\frac{H_y k_P}{J}}$ so that no additional damping is provided. The system cannot be made stable with proportional control alone.

- (b) Steady-state error to reference steps.

$$\begin{aligned} \frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s) \frac{1}{Js^2}}{1 + D_c(s) H_y \frac{1}{Js^2}}, \\ &= H_r \frac{(k_P + k_D s)}{Js^2 + (k_P + k_D s) H_y}. \end{aligned}$$

The parameters can be selected to make the (closed-loop) system stable. If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming the system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback).

- (c) Steady-state error to disturbance steps

$$\frac{\Theta(s)}{W(s)} = \frac{1}{Js^2 + (k_P + k_D s) H_y}.$$

If $W(s) = \frac{1}{s}$ then using the FVT (assuming system is stable), the error is $\theta_{ss} = -\frac{1}{k_P H_y}$.

- (d) The characteristic equation is

$$1 + H_y D_c(s) \frac{1}{Js^2} = 0.$$

With PI control,

$$Js^3 + H_y k_P s + H_y k_I = 0.$$

From the Hurwitz's test, with the s^2 term missing the system will always have (at least) one pole not in the LHP. Hence, this is not a good control strategy.

- (e) See (d) above.

(f) The characteristic equation with PID control is

$$1 + H_y \left(k_P + \frac{k_I}{s} + k_D s \right) \frac{1}{Js^2} = 0,$$

or

$$Js^3 + H_y k_D s^2 + H_y k_P s + H_y k_I = 0.$$

There is now control over all the three poles and the system can be made stable.

$$\begin{aligned} \frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s) \frac{1}{Js^2}}{1 + D_c(s) H_y \frac{1}{Js^2}}, \\ &= \frac{H_r (k_P + \frac{k_I}{s} + k_D s)}{Js^2 + (k_P + \frac{k_I}{s} + k_D s) H_y}, \\ &= \frac{H_r (k_D s^2 + k_P s + k_I)}{Js^3 + (k_D s^2 + k_P s + k_I) H_y}. \end{aligned}$$

If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback). In that case, the system is Type 3 and the (Jerk!) error constant is $K_J = \frac{k_I}{J}$.

2. (U4D: Three terms controller and Ziegler–Nichols Tuning)

4.37 A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s + 1},$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

- (a) Find the PID-controller parameters using the Ziegler–Nichols tuning rules.
- (b) The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 4.48. Find the optimal PID-controller parameters according to the Ziegler–Nichols tuning rules.

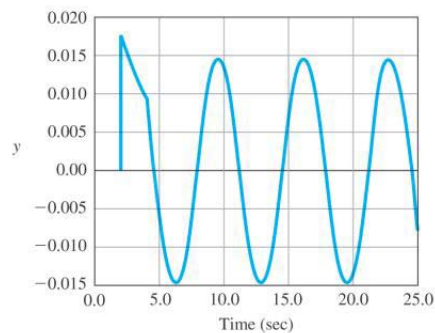


Figure 4.48: Unit impulse response for paper-machine in Problem 4.37

Solution:

- (a) From the transfer function: $L = \tau_d \simeq 2$ sec

$$R = \frac{1}{3} \simeq 0.33 \text{ sec}^{-1}.$$

From Table 4.1:

Controller Gain	P	:	$K = \frac{1}{RL} 1.5,$
	PI	:	$K = \frac{0.9}{RL} = 1.35 \quad T_I = \frac{L}{0.3} = 6.66,$
	PID	:	$K = \frac{1.2}{RL} = 1.8 \quad T_I = 2L = 4 \quad T_D = 0.5L = 1.0.$

- (b) From the impulse response: $P_u \simeq 7$ sec From Table 4.2:

Controller Gain	P	:	$K = 0.5K_u = 1.52,$
	PI	:	$K = 0.45K_u = 1.37 \quad T_I = \frac{1}{1.2}P_u = 5.83,$
	PID	:	$K = 0.6K_u = 1.82 \quad T_I = \frac{1}{2}P_u = 3.5 \quad T_D = \frac{1}{8}P_u = 0.875.$

3. (U4E: Feedforward Control)

- 4.38** Consider the DC motor speed-control system shown in Fig. 4.49 with proportional control. (a) Add feedforward control to eliminate the steady-state tracking error for a step reference input. (b) Also add feedforward control to eliminate the effect of a constant output disturbance signal, w , on the output of the system.

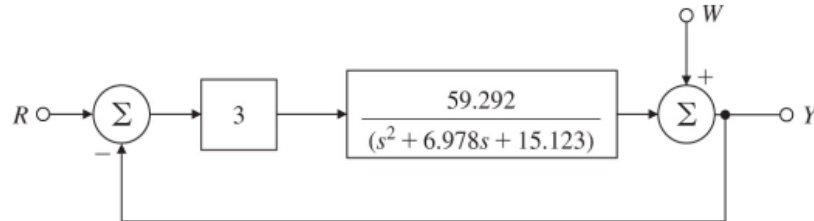
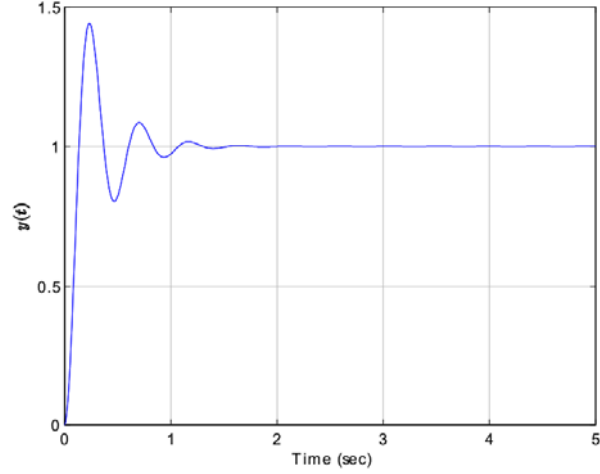


Figure 4.49: Block diagram for Problem 4.38

Solution: (a) In this case the plant inverse DC gain is $G^{-1}(0) = \frac{15.123}{59.292} = 0.2551$. We implement the closed-loop system as shown in Figure 4.22 (a) with $D_c(s) = k_p = 3$. The closed-loop transfer function is

$$\begin{aligned} Y(s) &= G(s)[k_p E(s) + G^{-1}(0)R(s)], \\ E(s) &= R(s) - Y(s), \\ \frac{Y(s)}{R(s)} &= T(s) = \frac{(G^{-1}(0) + k_p)G(s)}{1 + k_p G(s)}. \end{aligned}$$

Note that the closed-loop DC gain is unity ($T(0) = 1$). The following figure illustrates the effect of feedforward control in eliminating the steady-state tracking error. The addition of feedforward control results in zero steady-state tracking error for a step reference input.

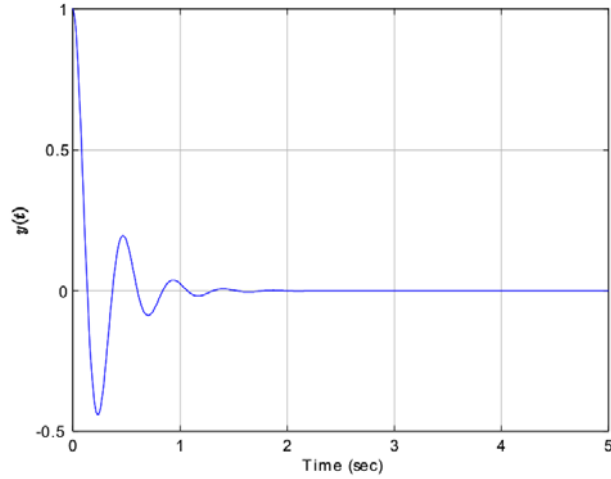


Tracking response with feedforward

(b) Similarly, we implement the closed loop system as shown Figure 4.22 (b). The closed-loop transfer function is

$$\begin{aligned} Y(s) &= W(s) + G(s)[k_p E(s) - G^{-1}(0)W(s)], \\ E(s) &= R(s) - Y(s) = 0 - Y(s), \\ \frac{Y(s)}{W(s)} &= \mathcal{T}_w(s) = \frac{1 - G^{-1}(0)G(s)}{1 + k_p G(s)}. \end{aligned}$$

Note that the closed-loop DC gain is zero ($\mathcal{T}_w(s) = 0$). The following figure illustrates the effect of feedforward control in eliminating the steady-state error for a step output disturbance.



Disturbance rejection response with feedforward

```

MATLAB code:
%FPE7e Problem 3.38
clf;
% Tracking
s=tf('s');
% plant
G=59.292/(s^2+6.978*s+15.123);
kp=3;
% Closed-loop Transfer function
dcgain1=dcgain(G);
T1=G*(1/dcgain1+kp)/(1+kp*G);
t=0:0.01:5;
% Step response

y1=step(T1,t);
figure()
plot(t,y1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;
% Disturbance rejection
kp=3;
Tw1=(1-1/dcgain1*G)/(1+kp*G);
yw1=step(Tw1,t);
figure()
plot(t,yw1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;

```

參考觀摩的作業

1. (Three terms controller)

無

參考觀摩的作業

2. (Three terms controller and Ziegler–Nichols Tuning)

作者：b08901085，施彥宇

理由：詳細解釋如何從 root locus 依照 Ziegler-Nichols 方法調整 PID 控制器參數來達到三階系統理想步階響應

作者：b09502033，朱本毅

理由：分別討論 RC 電路輸出延遲系統以及三階穩定系統用 Ziegler-Nichlos 方法調整 PID 參數的結果

作者：b10202032，卓然

理由：討論 Ziegler-Nicholas 兩種方法對 PID 參數調整的結果並做步階響應圖形，(b)部分表格討論 error 內容不太清楚

HW05 – Unit 4, Feedback Analysis

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系級：電機四

姓名：施彥宇

- **Question :** A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1}, \quad (1).$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

- Find the PID-controller parameters using the Ziegler-Nichols tuning rules.
- The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 1. Find the optimal PID-controller parameters according to the Ziegler-Nichols tuning rules.

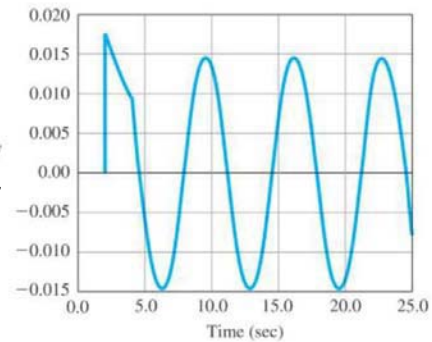


Fig. 1 : Unit impulse response for paper-machine

- **Solution :**

- Form the textbook, the first order transfer function with a time delay analyzed by Ziegler-Nichols' method is in the form of :

$$\frac{Y(s)}{U(s)} = \frac{Ae^{-st_d}}{\tau s + 1}, \quad (2).$$

the step response of this transfer function is shown in Fig. 2 :

In this case, the parameters

$$A = 1;$$

$$\tau = 3;$$

$$t_d = L = 2;$$

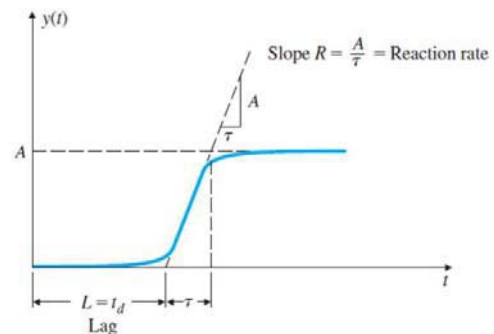


Fig. 2 : Step response with delay

And the max slope of the transient state of this transfer function is :

$$\text{Slope } R = \frac{A}{\tau} = \frac{1}{3} \text{ sec}^{-1}, \quad (3).$$

And then, the coefficients of controller gain are listed in Table. 1.

From this table, the controller gain :

$$P : \quad K = 1/RL = 1.5 \quad (4).$$

$$PI : \quad K = 0.9/RL = 1.35; \quad T_I = L/0.3 = 6.66 \quad (5).$$

$$PID : \quad K = 1.2/RL = 1.8; \quad T_I = 2L = 4; \quad T_D = 0.5L = 1 \quad (6).$$

- **Solution :**

Table. 1 : Ziegler-Nichols
Tuning for the Regulator $D(s) = k_p(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25.

Type of Controller	Optimum Gain
P	$k_p = 1/RL$
PI	$k_p = 0.9/RL$ $T_I = L/0.3$
PID	$k_p = 1.2/RL$ $T_I = 2L$ $T_D = 0.5L$

b) Observing from Fig. 1, the ultimate period $P_u \cong 7 \text{ sec}$. From Table. 2., we can calculate the controller gain to be

$$P : K = 0.5K_u = 1.52 \quad (7).$$

$$PI : K = 0.45K_u = 1.37; T_I = P_u/1.2 = 5.83 \quad (8).$$

$$PID : K = 0.6K_u = 1.82; T_I = P_u/2 = 3.5; T_D = P_u/8 = 0.875 \quad (9).$$

Table. 2 : Ziegler-Nichols
Tuning for the Regulator $D(s) = k_p(1 + 1/T_I s + T_D s)$, based on the Ultimate Sensitivity Method.

Type of Controller	Optimum Gain
P	$k_p = 0.5K_u$
PI	$k_p = 0.45K_u$ $T_I = P_u/1.2$
PID	$k_p = 0.6K_u$ $T_I = P_u/2$ $T_D = P_u/8$

- **What I can do more :**

We've already known that the Ziegler-Nichols method generated from experimental data. As a result, there can be many sets of parameters (k_p, T_I, T_d) . The following is some data sets I found from wikipedia.

Controller Type	k_p	T_I	T_d
P	$0.5K_u$	—	—
PI	$0.45K_u$	$0.83T_u$	—
PD	$0.8K_u$	—	$0.125T_u$
PID	$0.6K_u$	$0.5T_u$	$0.125T_u$
Some overshoot	$0.33K_u$	$0.5T_u$	$0.33T_u$
No overshoot	$0.2K_u$	$0.5T_u$	$0.33T_u$

Table. 3 : Some data sets of Ziegler-Nichols method

The parameters K_u and T_u means ultimate gain and ultimate period,

respectively. In the original question, these parameters are already given. However, in what way can we find these data by ourselves? It can be found by the root locus graph.

Taking a transfer function $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$ as an example, we can type “*rlocus*” command on MATLAB to get its root locus graph. Just as the following graph shown :

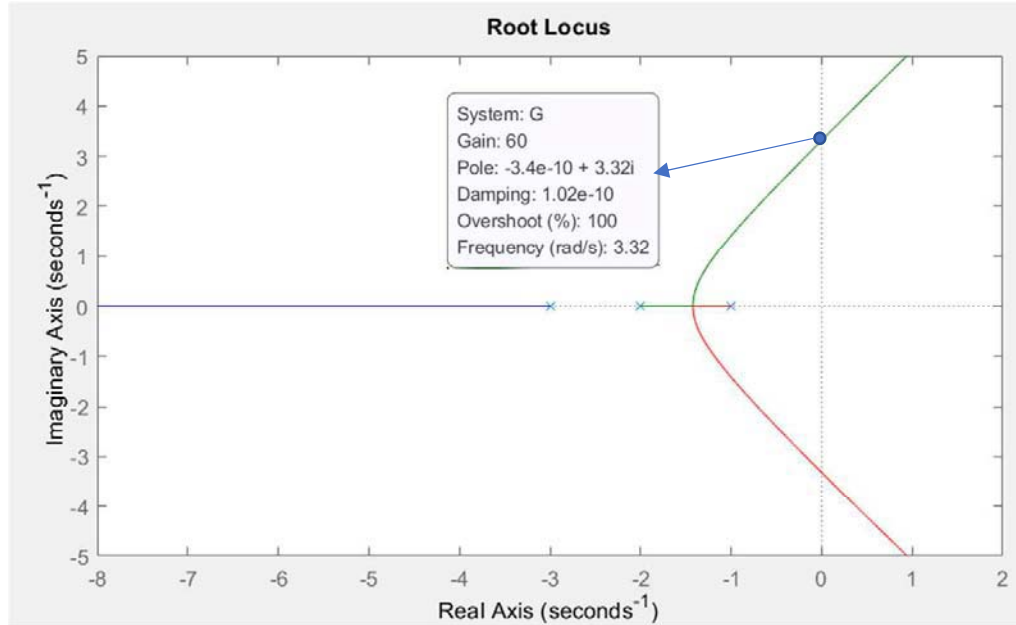


Fig. 3 : Root Locus Graph of Transfer Function $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$

From the above, finding the cross points of the graph and real axis, and then we can get the ultimate gain and ultimate frequency, we call K_u and W_u , respectively. Thus, we can generate K_u and T_u from this graph.

$$K_u = 60 ; T_u = \frac{2\pi}{W_u} = \frac{2\pi}{3.32} \cong 1.89 \text{ sec} \quad (10).$$

Substituting (10). Into Table. 3, and we can get a new table of controller parameters, we call it Table. 4.

Controller Type	k_p	T_I	T_d
P	30	—	—
PI	27	1.57	—
PD	48	—	0.236
PID	36	0.945	0.236
Some overshoot	20	0.945	0.236
No overshoot	12	0.945	0.236

Table. 4: Parameters of Different Controller Types with $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$

After this, we can analyze the step response with and without controller.

The following graphs are step responses with and without controllers in Table. 3. We can observe that with P-Controller, the steady-state error can be reduced.

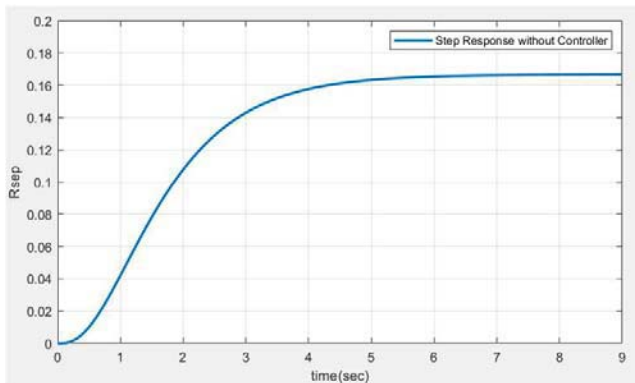


Fig. 4 : Response without Controller

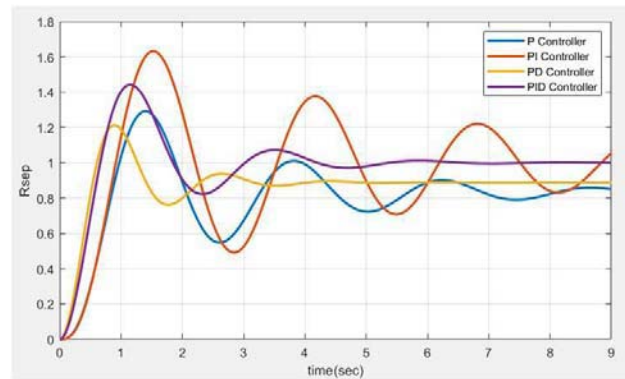


Fig. 5 : Responses with Controllers

And with PI-Controller, the system type rises but decreases damping. With PD-Controller can provide a sharper response to a suddenly changing signals (the overshoot is the smallest). These features are match to those written on textbook. In other words, I proved the description on textbook by mathematical calculation and simulation. Also, the following is the *stepinfo* of each response.

RiseTime: 2.7428	RiseTime: 0.5042	RiseTime: 0.5358	RiseTime: 0.3631	RiseTime: 0.4461
SettlingTime: 5.0039	SettlingTime: 9.0105	SettlingTime: 17.6777	SettlingTime: 3.6258	SettlingTime: 5.0058
SettlingMin: 0.1502	SettlingMin: 0.5486	SettlingMin: 0.4927	SettlingMin: 0.7624	SettlingMin: 0.8224
SettlingMax: 0.1665	SettlingMax: 1.2911	SettlingMax: 1.6327	SettlingMax: 1.2121	SettlingMax: 1.4429
Overshoot: 0	Overshoot: 54.9281	Overshoot: 63.2742	Overshoot: 36.3656	Overshoot: 44.2930
Undershoot: 0	Undershoot: 0	Undershoot: 0	Undershoot: 0	Undershoot: 0
Peak: 0.1665	Peak: 1.2911	Peak: 1.6327	Peak: 1.2121	Peak: 1.4429
PeakTime: 8.2586	PeakTime: 1.3954	PeakTime: 1.5343	PeakTime: 0.8977	PeakTime: 1.1418
No Ctrl	P-Ctrl	PI-Ctrl	PD-Ctrl	PID-Ctrl

Fig. 6 : Step Information of each Responses with and without Controller

Another interesting thing is Table. 3. from Wikipedia has parameter sets called “Some overshoot” and “No overshoot”. However, I simulated this result and found that both have overshooting. As Fig. 7. illustrated. I thought the reason why the “No overshoot” term exhibit overshooting is because Ziegler-Nichols tuning method comes from experiments and statistics. And there must be errors in the statistical results. As a result, we sometimes needs to adjust the tuning parameters on our own while using this method to design a controller.

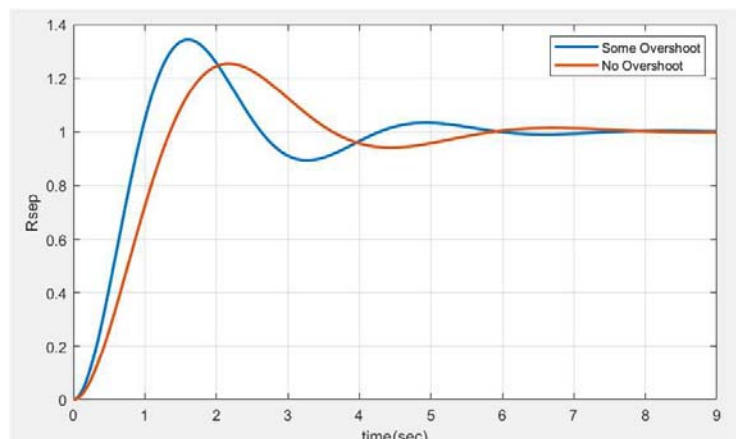
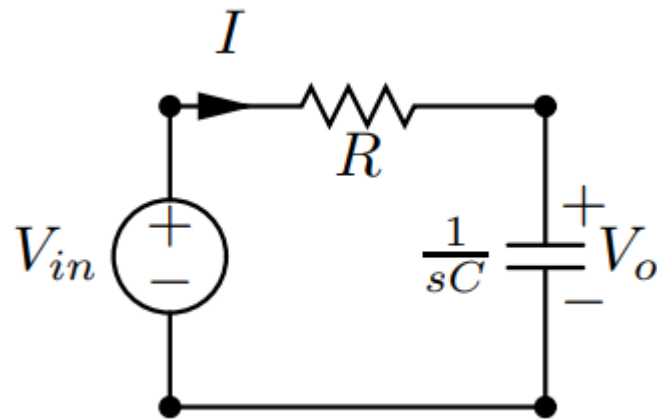


Fig. 7 : Responses of Controllers with and without overshoot theoretically

Problem (改自本週作業第二題)

A standard RC circuit, which is known as a first-order system, is shown in the following figure.



- (a) Find its transfer function $T(s) = \frac{V_o(s)}{V_{in}(s)}$
- (b) Briefly describe the physical meaning of this transfer function.
- (c) Find the PID-controller parameters using the quarter wave ratio method.

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Answer

(a)

$$\begin{cases} I = \frac{V_{in}}{R + \frac{1}{sC}} = \frac{V_{in} * sC}{1 + sCR} \\ V_o = V_{in} - IR \end{cases}$$

$$V_o = V_{in} \left(1 - \frac{sCR}{1 + sCR} \right) = V_{in} * \frac{1}{1 + sCR}$$

$$T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + sCR}$$

(b)

和標準式 $T(s) = \frac{Ae^{-t_d s}}{\tau s + 1}$ 比較，可以知道 time constant $\tau = \frac{1}{RC}$ 。

time delay 目前為 0，代表輸入電壓 V_{in} 在切換的一瞬間就會馬上對電路造成影響。

DC gain $A = 1$ 。

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(c)

寫完才發現沒有 time delay 的話好像不需要 PID 控制.....RC 電路好像比較會注意它在時域下的開關狀況。

考慮一個類似的 Transfer Function with time delay

$$T(s) = \frac{100e^{-0.001s}}{\tau s + 1}, \text{ where } R = 1M\Omega, C = 100\mu F, \text{ 就像 Vin 經過某個放}$$

大率為 40(dB in power)的放大器，但會產生 1ms 的 time delay。

先計算幾個參數

$$\tau = \frac{1}{RC} = \frac{1}{100} = 0.01$$

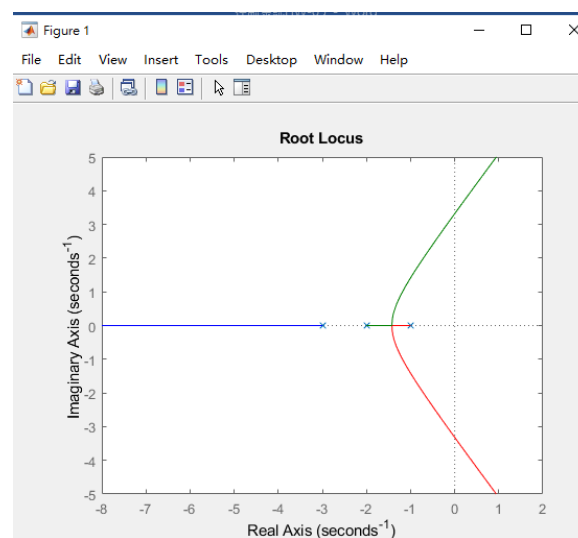
$$R_{slope} = \frac{A}{\tau} = 10000, L = 0.001$$

PID parameter table			
	P control	PI control	PID control
k_P	0.1	0.09	0.12
T_I		0.00333...	0.002
T_D			0.0005

(Bonus)

由於想試試看 Ultimate Sensitivity Method，我先隨便假設了一個

$$\text{stable system: } \text{sys}(s) = \frac{100}{(s+1)(s+2)(s+3)}$$

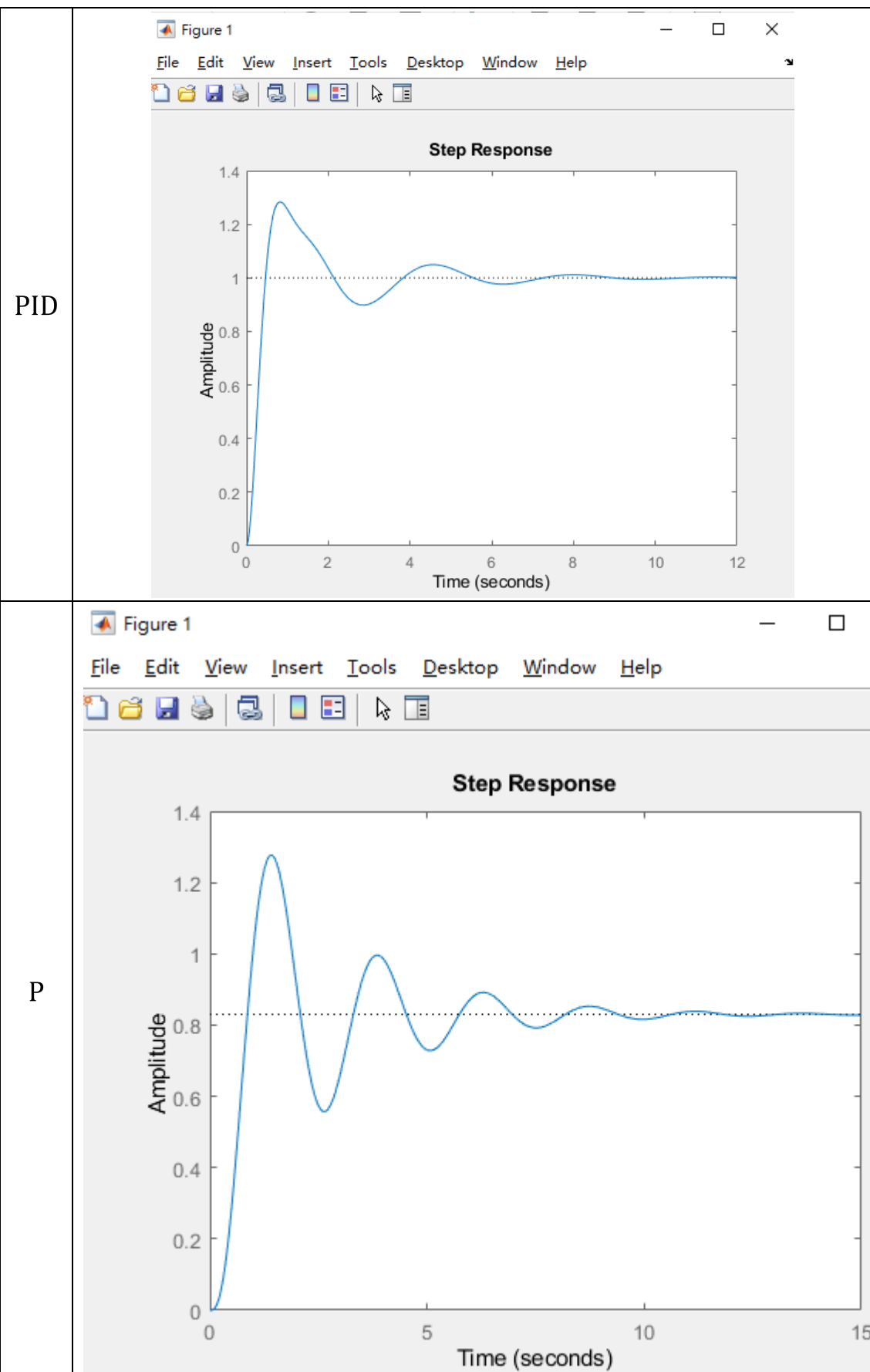


由 root locus 圖的綠線部分可以得知， $K_u=0.585$ $P_u=1.9098$

PID parameter table			
	P control	PI control	PID control
k_P	0.2925	0.26325	0.351
T_I		1.5915	0.9549
T_D			0.238725

分別利用這幾個參數進行 P、PI、PID 控制，以及略為加上

feedback，可以得到以下結果。



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看起來 PID 和 P 控制都很成功，但我在做 PI 控制的時候結果就有點奇怪了.....。

就結果而論，本次 PID 和 P 控制的 overshoot 差不多，但 PID 的 settling time 更短。下次應該再嘗試找一個更貼近實際狀況的 system 操作看看!

Control System HW

B10202032 物理 = 数学

2. (U4D: Three terms controller and Ziegler-Nichols Tuning)

4.37 A paper machine has the transfer function

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where the input is stock flow onto the wire and the output is basis weight or thickness.

- (a) Find the PID-controller parameters using the Ziegler-Nichols tuning rules.
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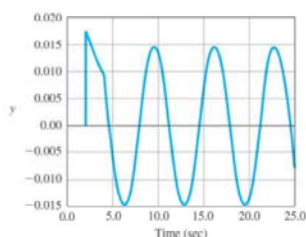


Figure 4.48: Unit impulse response for paper-machine in Problem 4.37

(a) Comparing $G(s) = \frac{e^{-2s}}{3s + 1}$ with $\frac{Ae^{-st_d}}{\tau s + 1}$, we can see that $\begin{cases} A=1 \\ \tau=3 \\ t_d=2 \end{cases}$. That is, $\begin{cases} \text{Lag} & L = t_d = 2s \\ \text{Reaction rate} & R = A/\tau = \frac{1}{3} s^{-1} \end{cases}$.

Using the "Quarter Decay Ratio" method,

the parameters for the PID controller are:

$$\begin{cases} K_p = 1.2/RL = 1.2/2 \cdot \frac{1}{3} = 1.8 \\ T_I = 2L = 2 \cdot 2 = 4 \\ T_D = 0.5L = 0.5 \cdot 2 = 1 \end{cases}$$

To visualize the lag and the reaction rate, figure 1 and figure 2 show the time domain step response. (Figure 1 is plotted using Matlab, and figure 2 is plotted using Desmos after manually calculating the inverse Laplace Transform.) ①

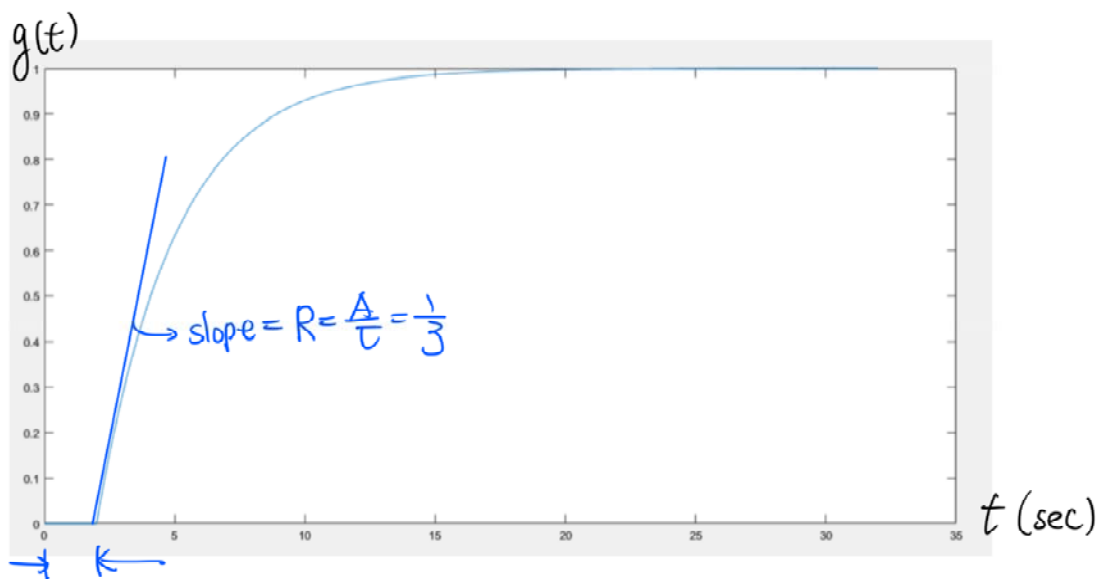


Figure 1 time domain step response plotted by Matlab.

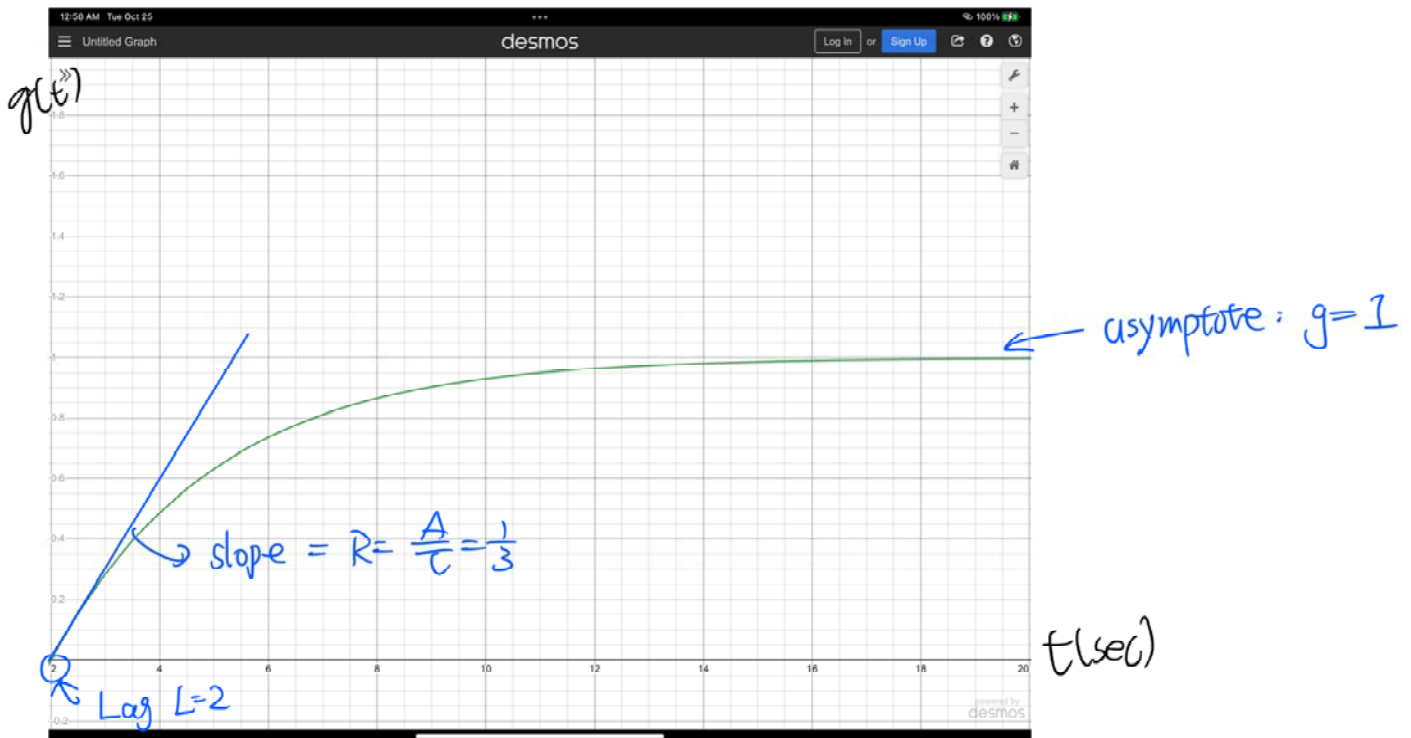


Figure 2 time domain step response plotted by Desmos.

The function in Figure 2 is $g(t) = \left(1 - e^{-\frac{t-2}{3}}\right) 1(t-2)$.

From the step response equation, we can easily see that $g(2) = 0$

and $\lim_{t \rightarrow \infty} g(t) = 1$ (which is guaranteed by the Final Value

Thm.). Also, $\frac{d}{dt}g(t) = \frac{1}{3}e^{-\frac{t-2}{3}}$, $\frac{d}{dt}g(t)\big|_{t=2} = \frac{1}{3} = R = \frac{A}{L}$.

These all meet the predictions of the determination method of R and L .

from this we see that the reaction rate is in fact the slope of $g(t)$ exactly at

$t = t_0$.

• Calculation of $g(t)$:

$$\mathcal{L}^{-1}\{e^{2s}[G(s) \cdot \mathcal{L}\{1(t)\}]\} = \mathcal{L}^{-1}\{e^{2s} \cdot \frac{e^{-2s}}{3s+1} \cdot \frac{1}{s}\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s(3s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{-3}{3s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+\frac{1}{3}}\right\} = (1 - e^{-\frac{1}{3}t})1(t)$$

$$\Rightarrow g(t) = \mathcal{L}^{-1}\{G(s)\mathcal{L}\{1(t)\}\} = (1 - e^{-\frac{1}{3}(t-2)})1(t-2) \#$$

$$\text{FVT: } \lim_{t \rightarrow \infty} g(t) = 1 = \lim_{s \rightarrow 0} s G(s) \mathcal{L}\{1(t)\} = \lim_{s \rightarrow 0} \frac{s e^{-2s}}{s(3s+1)} = 1.$$

(b) From the provided Figure, we can see that the period of the unit impulse response is approximately 7secs. $\Rightarrow P_u \approx 7s$. Therefore,

$\begin{cases} K_u = 3.044 \\ P_u \approx 7 \end{cases}$ By the "Ultimate Sensitivity Method", the parameters for the PID controller are

$$\begin{cases} K_p = 0.6 K_u = 0.6 \cdot 3.044 = 1.82 \\ T_I = 0.5 P_u = 0.5 \cdot 7 = 3.5 \\ T_D = 0.125 P_u = 0.125 \cdot 7 = 0.875 \end{cases}$$

Comparing the results (PID-controller parameters), we can see that the predictions of K_p are almost the same (1.8 vs 1.82), while that of T_I and T_D are much more different.

Method	Quarter Decay Ratio	Ultimate Sensitivity	error (based on QDR)	error (based on US)
K_p	1.8	1.82	1.1%	1.1%
T_I	4	3.5	12.5%	11.5%
T_D	1	0.875	12.5%	14.3%

It seems to be interesting to discuss the difference of the predictions by each method and study on which method is more accurate in which kind of conditions, but perhaps I will do this later, after my busy midterm month.

I'd like to discuss the impulse responses

produced by different values of K_u using Matlab, but Matlab tells me that the "impulse" command cannot be used for "continuous time models with internal delays", and thus cannot plot the impulse response I want. So, I tried manually:

$$H(s) = \mathcal{L}\{\delta(t)\} \cdot \frac{3.044 \cdot \left(\frac{e^{-2s}}{3s+1}\right)}{1 + 3.044 \left(\frac{e^{-2s}}{3s+1}\right)}$$
$$= \frac{3.044}{(3s+1)e^{2s} + 3.044}$$

I tried to calculate its inverse Laplace Transform but it's not possible even with online softwares.

參考觀摩的作業

3. (Feedforward Control)

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