Control System: Homework 05 for Unit 4C, 4D, 4E: Dynamic Response
Assigned: Oct 21, 2022 Due: Oct 27, 2022 (23:59)
Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

1. (U4C: Three terms controller)

4.34 Consider the satellite-attitude control problem shown in Fig. 4.45 where the normalized parameters are

$J = 10$ spacecraft inertia, N·m·sec^2/rad

θ_r = reference satellite attitude, rad.

θ = actual satellite attitude, rad.

$H_y = 1$ sensor scale, factor V/rad.

$H_r = 1$ reference sensor scale factor, V/rad.

w = disturbance torque, N·m.

(a) Use proportional control, P, with $D_c(s) = k_P$, and give the range of values for k_P for which the system will be stable.

(b) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to reference inputs.

(c) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to disturbance inputs.

(d) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to reference inputs.

(e) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to disturbance inputs.

(f) Use PID control, let $D_c(s) = (k_P + \frac{k_I}{s} + k_D s)$, and determine the system type and error constant with respect to reference inputs.

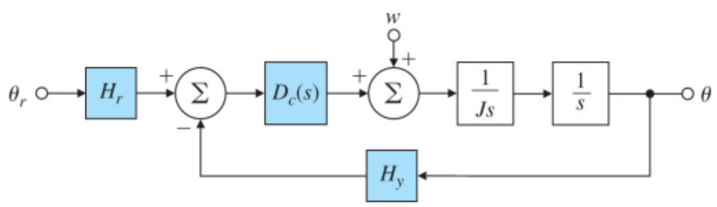


Figure 4.45: Satellite attitude control

Solution:

(a) $D_c(s) = k_P$; The characteristic equation is

$$1 + H_y D_c(s) \frac{1}{Js^2} = 0$$

$$Js^2 + H_y k_P = 0$$

or $s = \pm j\sqrt{\frac{H_y k_P}{J}}$ so that no additional damping is provided. The system cannot be made stable with proportional control alone.

(b) Steady-state error to reference steps.

$$\begin{aligned} \frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\frac{1}{Js^2}}{1 + D_c(s) H_y \frac{1}{Js^2}}, \\ &= H_r \frac{(k_P + k_D s)}{Js^2 + (k_P + k_D s) H_y}. \end{aligned}$$

The parameters can be selected to make the (closed-loop) system stable. If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming the system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback).

(c) Steady-state error to disturbance steps

$$\frac{\Theta(s)}{W(s)} = \frac{1}{Js^2 + (k_P + k_D s) H_y}.$$

If $W(s) = \frac{1}{s}$ then using the FVT (assuming system is stable), the error is $\theta_{ss} = -\frac{1}{k_P H_y}$.

(d) The characteristic equation is

$$1 + H_y D_c(s) \frac{1}{Js^2} = 0.$$

With PI control,

$$Js^3 + H_y k_P s + H_y k_I = 0.$$

From the Hurwitz's test, with the s^2 term missing the system will always have (at least) one pole not in the LHP. Hence, this is not a good control strategy.

(e) See (d) above.

(f) The characteristic equation with PID control is

$$1 + H_y \left(k_P + \frac{k_I}{s} + k_D s \right) \frac{1}{Js^2} = 0,$$

or

$$Js^3 + H_y k_D s^2 + H_y k_P s + H_y k_I = 0.$$

There is now control over all the three poles and the system can be made stable.

$$\begin{aligned}
\frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\frac{1}{Js^2}}{1 + D_c(s) H_y \frac{1}{Js^2}}, \\
&= \frac{H_r(k_P + \frac{k_I}{s} + k_D s)}{Js^2 + (k_P + \frac{k_I}{s} + k_D s) H_y}, \\
&= \frac{H_r(k_D s^2 + k_P s + k_I)}{Js^3 + (k_D s^2 + k_P s + k_I) H_y}.
\end{aligned}$$

If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback). In that case, the system is Type 3 and the (Jerk!) error constant is $K_J = \frac{k_I}{J}$.

2. (U4D: Three terms controller and Ziegler–Nichols Tuning)

4.37 A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1},$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

(a) Find the PID-controller parameters using the Ziegler–Nichols tuning rules.

(b) The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 4.48. Find the optimal PID-controller parameters according to the Ziegler–Nichols tuning rules.

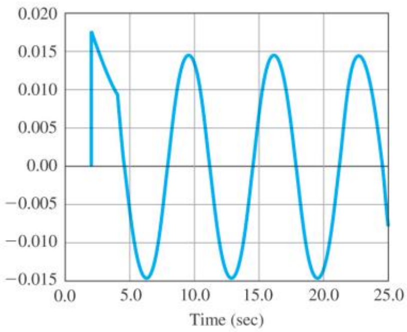


Figure 4.48: Unit impulse response for paper-machine in Problem 4.37

Solution:

(a) From the transfer function: $L = \tau_d \simeq 2$ sec

$$R = \frac{1}{3} \simeq 0.33 \quad \text{sec}^{-1}.$$

From Table 4.1:

Controller Gain P : $K = \frac{1}{RL}1.5,$

PI : $K = \frac{0.9}{RL} = 1.35 \quad T_I = \frac{L}{0.3} = 6.66,$

PID: $K = \frac{1.2}{RL} = 1.8 \quad T_I = 2L = 4 \; T_D = 0.5L = 1.0.$

(b) From the impulse response: $P_u \simeq 7$ sec From Table 4.2:

Controller Gain P : $K = 0.5K_u = 1.52,$

PI : $K = 0.45K_u = 1.37 \quad T_I = \frac{1}{1.2}P_u = 5.83,$

PID : $K = 0.6K_u = 1.82 \quad T_I = \frac{1}{2}P_u = 3.5T_D = \frac{1}{8}P_u = 0.875.$

3. (U4E: Feedforward Control)

4.38 Consider the DC motor speed-control system shown in Fig. 4.49 with proportional control. (a) Add feedforward control to eliminate the steady-state tracking error for a step reference input. (b) Also add feedforward control to eliminate the effect of a constant output disturbance signal, w, on the output of the system.

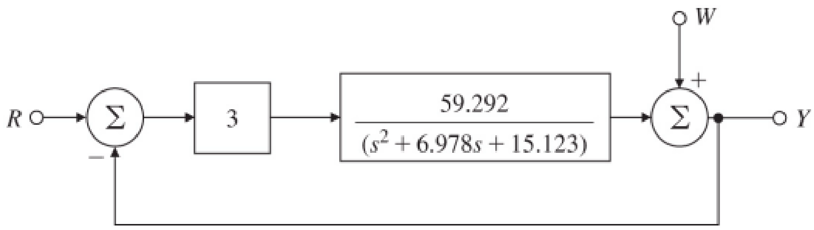
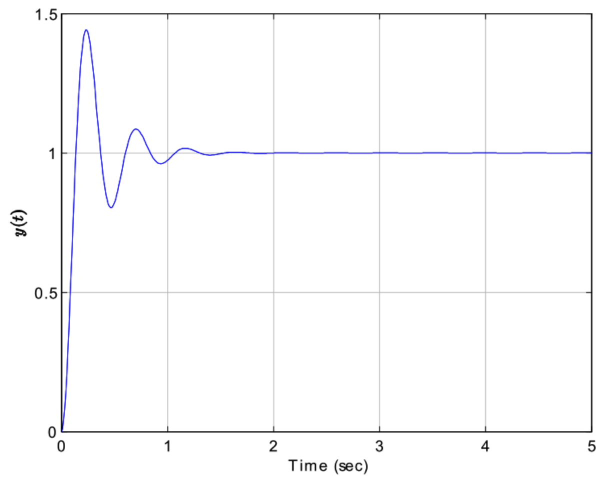


Figure 4.49: Block diagram for Problem 4.38

Solution: (a) In this case the plant inverse DC gain is $G^{-1}(0) = \frac{15.123}{59.292} = 0.2551$. We implement the closed-loop system as shown in Figure 4.22 (a) with $D_c(s) = k_p = 3$. The closed-loop transfer function is

$$
\begin{aligned}
Y(s) &= G(s)[k_p E(s) + G^{-1}(0)R(s)], \\
E(s) &= R(s) - Y(s), \\
\frac{Y(s)}{R(s)} &= \mathcal{T}(s) = \frac{(G^{-1}(0) + k_p)G(s)}{1 + k_p G(s)}.
\end{aligned}
$$

Note that the closed-loop DC gain is unity ($\mathcal{T}(0) = 1$). The following figure illustrates the effect of feedforward control in eliminating the steady-state tracking error. The addition of feedforward control results in zero steady-state tracking error for a step reference input.

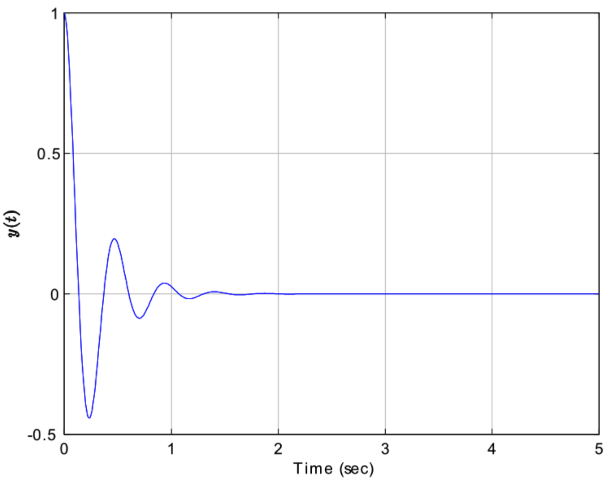


Tracking response with feedforward

(b) Similarly, we implement the closed loop system as shown Figure 4.22 (b). The closed-loop transfer function is

$$\begin{aligned} Y(s) &= W(s) + G(s)[k_p E(s) - G^{-1}(0)W(s)], \\ E(s) &= R(s) - Y(s) = 0 - Y(s), \\ \frac{Y(s)}{W(s)} &= \mathcal{T}_w(s) = \frac{1 - G^{-1}(0)G(s)}{1 + k_p G(s)}. \end{aligned}$$

Note that the closed-loop DC gain is zero ($\mathcal{T}_w(s) = 0$). The following figure illustrates the effect of feedforward control in eliminating the steady-state error for a step output disturbance.



Disturbance rejection response with feedforward

MATLAB code:

```
%FPE7e Problem 3.38
clf;
% Tracking
s=tf('s');
% plant
G=59.292/(s^2+6.978*s+15.123);
kp=3;
% Closed-loop Transfer function
dcgain1=dcgain(G);
T1=G*(1/dcgain1+kp)/(1+kp*G);
t=0:.01:5;
% Step response

y1=step(T1,t);
figure()
plot(t,y1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;
% Disturbance rejection
kp=3;
Tw1=(1-1/dcgain1*G)/(1+kp*G);
yw1=step(Tw1,t);
figure()
plot(t,yw1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;
```