Control System: Homework 04 for Unit 4A, 4B: Feedback Analysis

Assigned: Oct 14, 2022

Due: Oct 20, 2022 (23:59)

Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

1. (U4A: Sensitivity)

4. A unity feedback control system has the open-loop transfer function

$$G(s) = \frac{A}{s(s+a)}.$$

(a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter A.

(b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter a.

(c) If the unity gain in the feedback changes to a value of $\beta \neq 1$, compute the sensitivity of the closed-loop transfer function with respect to β.

Solution:

(a)

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\dfrac{A}{s(s+a)}}{1+\dfrac{A}{s(s+a)}} = \frac{A}{s^2+as+A},$$

$$\frac{dT}{dA} = \frac{(s^2+as+A)-A}{(s^2+as+A)^2},$$

$$S_A^T = \frac{A}{T}\frac{dT}{dA} = \frac{A(s^2+as+A)}{A}\frac{s^2+as}{(s^2+as+A)^2} = \frac{s(s+a)}{s(s+a)+A}.$$

(b)

$$\frac{dT}{da} = \frac{-sA}{(s^2+as+A)^2}.$$

$$\frac{a}{T}\frac{dT}{da} = \frac{a(s^2+as+A)}{A}\frac{-sA}{(s^2+as+A)^2}.$$

$$\mathcal{S}_a^T = \frac{-as}{s(s+a)+A}.$$

(c) In this case,

$$T(s) = \frac{G(s)}{1+\beta G(s)},$$

$$\frac{dT}{d\beta} = \frac{-G(s)^2}{(1+\beta G(s))^2},$$

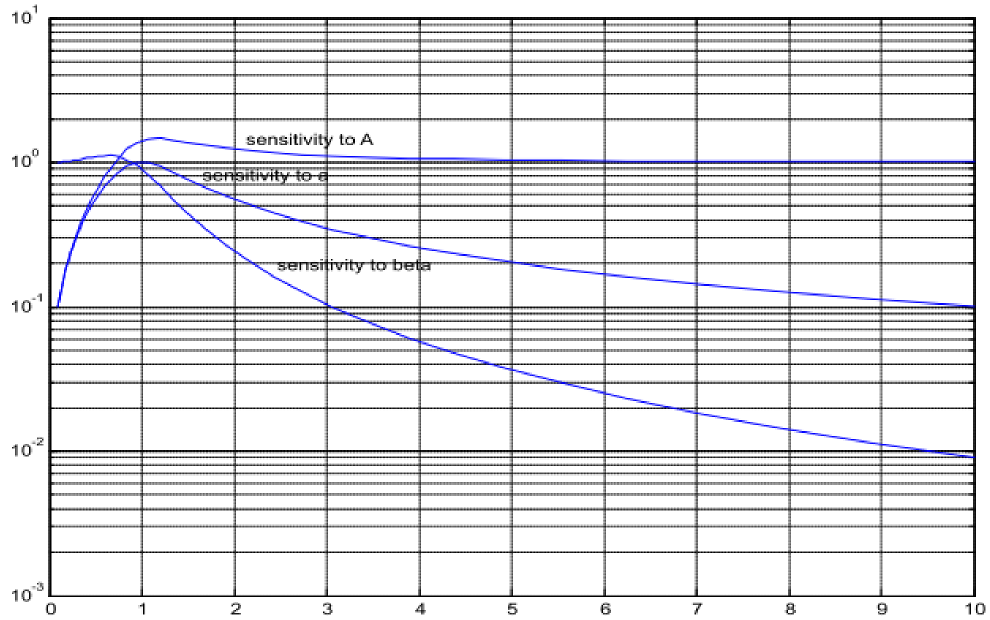
$$\frac{\beta}{T}\frac{dT}{d\beta} = \frac{\beta(1+\beta G)}{G}\frac{-G^2}{(1+\beta G)^2} = \frac{-\beta G}{1+\beta G},$$

$$\mathcal{S}_\beta^T = \frac{\dfrac{-\beta A}{s(s+A)}}{1+\dfrac{\beta A}{s(s+a)}} = \frac{-\beta A}{s(s+a)+\beta A}.$$

- If $a = A = 1$,the transfer function is most sensitive to variations in a and A near $\omega = 1$ rad/sec .
- The steady-state response is not affected by variations in A and a ($\mathcal{S}_A^T(0)$ and $\mathcal{S}_a^T(0)$ are both zero).
- The steady-state response *is* heavily dependent on β since $|\mathcal{S}_\beta^T(0)|$ $= 1.0$

See attached plots of sensitivities versus radian frequency for $a = A = 1.0$.

Sensitivity function frequency response follows.

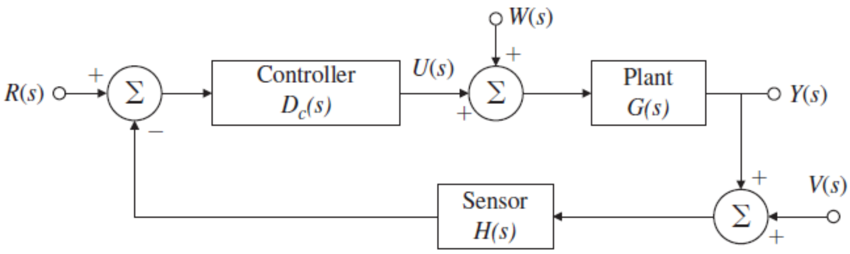

sensitivity to A
sensitivity to a
sensitivity to beta
10^1
10^0
10^-1
10^-2
10^-3
0
1
2
3
4
5
6
7
8
9
10

2. (U4B: Steady-State Error and System Type)

8. A standard feedback control block diagram is shown in Fig. 4.5 with

$$G(s) = \frac{1.5}{s};\ \ D_c(s) = \frac{(s+9)}{(s+3)}; H(s) = \frac{70}{(s+70)}; V(s) = 0.$$

(a) Let $W = 0$ and compute the transfer function from R to Y.
(b) Let $R = 0$ and compute the transfer function from W to Y.
(c) What is the tracking error if R a unit-step input and W = 0?
(d) What is the tracking error if R is a unit-ramp input and W = 0?
(e) What is the system type with respect to the reference inputs and the corresponding error coefficient?



Solution:

(a)

$$\frac{Y(s)}{R(s)} = T(s) = F(s)\frac{D_c(s)G(s)}{1 + D_c(s)G(s)H(s)} = \frac{1.5(s+9)(s+70)}{s^3 + 73s^2 + 315s + 945}$$

(b)

$$\frac{Y(s)}{W(s)} = T(s) = \frac{G(s)}{1 + D_c(s)G(s)H(s)} = \frac{1.5(s+3)(s+70)}{s^3 + 73s^2 + 315s + 945}$$

(c)

$$\frac{E(s)}{R(s)} = 1 - T(s) = S(s) = \frac{s^3 + 71.5s^2 + 196.5s}{s^3 + 73s^2 + 315s + 945}.$$

With $R(s) = \frac{1}{s}$,

$$e_{step}(\infty) = \lim_{s\to 0} sE(s) = 0.$$

(d)

With $R(s) = \frac{1}{s^2}$,

$$e_{ramp}(\infty) = \lim_{s\to 0} sE(s) = 0.2.$$

(e) System is Type 1 and K_v is

$$K_v = \frac{1}{|e_{ramp(\infty)}|} = 5\ sec^{-1}.$$

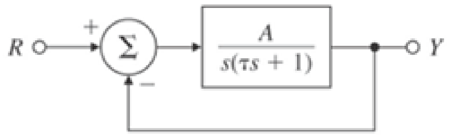
3. (U4B: System Type and Tracking)

14. Consider the system shown in Fig. 4.36(a).

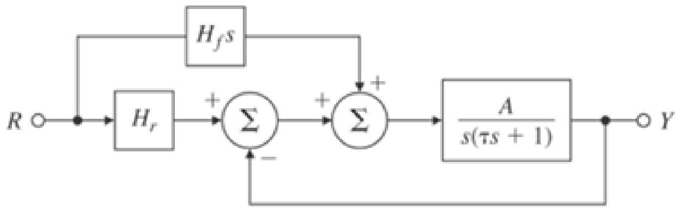
   (a) What is the system type? Compute the steady-state tracking error due to a ramp input $r(t) = r_o t1(t)$.

   (b) For the modified system with a feed forward path shown in Fig.4.36(b), give the value of H_f so the system is Type 2 for reference inputs and compute the K_a in this case.

   (c) Is the resulting Type 2 property of this system robust with respect to changes in H_f i.e., will the system remain Type 2 if H_f changes slightly?



(a)



(b)

Solution:

(a) System is Type 1 since it is unity feedback and has a pole at $s = 0$in the forward path. Also,

$$\begin{aligned} E(s) &= [1 - \mathcal{T}(s)]R(s), \\ &= \left[\frac{1}{1 + G(s)}\right] R(s), \\ &= \frac{s(\tau s + 1)}{s(\tau s + 1) + A} \frac{r_o}{s^2}. \end{aligned}$$

The steady-state tracking error using the FVT (assuming stability) is,

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{r_o}{A}.$$

(b)

$$Y(s) = \frac{A}{s(\tau s + 1)} U(s),$$

$$U(s) = H_f s R(s) + H_r R(s) - Y(s),$$

$$Y(s) = \frac{A(H_f s + H_r)}{s(\tau s + 1) + A} R(s).$$

The tracking error is,

$$E(s) = R(s) - Y(s),$$

$$= \frac{s(\tau s + 1) + A - A(H_f s + H_r)}{s(\tau s + 1) + K} R(s),$$

$$= \frac{\tau s^2 + (1 - AH_f)s + A(1 - H_r)}{s(\tau s + 1) + A}.$$

To get zero steady-state error with respect to a ramp, the numerator in the above equation must have a factor s^2. For this to happen, let

$$H_r = 1,$$
$$AH_f = 1.$$

Then

$$E(s) = \frac{\tau s^2}{s(\tau s + 1) + A} R(s)$$

and, with $R(s) = \frac{r_o}{s^2}$, apply the FVT (assuming stability) to obtain

$$e_{ss} = 0.$$

Thus the system will be Type 2 with ~~$K_a = \frac{\tau}{A}$.~~ $K_a = \frac{A}{\tau}$

(c) No, the system is not robust Type 2 because the property is lost if either H_r or H_f changes slightly.