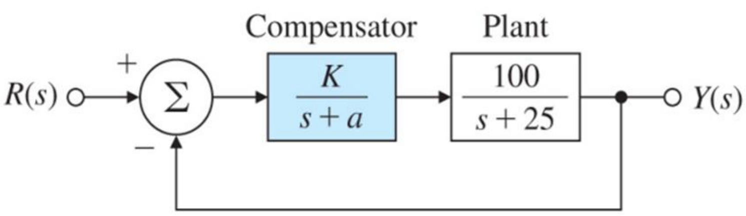
Midterm Exam, Control Systems, 111-1 (2022)	姓名：
Date: Friday, November 4, 2022. Time: 9:30-11:30am.	學號：
Closed books, closed notes, no calculators. Only pens and erasers are allowed.	系級：

(1) (20% = 5% * 4)

For the unity feedback system shown in the following figure.
Please specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 10 %, and a 1% settling time of no more than 0.5 sec. That is, please find the value of the following terms:
(a) Damping ratio; (b) Un-damped natural frequency; (c) *a*; (d) *K*.



Solution:

After calculation, the transfer function from R(s) to Y(s) can be described as follows:

$$\frac{Y(s)}{R(s)} = \frac{100K}{s^2 + (25 + a)s + 25a + 100K} = \frac{100K}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

And R(s) = 1/s (unit step).

(a)
Look at the plot of overshoop M_p vs damping ratio ζ (zeta).
For M_p = 10%, then ζ=0.6 (roughly).

(b)
t_s = 4.6/(w_n*zeta)
➩ 0.5 = 4.6 / (w_n*0.6)
➩ w_n = 15.33

(c)
(25 + a) = 2*zeta*w_n
➩ a = 2*0.6*15.33 – 25 = -6.6

(d)

(25a + 100K) = w_n^2

⇨ K = [15.33^2 – 25*(-6.6)] / (100) = 4

(2) (20% = 5%*4)

For a second-order system with transfer function:

$$G(s) = \frac{2s + 1}{s^2 + 3s + 2}$$

(a) Determine the poles and zeros of the system.
(b) Determine whether the system is stable and why?
(c) Find the DC gain of the system.
(d) Find the final value of the output of the system if the input is unit-step function.

Solution:

(a)

zeors: $2s + 1 = 0, \rightarrow s = -1/2$

poles: $s^2 + 3s + 2 = 0, \rightarrow s = -1, -2$

(b)

Poles: s = -1, -2, are on the left-hand side of the s-plane. Hence, the system is stable.

(c)

DC gain = G(0) = 1/2.

(d)

Use Final-Value Theorem.

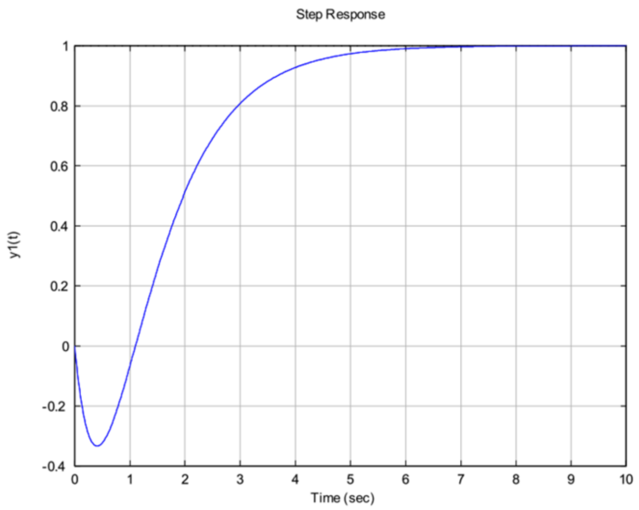
$$\lim_{s \to 0} s\, G(s)\, U(s) = \lim_{s \to 0} s \frac{2s + 1}{s^2 + 3s + 2} \frac{1}{s} = \frac{1}{2}$$

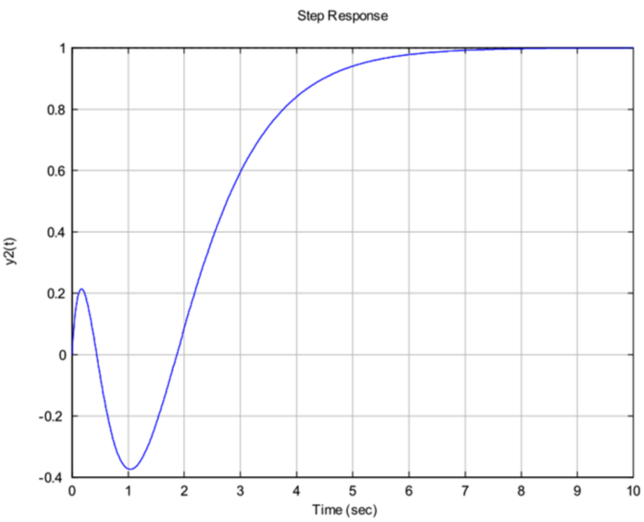
(3) (10% = 5%*2)

Consider the two non-minimum phase systems:

$$G_1(s) = -\frac{2(s-1)}{(s+1)(s+2)},$$
$$G_2(s) = \frac{3(s-1)(s-2)}{(s+1)(s+2)(s+3)}.$$

The unit step responses for G1(s) and G2(s) are shown as follows (Left-G1, Right-G2):





Please explain the difference in the behavior of the two responses as it relates to the zero locations.

Solution:

For G1(s)

$$Y_1(s) = \frac{1}{s}G_1(s) = \frac{-2(s-1)}{s(s+1)(s+2)},$$

$$Y_1(s) = \frac{-2(s-1)}{s(s+1)(s+2)} = \frac{R_0}{s} + \frac{R_{-1}}{(s+1)} + \frac{R_{-2}}{(s+2)} = \frac{1}{s} - \frac{4}{s+1} + \frac{3}{s+2},$$
$$y_1(t) = 1 - 4e^{-t} + 3e^{-2t}.$$

For G2(s), similarly,

$$Y_2(s) = \frac{3(s-1)(s-2)}{s(s+1)(s+2)(s+3)} = \frac{1}{s} + \frac{-9}{(s+1)} + \frac{18}{(s+2)} + \frac{-10}{(s+3)},$$
$$y_2(t) = 1 - 9e^{-t} + 18e^{-2t} - 10e^{-3t}.$$

The first system presents an "undershoot". The second system, on the other hand, starts off in the right direction.

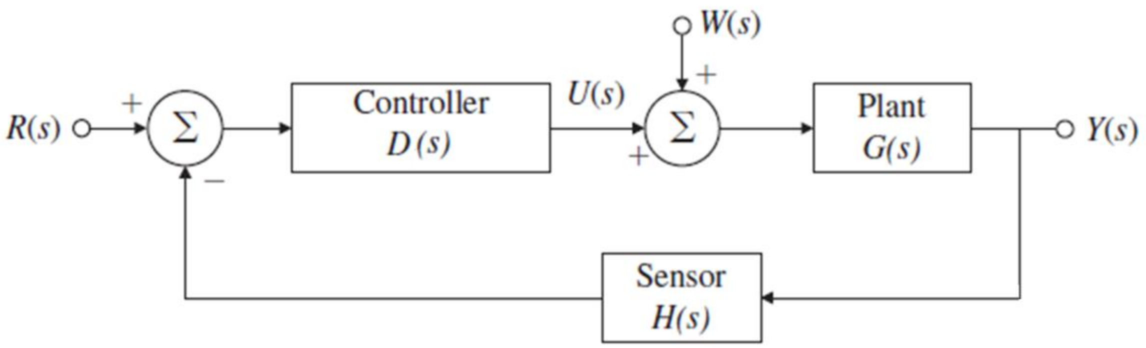
The reasons for this initial behavior of the step response will be analyzed in part c.

In $y_1(t)$: dominant at $t = 0$ the term $-4e^{-t}$

In $y_2(t)$: dominant at $t = 0$ the term $18e^{-2t}$

(4) (20% = 5%*4)

A standard feedback control block diagram is shown as follows:



where $$G(s) = \frac{2}{s}; \quad D(s) = \frac{(s+8)}{(s+2)}; \quad H(s) = 1.$$

(a) Let $W = 0$ and compute the transfer function from R to Y.
(b) What is the tracking error if R is a unit-step input and $W = 0$?
(c) What is the tracking error if R is a unit-ramp input and $W = 0$?
(d) What is the system type with respect to the reference input R and the corresponding error coefficient?

Solution:

(a)

$$G(s) = \frac{2}{s}; \quad D(s) = \frac{(s+8)}{(s+2)}; \quad H(s) = 1$$

$$T_{RY}(s) = \frac{D(s)\,G(s)}{1 + D(s)\,G(s)\,H(s)} = \frac{\frac{(s+8)}{(s+2)}\frac{2}{s}}{1 + \frac{(s+8)}{(s+2)}\frac{2}{s}}$$

$$= \frac{2s + 16}{s^2 + 4s + 16}$$

(b)

$$E(s) = R(s) - Y(s) = R(s) - T_{RY}(s)\,R(s)$$

$$= R(s)\,(1 - T_{RY}(s)\,)$$

$$= R(s)\,(1 - \frac{2s + 16}{s^2 + 4s + 16})$$

$$= R(s)\,(\frac{s^2 + 2s}{s^2 + 4s + 16})$$

$$e_{ss} = \lim_{s\to 0} s\,\frac{1}{s}\,(\frac{s^2 + 2s}{s^2 + 4s + 16}) = 0$$

(c)

$$e_{ss} = \lim_{s\to 0} s\,\frac{1}{s^2}\,(\frac{s^2 + 2s}{s^2 + 4s + 16}) = \frac{2}{16} = 0.125$$

(d)

From (b) and (c), the system is System Type 1.

$$k_v = \frac{1}{|e_{ramp}|} = \frac{1}{|0.125|} = 8$$

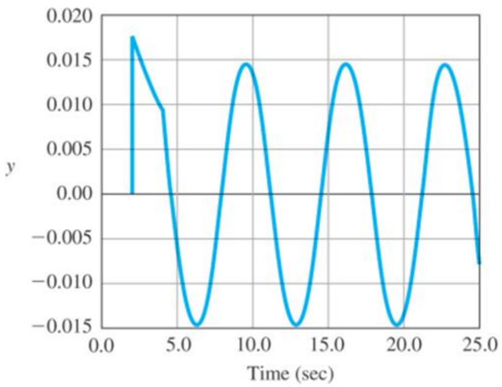
(5) (10% = 5%*2)

A dynamical system has the transfer function from the input signal to the output signal:

$$G(s) = \frac{e^{-3s}}{2s + 1}$$

(a) Find the P, PI, PID-controller parameters using the Ziegler-Nichols tuning rules.

(b) The system becomes marginally stable for a proportional gain of Ku = 1.2 as shown by the unit impulse response in the following plot. Find the optimal P, PI, PID-controller parameters according to the Ziegler-Nichols tuning rules.



Solution:

(a)

From the transfer function: L = tau_d = 3 sec.

R = 1/2 = 0.5 sec^(-1)

From the Table Left,

P:

Kp = 1/RL = 1/(0.5*3) = 1/1.5 = 0.67

PI:

Kp = 0.9/RL = 0.9/(0.5*3) = 6

T_I = L/0.3 = 3/0.3 = 10

PID:

Kp = 1.2/RL = 1.2/(0.5*3) = 0.8

T_I = 2L = 2*3 = 6

T_D = 0.5L = 0.5*3 = 1.5

(b)

From the impulse response: Pu = 7 sec and Ku = 1.2:

From the Table Right,

P:

Kp = 0.5 Ku = 0.5*1.2 = 6

PI:

Kp = 0.45 Ku = 0.45*1.2 = 0.54

T_I = Pu/1.2 = 7/1.2 = 5.83

PID:

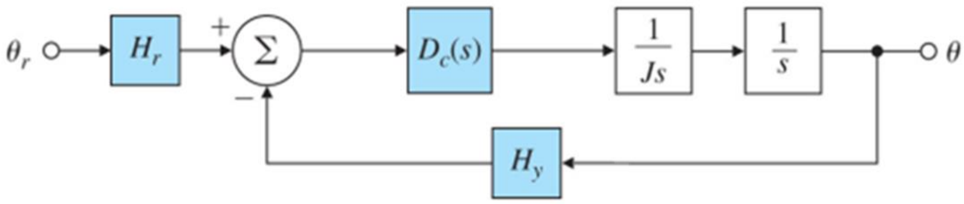
Kp = 0.6 Ku = 0.6*1.2 = 0.72

T_I = 0.5 Pu = 0.5*7 = 3.5

T_D = 0.125 Pu = 0.125*7 = 0.875

(6) (20% = 5%*4)

Consider a control system shown in the block diagram:



$J = 10$ spacecraft inertia, N-m-sec$^2/rad$
θ_r = reference satellite attitude, rad.
θ = actual satellite attitude, rad.
$H_y = 1$ sensor scale, factor volts/rad.
$H_r = 1$ reference sensor scale factor, volts/rad.

(a) Use proportional control, P, with Dc(s) = Kp, and give the range of values for Kp for which the system will be stable.
(b) Use PD control, let Dc(s) = (Kp + Kd s), and determine the system type and error constant of E with respect to reference inputs.
(c) Use PI control, let Dc(s) = (Kp + Ki/s), and determine the system type and error constant of E with respect to reference inputs.
(d) Use PID control, let Dc(s) = (Kp + Ki/s + Kd s), and determine the system type and error constant of E with respect to reference inputs.

Solution:

(a)

$D_c(s) = k_P$; The characteristic equation is

$$1 + H_y D_c(s)\frac{1}{Js^2} = 0$$

$$Js^2 + H_y k_P = 0$$

or $s = \pm j\sqrt{\frac{H_y k_P}{J}}$ so that no additional damping is provided. The system cannot be made stable with proportional control alone.

(b)

Steady-state error to reference steps.

$$\begin{aligned}\frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\dfrac{1}{Js^2}}{1 + D_c(s)H_y\dfrac{1}{Js^2}},\\ &= H_r \frac{(k_P + k_D s)}{Js^2 + (k_P + k_D s)H_y}.\end{aligned}$$

The parameters can be selected to make the (closed-loop) system stable. If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming the system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback).

(c)

The characteristic equation is

$$1 + H_y D_c(s)\frac{1}{Js^2} = 0.$$

With PI control,

$$Js^3 + H_y k_P s + H_y k_I = 0.$$

From the Hurwitz's test, with the s^2 term missing the system will always have (at least) one pole not in the LHP. Hence, this is not a good control strategy.

(d)

The characteristic equation with PID control is

$$1 + H_y\left(k_P + \frac{k_I}{s} + k_D s\right)\frac{1}{Js^2} = 0,$$

or

$$Js^3 + H_y k_D s^2 + H_y k_P s + H_y k_I = 0.$$

There is now control over all the three poles and the system can be made stable.

$$\begin{aligned}
\frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\dfrac{1}{Js^2}}{1 + D_c(s)H_y\dfrac{1}{Js^2}}, \\
&= \frac{H_r(k_P + \dfrac{k_I}{s} + k_D s)}{Js^2 + (k_P + \dfrac{k_I}{s} + k_D s)H_y}, \\
&= \frac{H_r(k_D s^2 + k_P s + k_I)}{Js^3 + (k_D s^2 + k_P s + k_I)H_y}.
\end{aligned}$$

If $\Theta_r(s) = \dfrac{1}{s}$ then using the FVT (assuming system is stable)

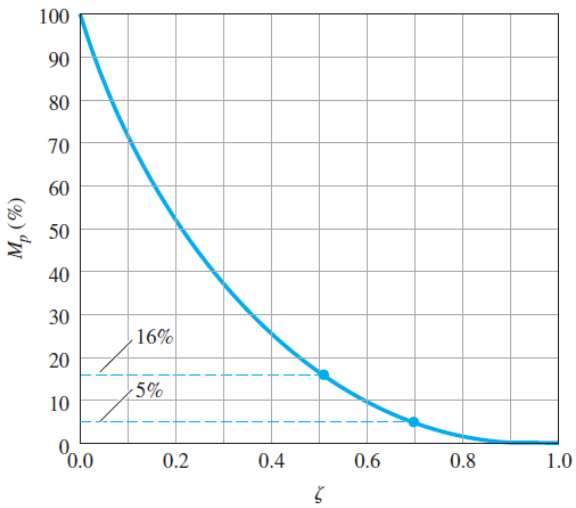
$$\theta_{ss} = \frac{H_r}{H_y},$$

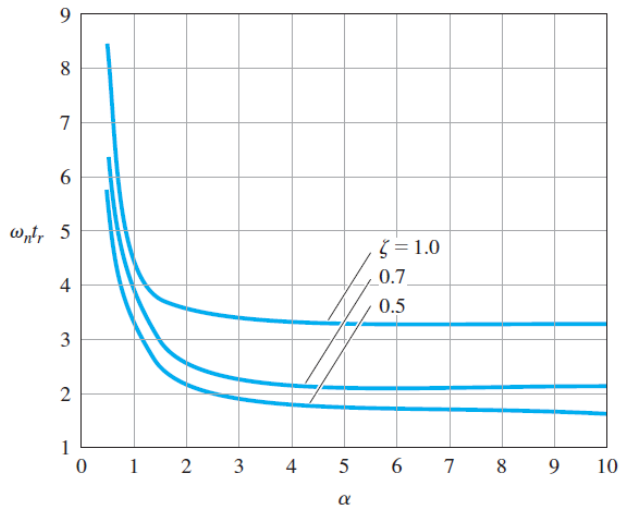
and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback). In that case, the system is Type 3 and the (Jerk!) error constant is $K_J = \dfrac{k_I}{J}$.

[Helpful Information]

$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem

$$t_r \cong \frac{1.8}{w_n},\quad t_p = \frac{\pi}{w_d},\quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}},\quad t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$





Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

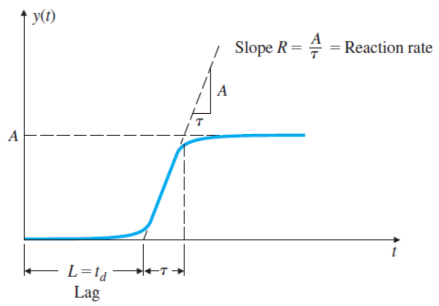
Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$k_P = 0.9/RL$; $T_I = L/0.3$
PID	$k_P = 1.2/RL$; $T_I = 2L$; $T_D = 0.5L$

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$k_P = 0.45K_u$; $T_I = \frac{P_u}{1.2}$
PID	$k_P = 0.6K_u$; $T_I = 0.5P_u$; $T_D = 0.125P_u$

Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$



$$\frac{Y(s)}{U(s)} = \frac{A\, e^{-s\, t_d}}{\tau\, s + 1}$$