Midterm Exam, Control System, 109-2 (2021)

Date: Friday, April 30, 2021. Time: 2pm-4pm.

Closed books, closed notes, no calculators. Only pens and erasers are allowed.

Part A. [20%] Find the best choice. (評分標準：只需寫出正確選項)

Consider the following figures and answer the following questions.

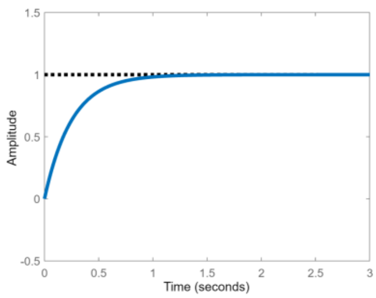


Figure 1

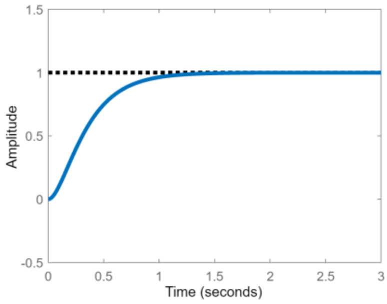


Figure 2

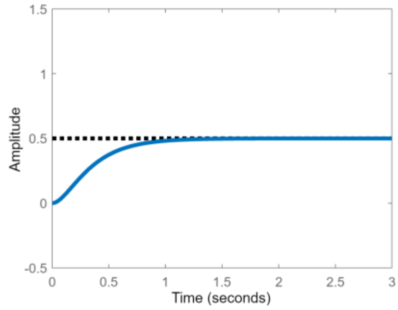


Figure 3

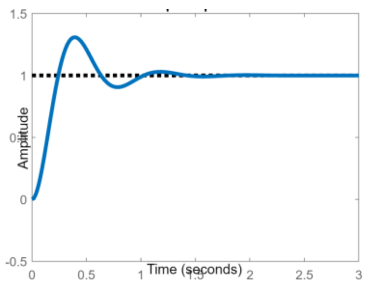


Figure 4

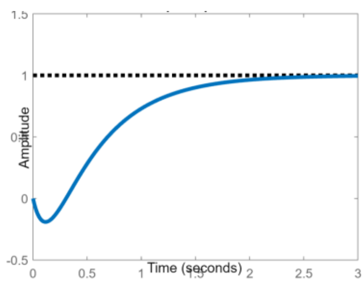


Figure 5

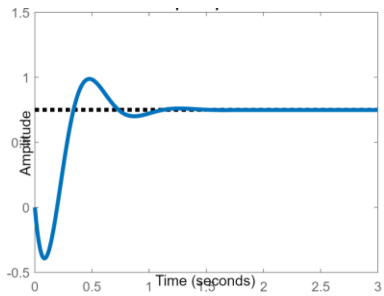


Figure 6

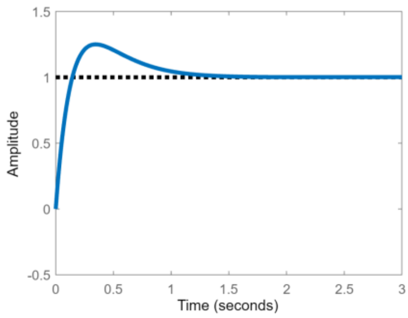


Figure 7

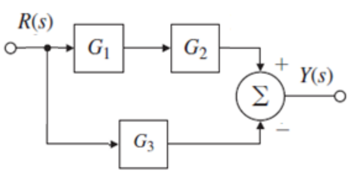


Figure 8

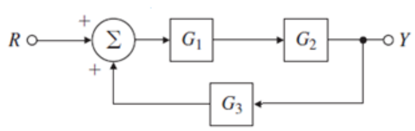


Figure 9

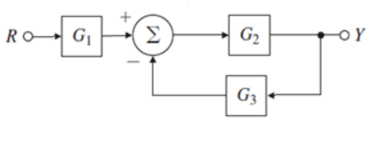


Figure 10

(A1) For the step response shown in **Figure 1**, please find the best possible transfer function:

(A) $\frac{1}{(s+1)}$; (B) $\frac{4}{(s+4)}$; (C) $\frac{16}{(s+16)}$; (D) $\frac{64}{(s+64)}$;

(A2) For the step response shown in **Figure 2**, please find the best possible transfer function:

(A) $\frac{4}{(s+4)}$; (B) $\frac{8}{(s+8)}$; (C) $\frac{16}{(s^2+12s+32)}$; (D) $\frac{32}{(s^2+33s+32)}$;

(A3) For the step response shown in **Figure 3**, please find the best possible transfer function:

(A) $\frac{16}{(s^2+12s+32)}$; (B) $\frac{32}{(s^2+12s+32)}$; (C) $\frac{16}{(s^2+33s+32)}$; (D) $\frac{32}{(s^2+33s+32)}$;

(A4) For the step response shown in **Figure 4**, please find the best possible transfer function:

(A) $\frac{73}{(s^2+6s+73)}$; (B) $\frac{72}{(s^2+17s+72)}$; (C) $\frac{400}{(s^2+6s+409)}$; (D) $\frac{400}{(s^2+40s+375)}$;

(A5) For the step response shown in **Figure 5**, please find the best possible transfer function:

(A) $\frac{(4s+16)}{(s^2+10s+16)}$; (B) $\frac{(4s-16)}{(s^2+10s+16)}$; (C) $\frac{(-4s-16)}{(s^2+10s+16)}$; (D) $\frac{(16-4s)}{(s^2+10s+16)}$;

(A6) For the step response shown in **Figure 6**, please find the best possible transfer function:

(A) $\frac{(60-10s)}{(s^2+8s+80)}$; (B) $\frac{(20-10s)}{(s^2+8s+80)}$; (C) $\frac{(20-10s)}{(s^2+12s+32)}$; (D) $\frac{(60-10s)}{(s^2+12s+32)}$;

(A7) For the step response shown in **Figure 7**, please find the best possible transfer function:

(A) $\frac{(12s+24)}{(s^2+10s+24)}$; (B) $\frac{(12s-24)}{(s^2+10s+24)}$; (C) $\frac{(4s+17)}{(s^2+2s+17)}$; (D) $\frac{(4s-17)}{(s^2+2s+17)}$;

(A8) For the block diagram shown in **Figure 8**, please find the transfer function from *R* to *Y*:

(A) $G_1 + G_2 + G_3$; (B) $G_1G_2 - G_3$; (C) $G_1 + G_2G_3$; (D) $G_1G_2 + G_3$;

(A9) For the block diagram shown in **Figure 9**, please find the transfer function from *R* to *Y*:

(A) $G_1G_2 + G_3$; (B) $\frac{G_1G_2}{G_3}$; (C) $\frac{G_1G_2}{1+G_1G_2G_3}$; (D) $\frac{G_1G_2}{1-G_1G_2G_3}$;

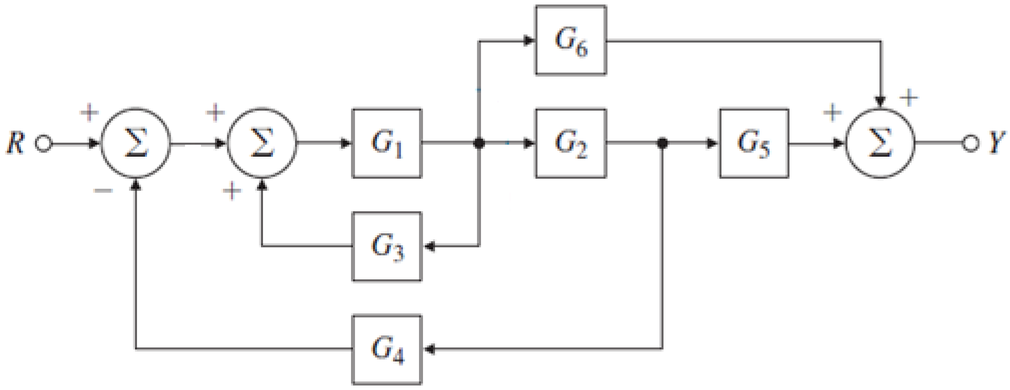
(A10) For the block diagram shown in **Figure 10**, please find the transfer function from *R* to *Y*:

(A) $\frac{G_1G_2}{1+G_1G_2G_3}$; (B) $\frac{G_1G_2}{1+G_2G_3}$; (C) $\frac{G_1G_2G_3}{1+G_1G_2G_3}$; (D) $\frac{G_1}{1+G_2G_3}$;

Part B. [80%] Write down proper description for the following problems.

(B1) (8%)

Find the transfer function (from R to Y) of the following block diagram, in terms of $G_i, i = 1, 2, \ldots, 6$:



(B2) (12%=3%*4)

For a second-order system with transfer function:

$$G(s) = \frac{2s + 1}{s^2 + 3s + 2}$$

(a) Determine the poles and zeros of the system.
(b) Determine whether the system is stable and why?
(c) Find the DC gain of the system.
(d) Find the final value of the output of the system if the input is unit-step function.

(B3) (10%=5%+5%)

A unit negative feedback system has the following open-loop transfer function:

$$G(s) = \frac{b}{(s + a)}$$

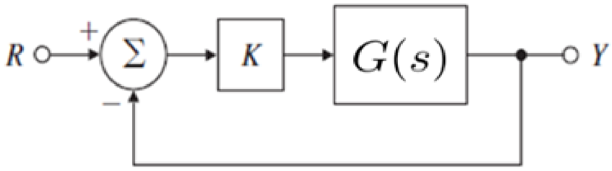
(a) Compute the sensitivity of the closed-loop transfer function to changes in parameter a.
(b) Compute the sensitivity of the closed-loop transfer function to changes in parameter b.

(B4) (10%=3%+3%+4%)

For a second-order system with transfer function:

$$G(s) = \frac{1}{(s - 1)(s + 2)}$$

(a) Find the poles and zeros of system *G(s)*.
(b) Determine whether the system is stable and why?
(c) If system *G(s)* is added into the following block diagram,



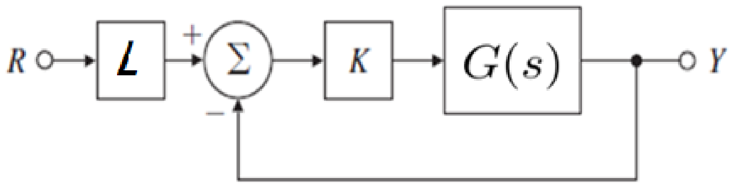
determine the value of *K*, such that the poles of the overall closed-loop system are placed at s = -0.5+j and s = -0.5-j.

(B5) (10%=5%*2)

For a second-order system with transfer function:

$$G(s) = \frac{4}{s^2 + s + 4}$$

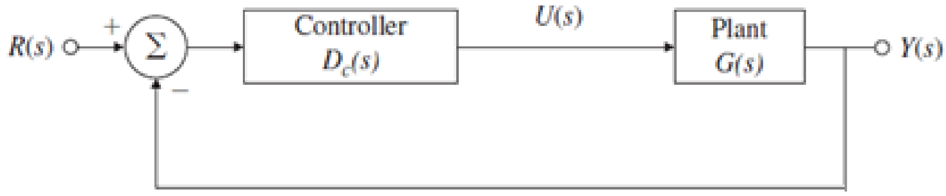
(a) Determine the overshoot and rise time of system *G(s)* if unit-step input is applied.
(b) If system *G(s)* is added into the following block diagram,



determine the values of *K* and *L*, such that the overshoot the overall system from *R* to *Y* is about 16% if unit-step input is applied to *R*.

(B6) (20%=5%*4)

A standard feedback control block diagram is shown as follows:



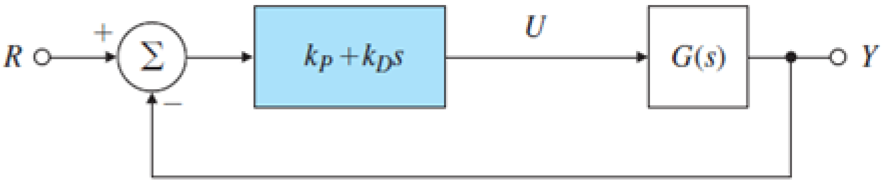
where $G(s) = \frac{2}{s}$ $D_c(s) = \frac{(s+9)}{(s+3)}$.

(a) Compute the transfer function from *R* to *Y*.
(b) What is the tracking error if *R* is a unit-step input?
(c) What is the tracking error if *R* is a unit-ramp input?
(d) What is the system type with respect to the reference inputs and the corresponding error coefficients?

(B7) (10%=5%*2)

Consider the transfer function: $G(s) = \frac{A}{s^2}$

and in the following control block diagram system:

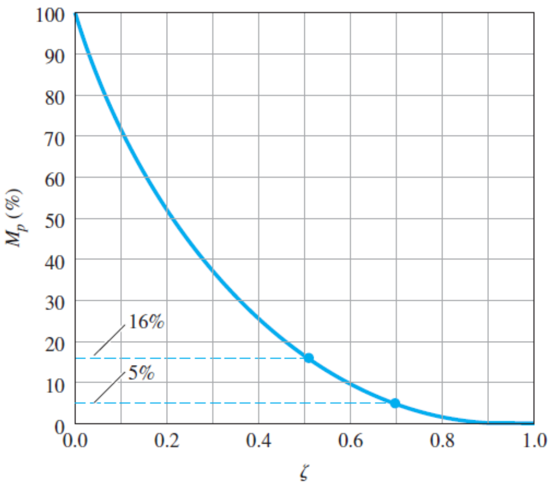


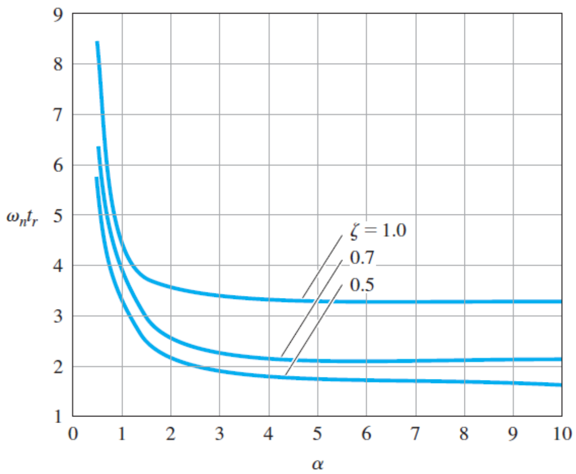
determine the steady-state values of the plant output *y(t)* and the plant input *u(t)* if the reference input *r(t)* is the unit-step function.

[Helpful formula]

$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem

$$t_r \cong \frac{1.8}{w_n},\ t_p = \frac{\pi}{w_d},\ M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}},\ t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$





Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$k_P = 0.9/RL$, $T_I = L/0.3$
PID	$k_P = 1.2/RL$, $T_I = 2L$, $T_D = 0.5L$

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$k_P = 0.45K_u$, $T_I = \frac{P_u}{1.2}$
PID	$k_P = 1.6K_u$, $T_I = 0.5P_u$, $T_D = 0.125P_u$