Midterm Exam, Control Systems, 108-2 (2020)
Date: Friday, April 24, 2020. Time: 2pm-4pm.
Closed books, closed notes, no calculators. Only pens and erasers are allowed.

[Helpful formula]

$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem
- $t_r \cong \frac{1.8}{w_n},\ t_p = \frac{\pi}{w_d},\ M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}},\ t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$
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Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

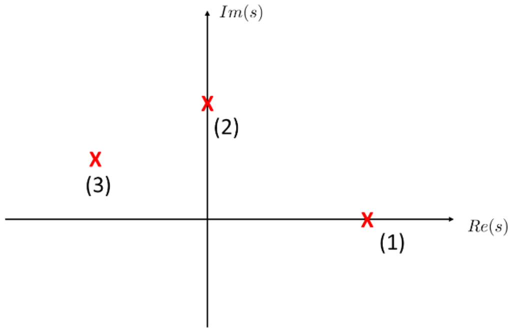
Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$k_P = 0.9/RL$, $T_I = L/0.3$
PID	$k_P = 1.2/RL$, $T_I = 2L$, $T_D = 0.5L$

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

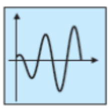
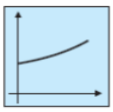
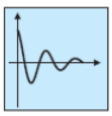
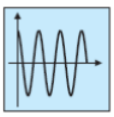
Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$k_P = 0.45K_u$, $T_I = \frac{P_u}{1.2}$
PID	$k_P = 1.6K_u$, $T_I = 0.5P_u$, $T_D = 0.125P_u$

Part A. [30%] Find the best choice.

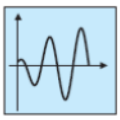
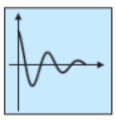
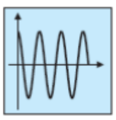
1. (Pole location, U33) For the pole locations shown in the figure below, please identify the corresponding responses.



The response corresponds to position (1) is

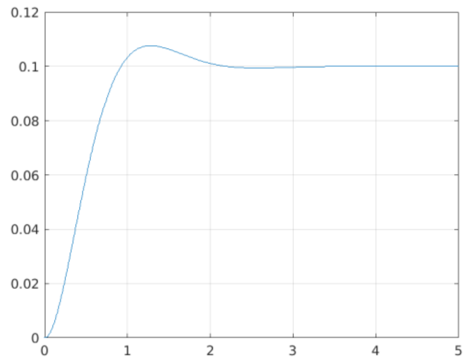
(A)  (B)  (C)  (D) 

2. (Pole location, U33) Continued, the response corresponds to position (3) is

(A)  (B)  (C)  (D) 

3. (Time spec, U34) Consider a second order system $H(s) = \dfrac{1}{s^2 + bs + k}$ whose unit-step response is shown as the following figure. What is the value of parameter k?

   (A) 1 (B) 2 (C) 10 (D) 20

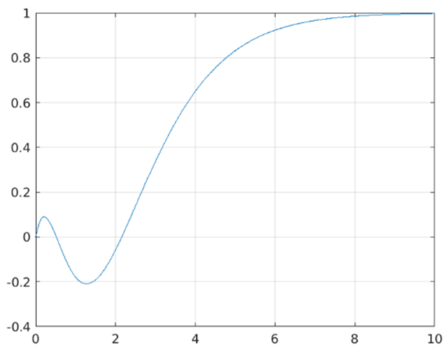


4. (Time spec, U34) Continued with question 3, what is the possible value of b?

   (A) -1.72 (B) 4.08 (C) 7.26 (D) 10.82

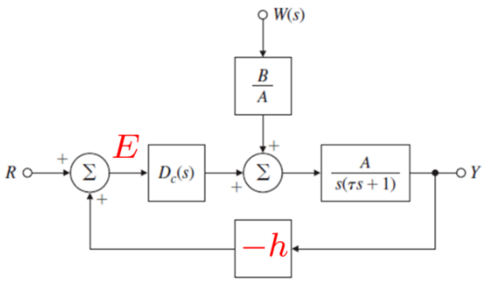
5. (Zero effect, U35) Consider the unit-step-response of a second system with the dynamical response in the following figure. The possible transfer function is

   (A) $\dfrac{2}{(s+2)(s+1)}$ (B) $\dfrac{s+2}{(s-2)(s+1)}$ (C) $\dfrac{s-2}{(s+2)(s+1)}$ (D) $\dfrac{(s-2)(s-1)}{(s+2)(s+1)^2}$



6. (System type and PID, U43) Consider a system with the following blocks, where the function $D_c(s) = k_P + \dfrac{k_I}{s}$ and reference R=0.
   Suppose the disturbance input $W(s)$ is a *unit step*, what is the steady state error?

   (A) 0 (B) $\dfrac{B}{Ak_I h}$ (C) $-\dfrac{B}{Ak_P h}$ (D) $-\dfrac{B}{Ak_I h}$



Part B. [70%] Answer the problems. In order to obtain the score, you need to provide all the detailed derivations, explanations, or descriptions.

1. (10%, DC gain and Final value, U31)

   For the transfer function of a system

$$G(s) = \frac{2}{s^2 + 3s + 2}$$

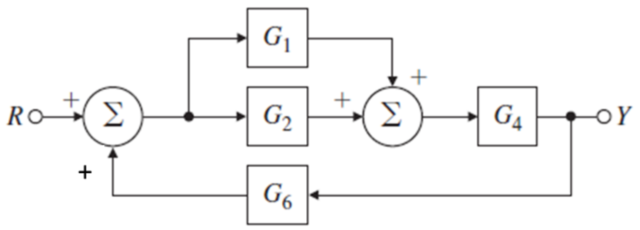
   (a) Find the DC gain.
   (b) Find the final value of the system with respect to unit-step input.

2. (10%, Block diagram, U32)

   Consider a system with the following block diagram, where

$$G_1(s) = 2, \quad G_2(s) = \frac{2}{s}, \quad G_4(s) = \frac{3}{s}, \quad G_6(s) = 1.$$

   Please write the closed-loop transfer function from R to Y.



3. (10%, Stability, U36)

   The transfer function of a system is given by

$$KG(s) = \frac{K(s+2)}{s[(s+4)(s+1)]}$$

   where time is measured in milliseconds. Using Routh's stability criterion to determine the range of K for which this system is stable when the characteristic equation is $1 + KG(s) = 0$.

4. (10%, Sensitivity, U41)

A unit feedback system has the open-loop transfer function

$$G(s) = \frac{K}{s+a}$$

   (a) Compute the sensitivity of the closed-loop transfer function to changes in the parameter a.

   (b) Compute the sensitivity of the closed-loop transfer function to changes in the parameter K.

5. (10%, Tracking, U42)

Consider a system as shown in the figure below, where

$$G(s) = \frac{1}{s(\tau s + 1)}, \quad D_c(s) = k_p, \quad H(s) = 1 + k_t s$$

Assume $W(s) = 0, \ V(s) = 0$.

(a) Please find the transfer function $E(s)$ from R to E in terms of $s, \ \tau, \ k_p, \ k_s$.

(b) Suppose $R(s) = \frac{1}{s^2}$, find the value of $e_{ss} = \lim_{t\to\infty} e(t)$, where $e(t) = y(t) - r(t)$. What is the type of the system?

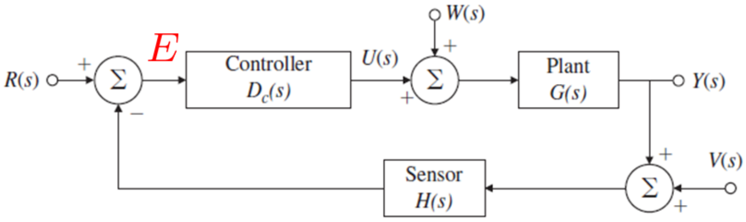


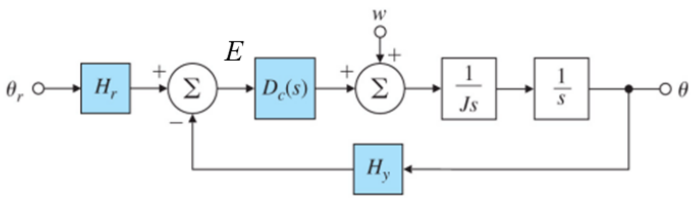
Figure of Problem 5.

6. (10%, PID control, U43)

Consider a control system shown in the figure below.

(a) Use PD control, let $D_c(s) = (k_p + K_D s)$ and determine the system type and error constant of E with respect to reference inputs.

(b) Use PID control, let $D_c(s) = (k_p + \frac{K_I}{s} + K_D s)$ and determine the system type and error constant of E with respect to distrubance inputs. $(\theta_r = 0)$



$$J = 10 \quad \text{spacecraft inertia, N-m-sec}^2/rad$$
$$\theta_r = \text{reference satellite attitude, rad.}$$
$$\theta = \text{actual satellite attitude, rad.}$$
$$H_y = 1 \quad \text{sensor scale, factor volts/rad.}$$
$$H_r = 1 \quad \text{reference sensor scale factor, volts/rad.}$$
$$w = \text{disturbance torque. N-m}$$

7. (10%, PID with Ziegler–Nichols Tuning, U44)

A system has the transfer function $G(s) = \frac{2e^{-s}}{2s+1}$. Find the PID-controller parameters using the Zieler-Nichols tuning rules.