Fall 2022 (111-1)

控制系統
Control Systems

Unit 6K
PID Lead-Lag Compensation

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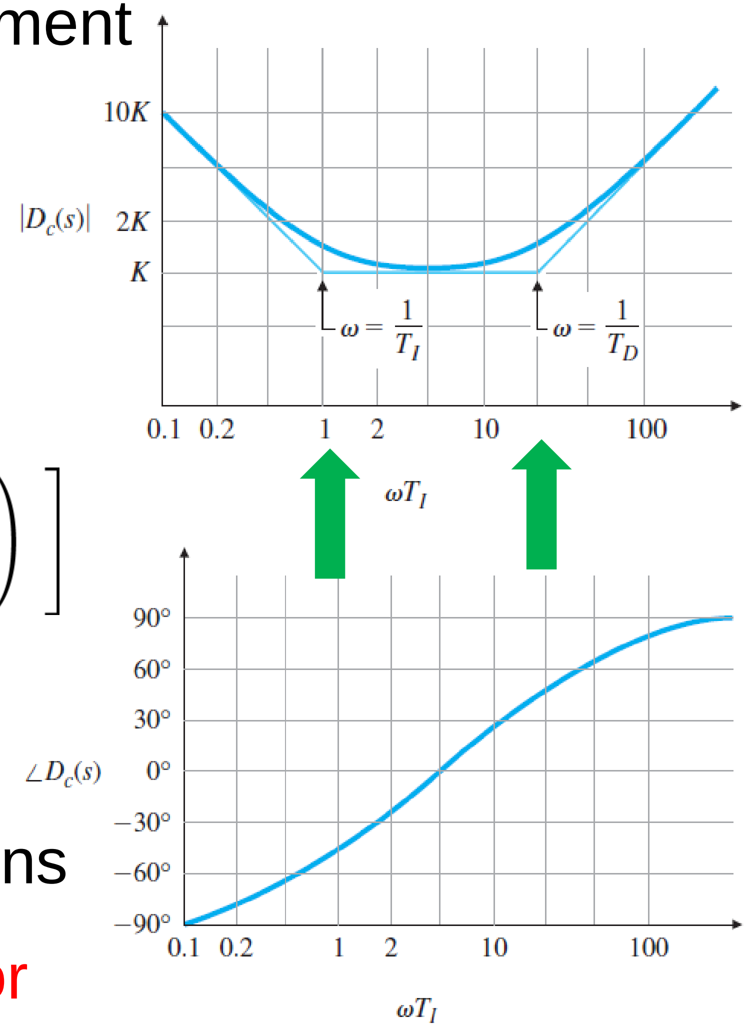
Sep 2022 – Dec 2022

- PID Compensation
  - Need PM improvement at ω_c
  - Need low-frequency gain improvement
- A common transfer function:

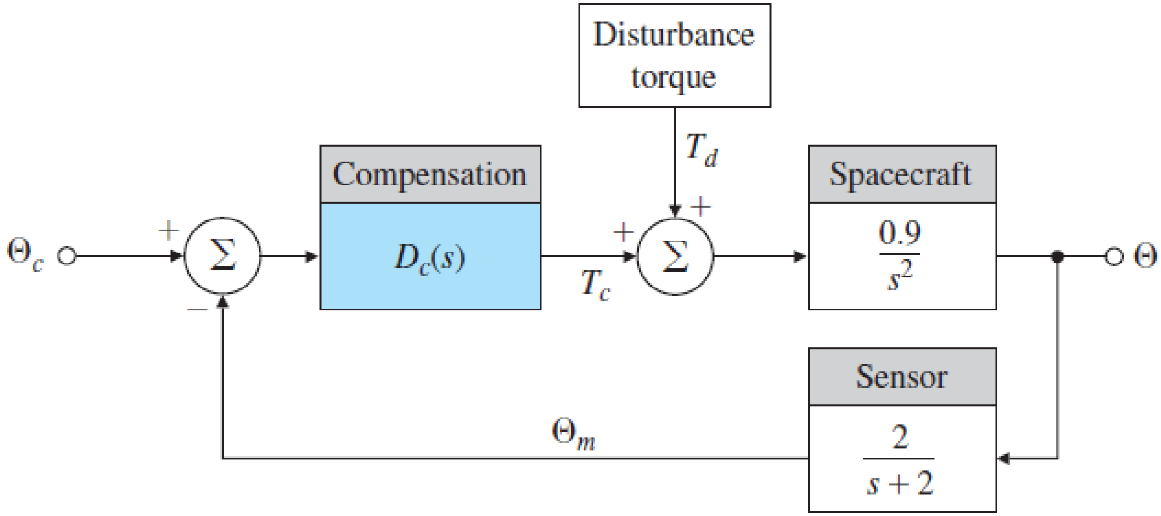
$$D_c(s) = k_P + \frac{k_I}{s} + k_D\, s$$

$$D_c(s) = \frac{K}{s}\left[(T_D\, s + 1) \left(s + \frac{1}{T_I} \right) \right]$$

- Roughly equivalent to combining lead and lag compensations referred to as a lead-lag compensator



- Example 6.20: PID Compensation for Spacecraft Attitude Control



- Design PID controller for:
  - Zero steady-state error
  - PM = 65^o
  - As high a bandwidth as possible
  - Torque disturbance ω = 0.001 rad/sec

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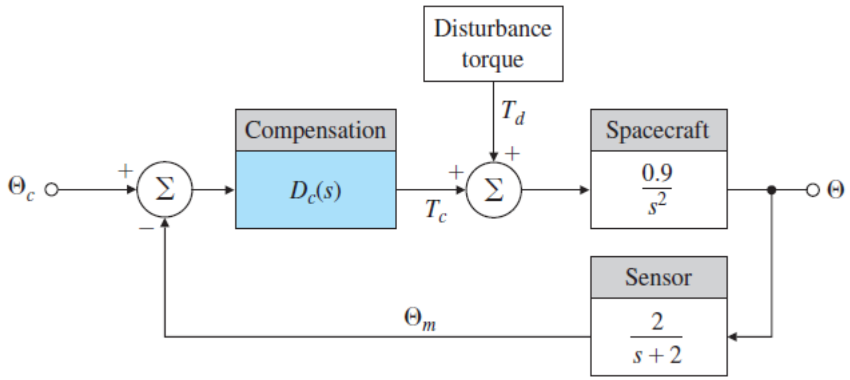


- For Final Steady Value

  → $T_d + T_c = 0$

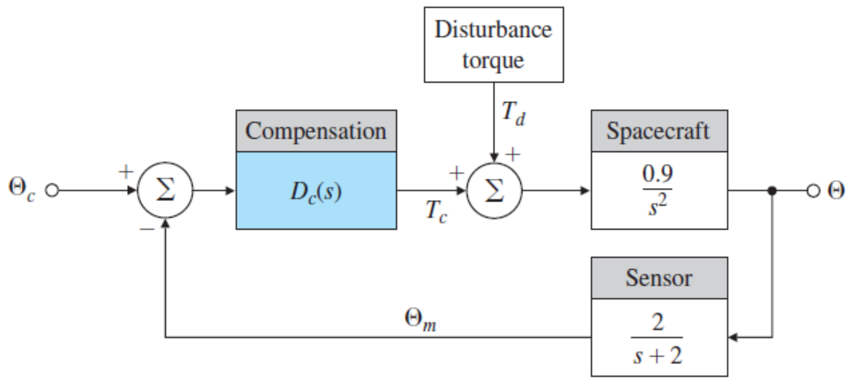
- Therefore, if $T_d =/= 0$

  → $T_c = -\,T_d$

- The only way this can be true with no error ($e = 0$) is:

  for $D_c(s)$ to contain an integral term.

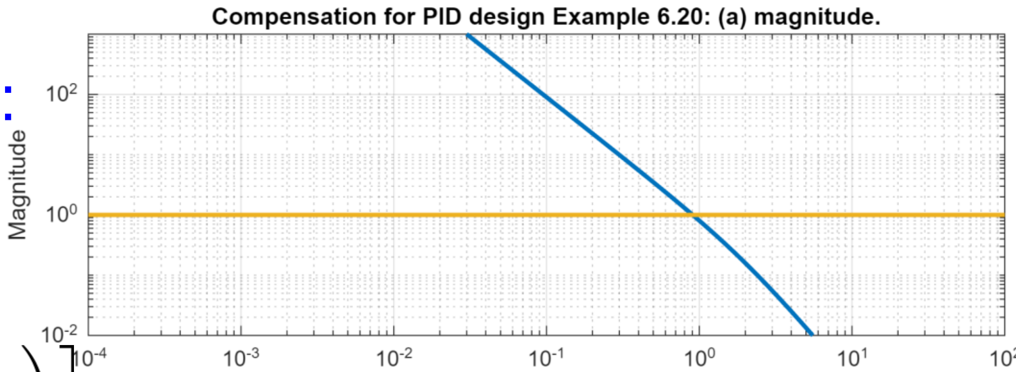
- Example 6.20: PID Compensation for Spacecraft Attitude Control



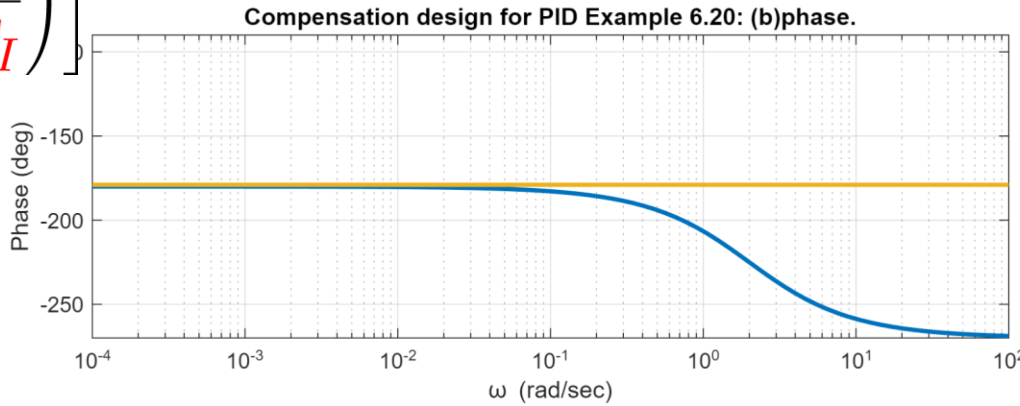
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

- Frequency Response of GH:
- Unstable



$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$



- Select: $K,\ T_D,\ T_I$
- PM = 65^o at high frequency
  - By adjusting T_D

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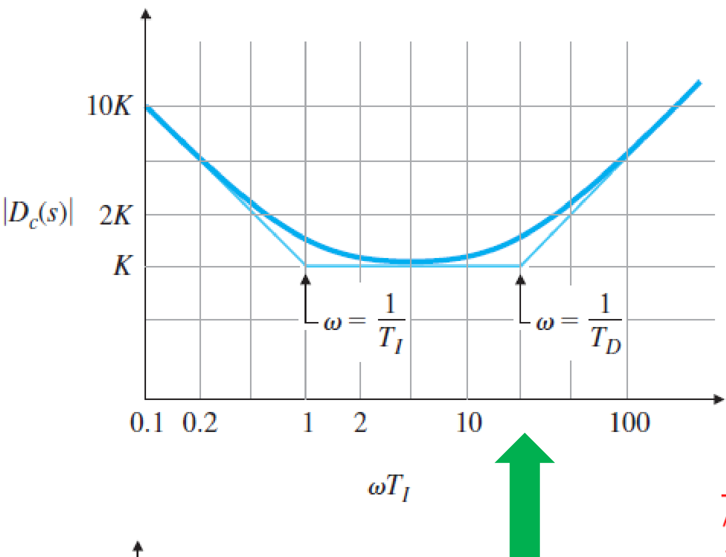
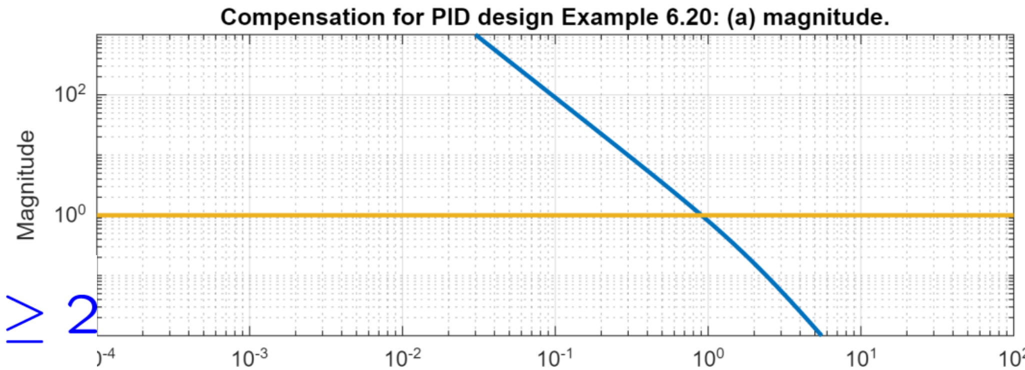
$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

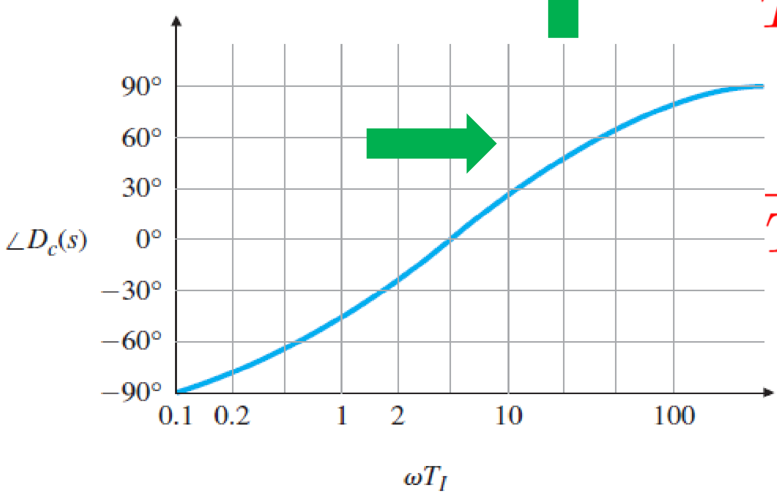
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

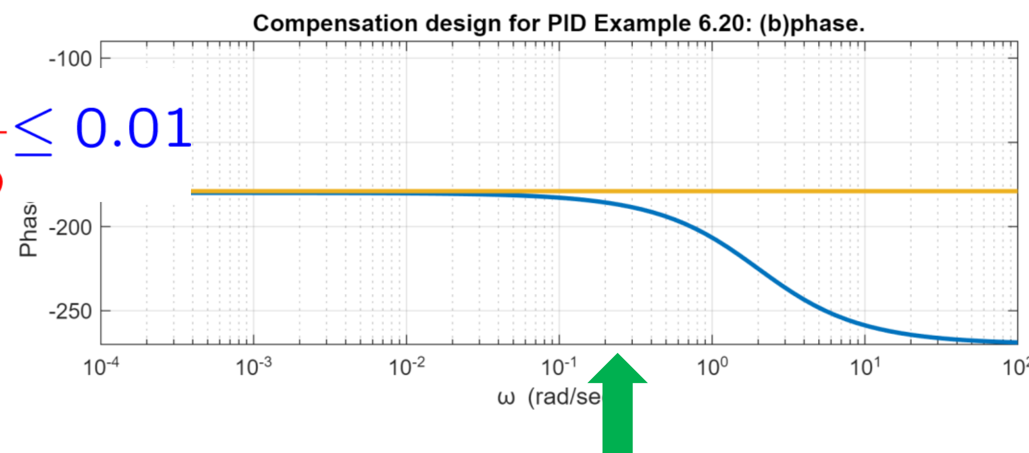




$$\frac{1}{T_D} \geq 2$$



$$\frac{1}{T_D} \leq 0.01$$



- Example 6.20: PID Compensation for Spacecraft Attitude Control

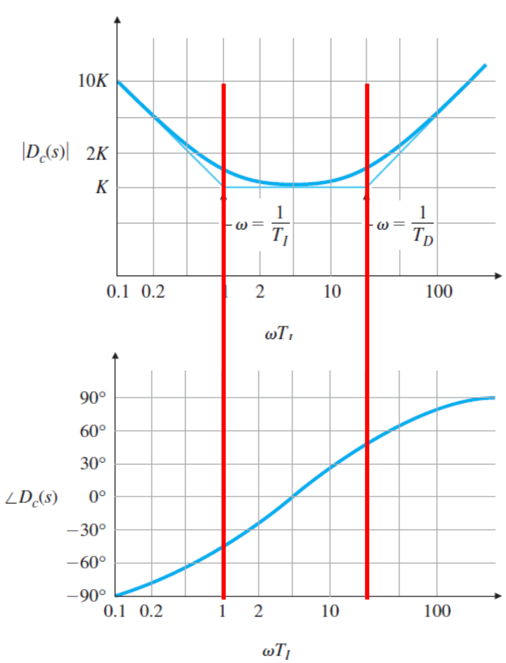
$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

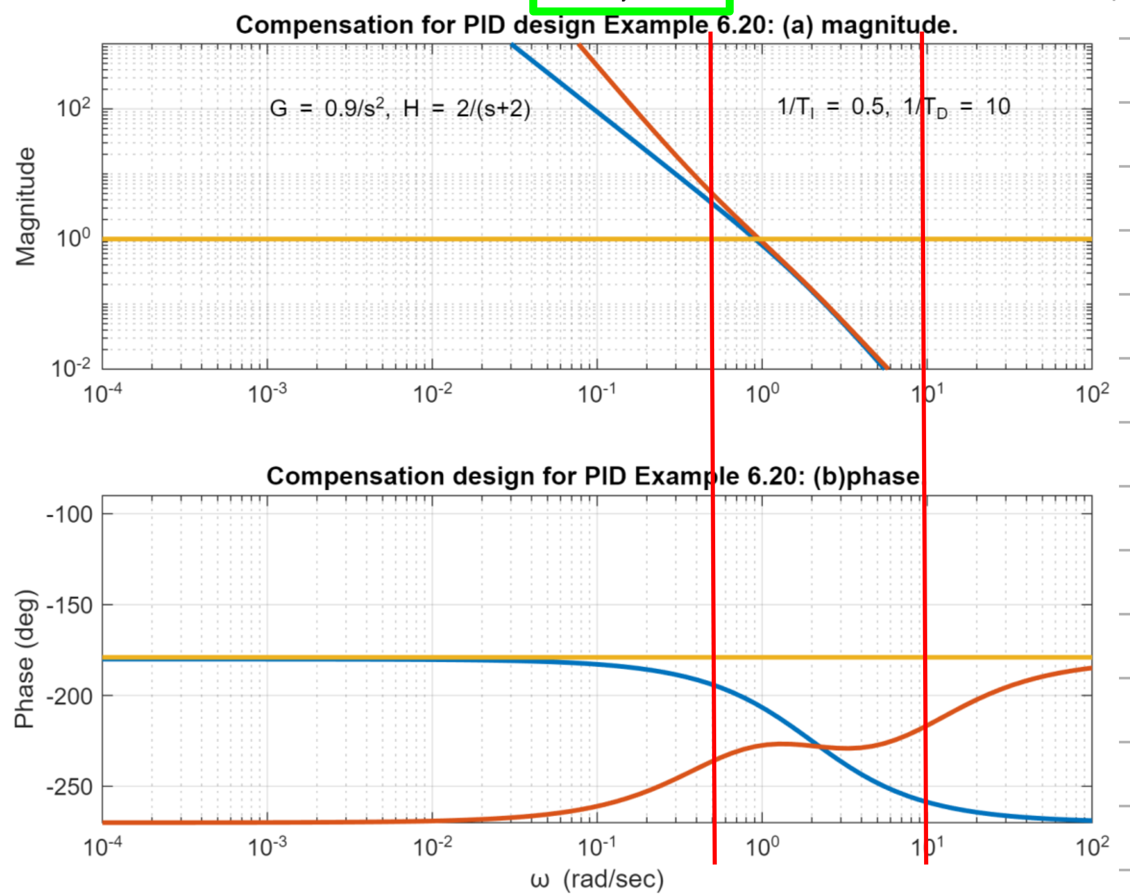
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

$$\frac{1}{T_I} = 0.5$$

$$\frac{1}{T_D} = 10$$





- Example 6.20: PID Compensation for Spacecraft Attitude Control

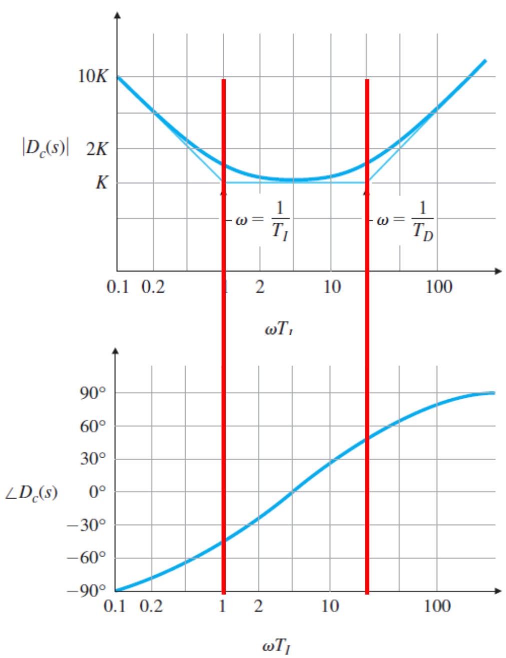
$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

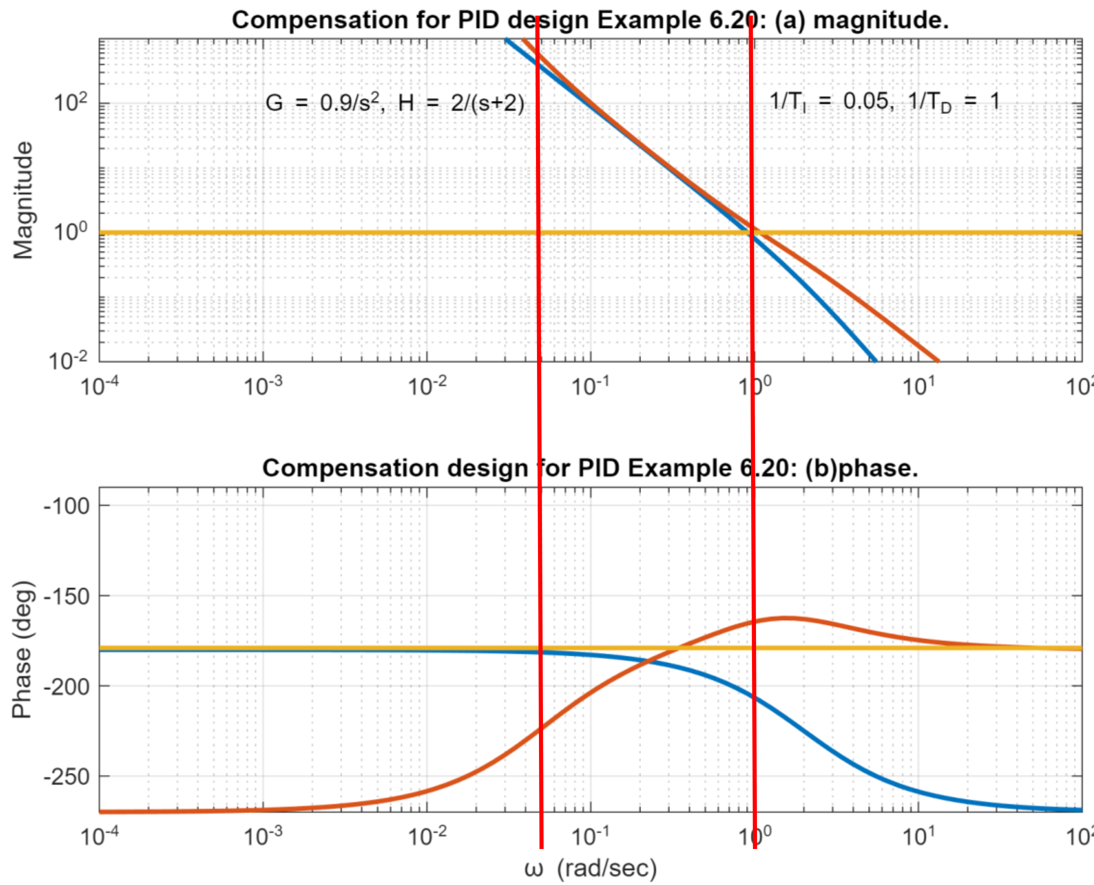
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

$$\frac{1}{T_I} = 0.05$$

$$\frac{1}{T_D} = 1$$





Example 6.20: PID Compensation for Spacecraft Attitude Control

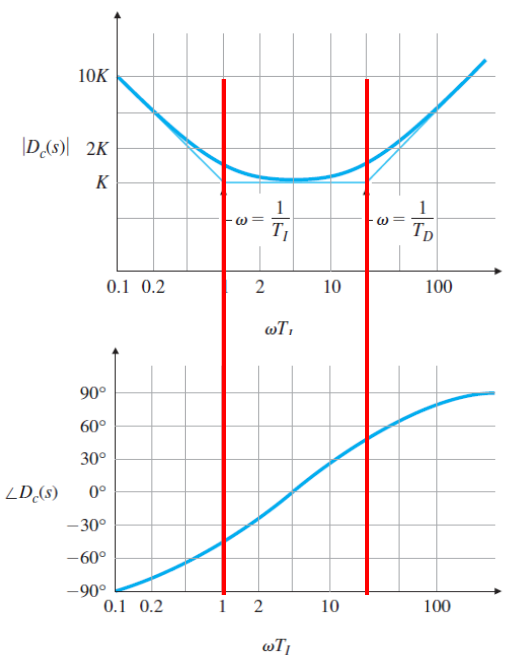
$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

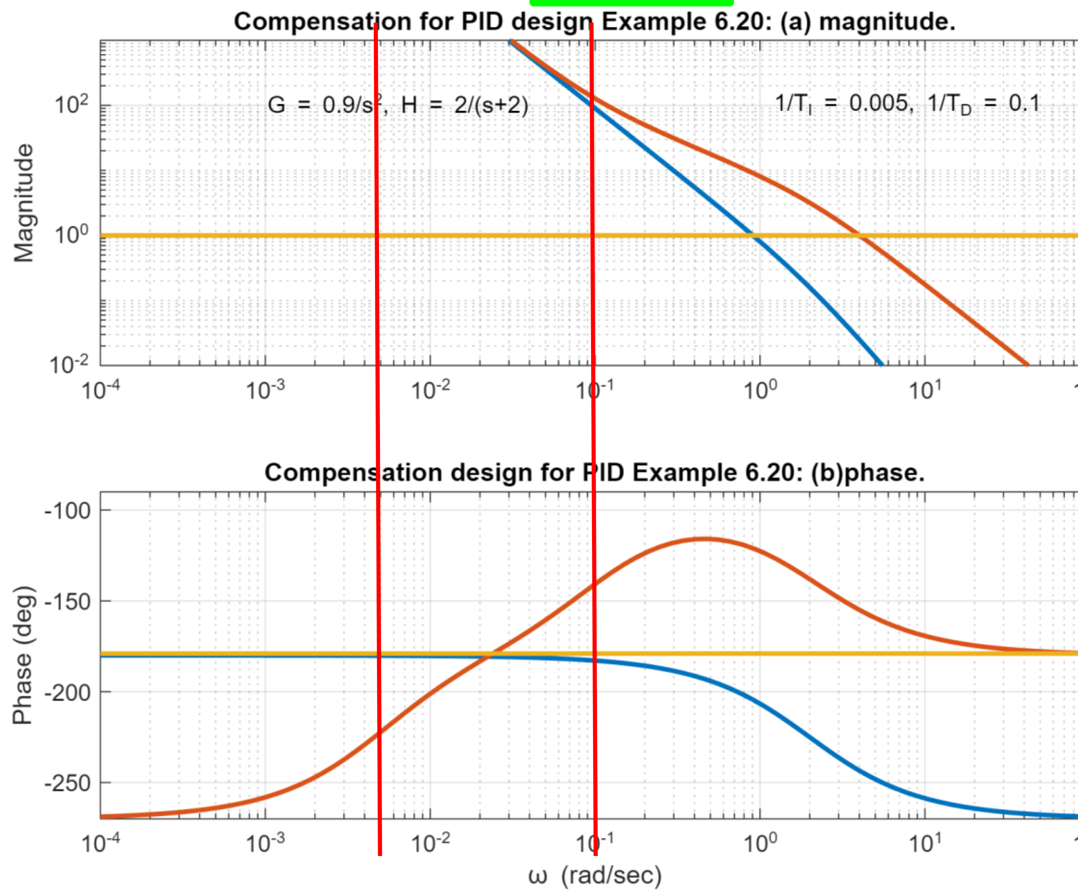
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

$$\frac{1}{T_I} = 0.005$$

$$\frac{1}{T_D} = 0.1$$





- Example 6.20: PID Compensation for Spacecraft Attitude Control

$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

$$G(s) = \frac{0.9}{s^2}$$

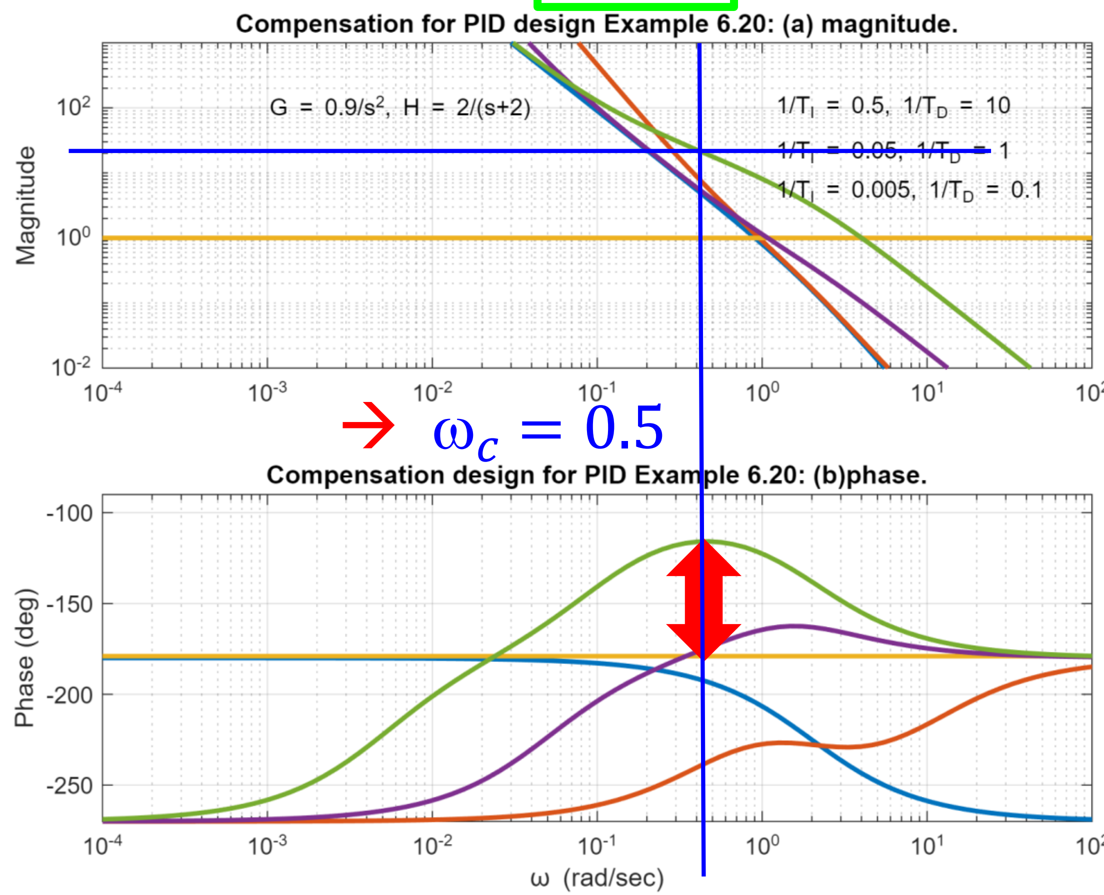
$$H(s) = \frac{2}{s+2}$$

$$\frac{1}{T_I} = 0.5 \qquad \frac{1}{T_D} = 10$$

$$\frac{1}{T_I} = 0.05 \qquad \frac{1}{T_D} = 1$$

$$\frac{1}{T_I} = 0.005 \qquad \frac{1}{T_D} = 0.1$$

- PM = 65^o
- → Find K
- $|\ D_c(s)\, G(s)\ | = 20$
- 1/K = 20
- K = 0.05



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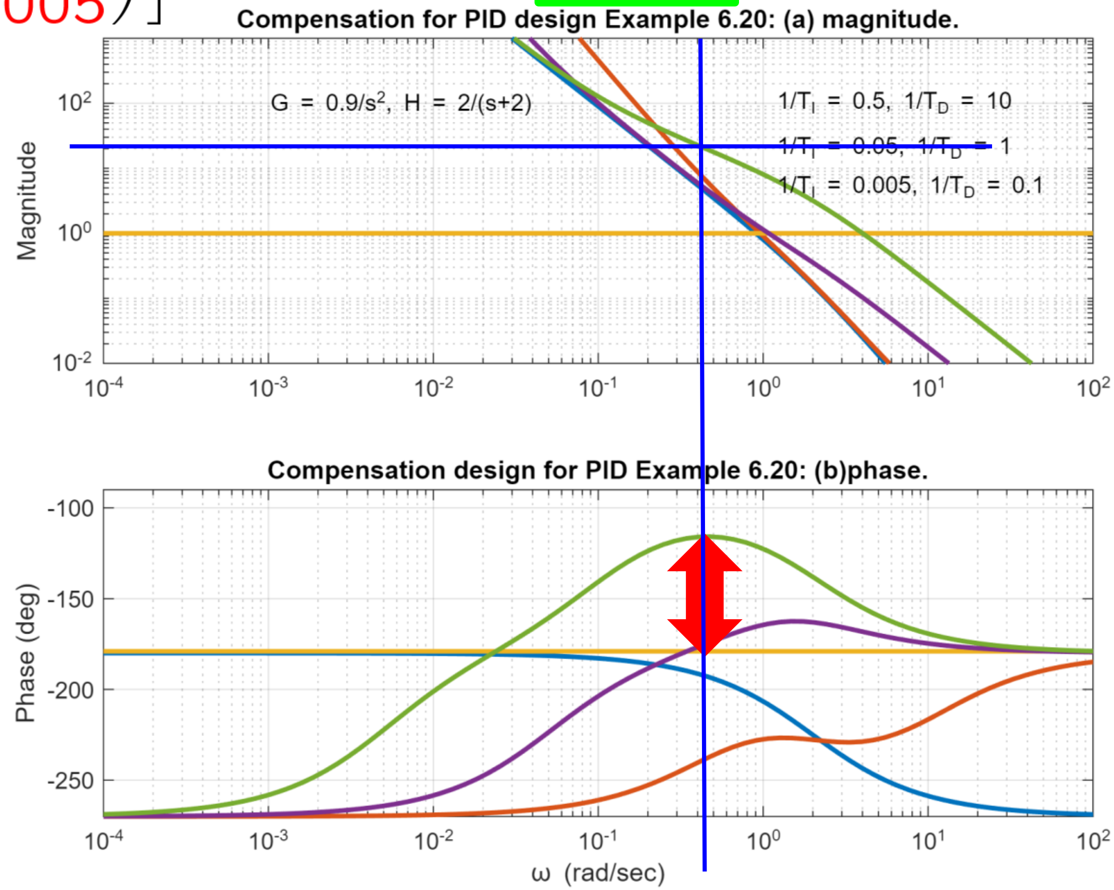
$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

$$= \frac{0.05}{s}\left[(10s + 1)\left(s + \frac{1}{0.005}\right)\right]$$

$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$



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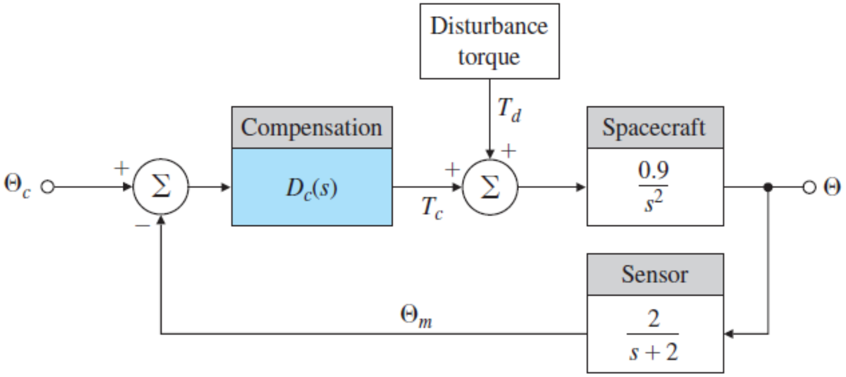
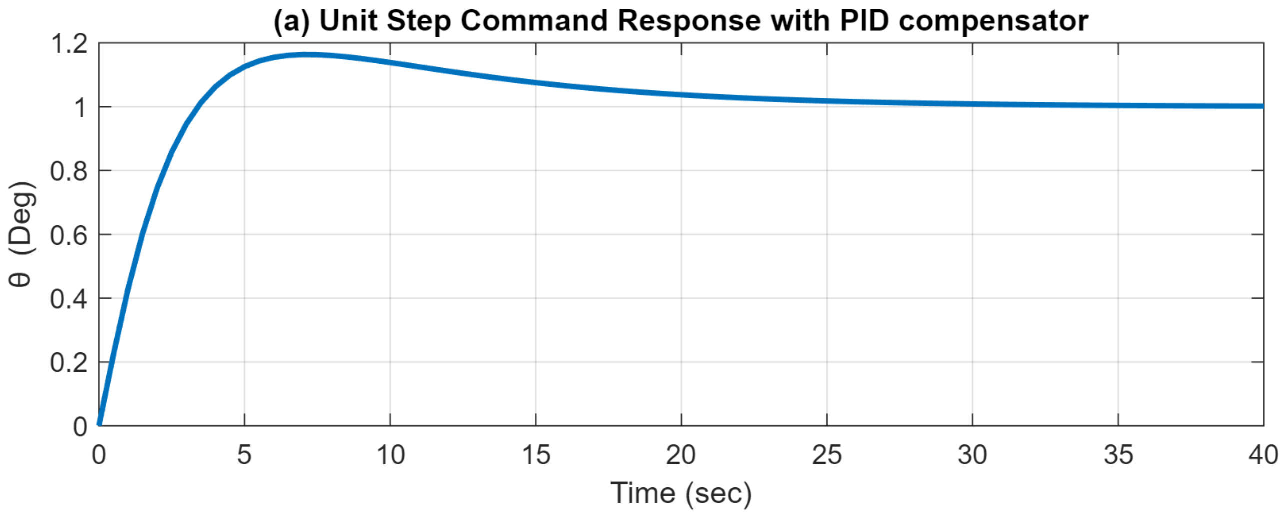


- Response of the system for a unit step θ_{com} is found from:

$$\mathcal{T}(s) = \frac{\Theta}{\Theta_{com}} = \frac{D_c\, G}{1 + D_c\, G\, H}$$



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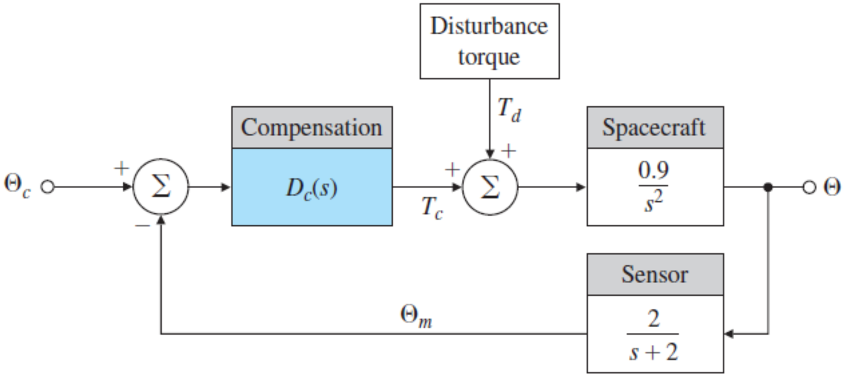
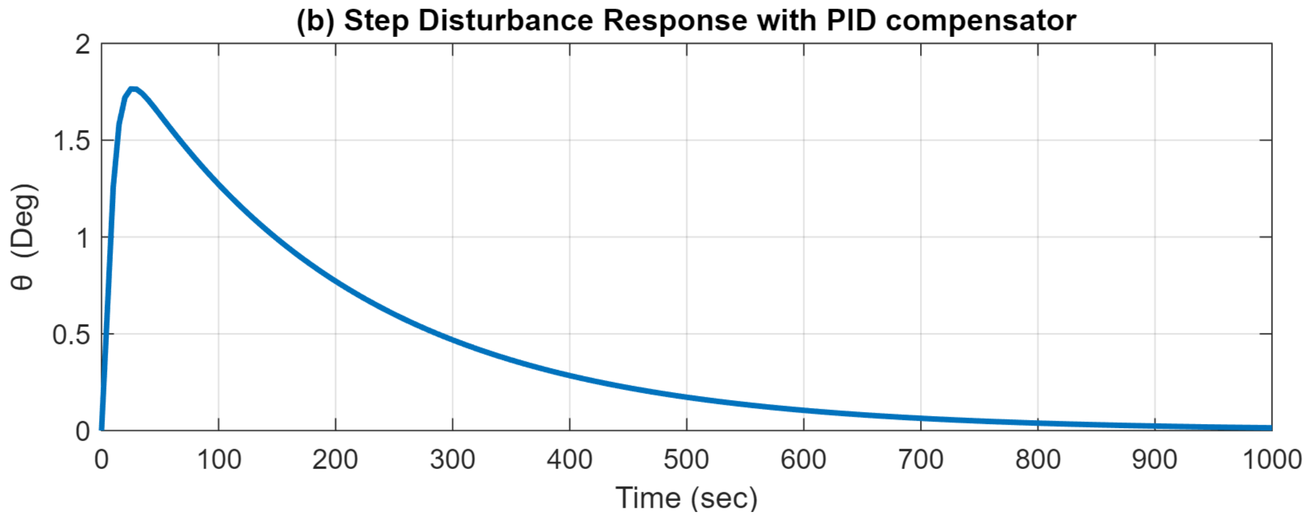


- Response for a step disturbance torque of T_d is found from:

$$\frac{\Theta}{T_d} = \frac{G}{1 + D_c G H}$$



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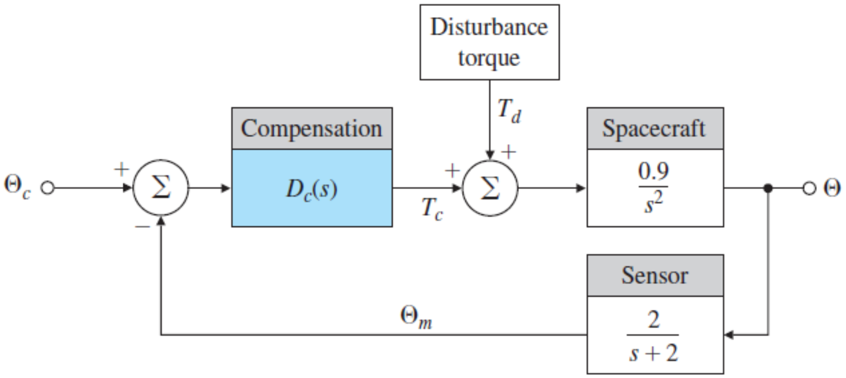

- Frequency Response of
  $T(s)$ and $S(s)$ are shown: