Fall 2022 (111-1)

控制系統
Control Systems

Unit 6I
PD Compensation and Lead Compensation

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Compensation

- Dynamic elements (or compensation)

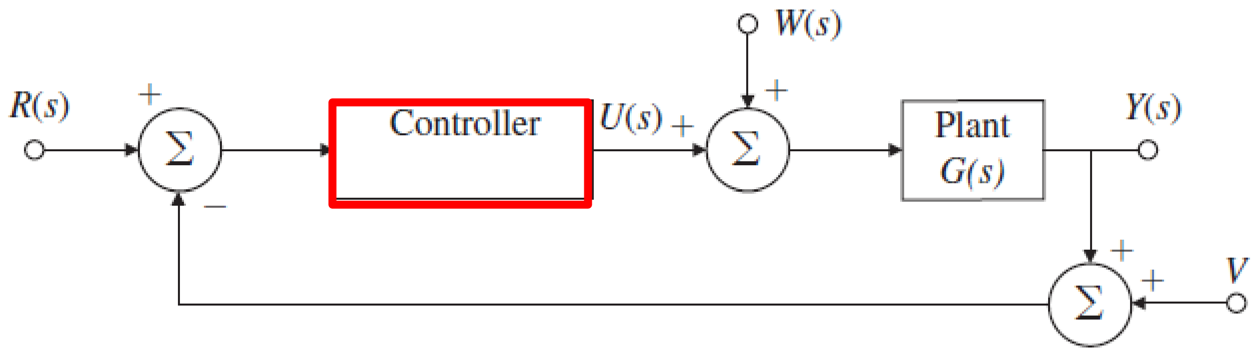
  are typically added to feedback controllers
- To improve the system's stability and error characteristics
- Because the process itself cannot be made

  to have acceptable characteristics

  with proportional feedback alone.
- Unit 4C:
  - 3 basic types of feedback: P, I, D.
- Unit 5F:
  - 3 kinds of dynamic compensation: Lead-PD, Lag-PI, Notch.

PD Compensation

- The closed-loop system:



- Controller = K $\qquad 1 + K\, G(s) = 0$

- Controller = $K\, D_c(s)$ $\qquad 1 + K\, D_c(s)\, G(s) = 0$

- PD Control or PD Compensation $\qquad D_c(s) = (\, T_D\, s + 1\,)$

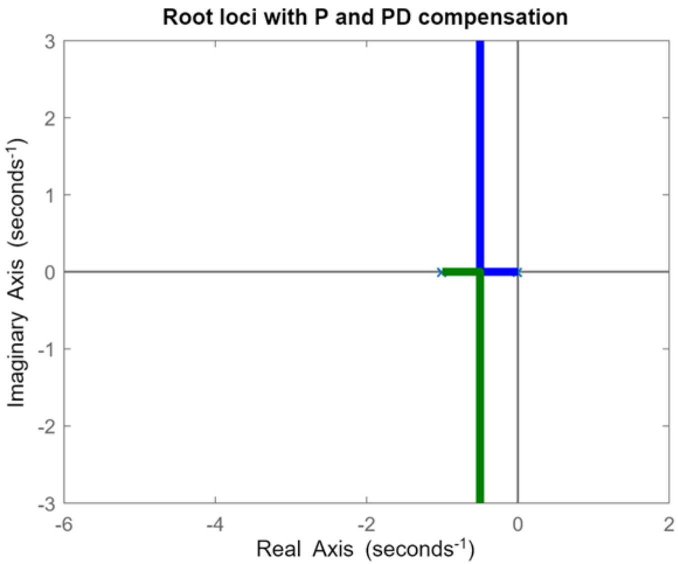
▪ Root Locus of PD Compensation

$$D_c(s) = K$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K \frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K = 0$$

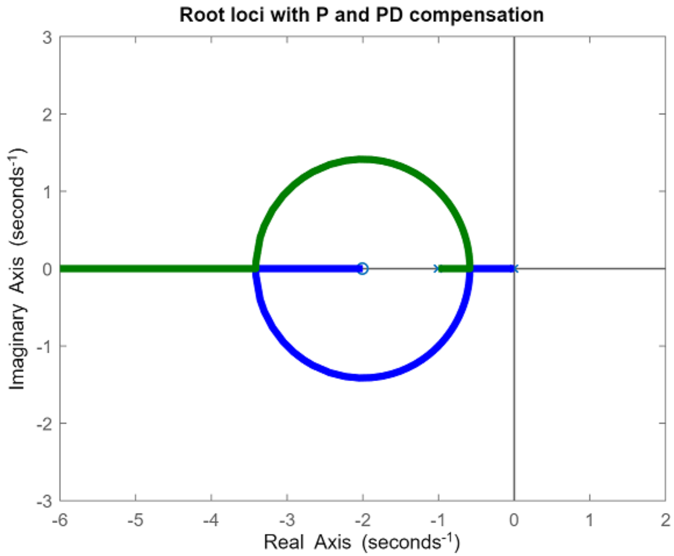


$$D_c(s) = K\,(s+2)$$

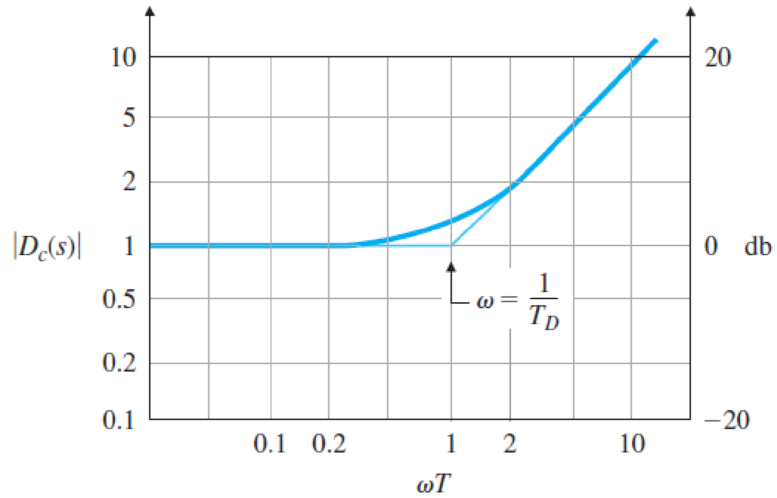
$$G(s) = \frac{1}{s(s+1)}$$

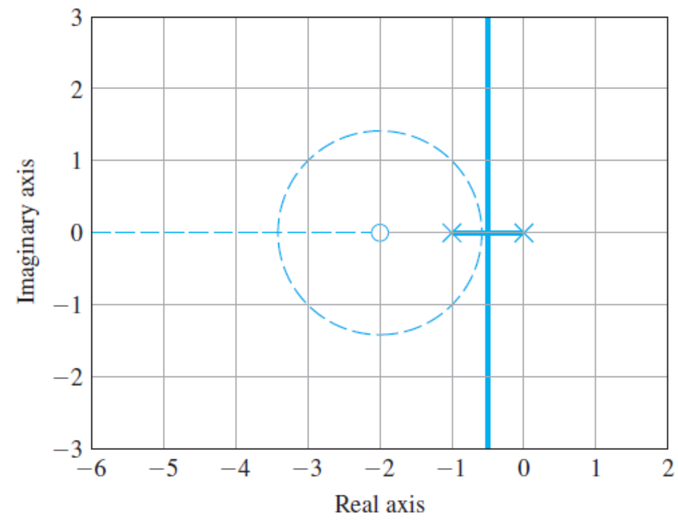
$$\Rightarrow 1 + K\,(s+2) \frac{1}{s(s+1)} = 0$$

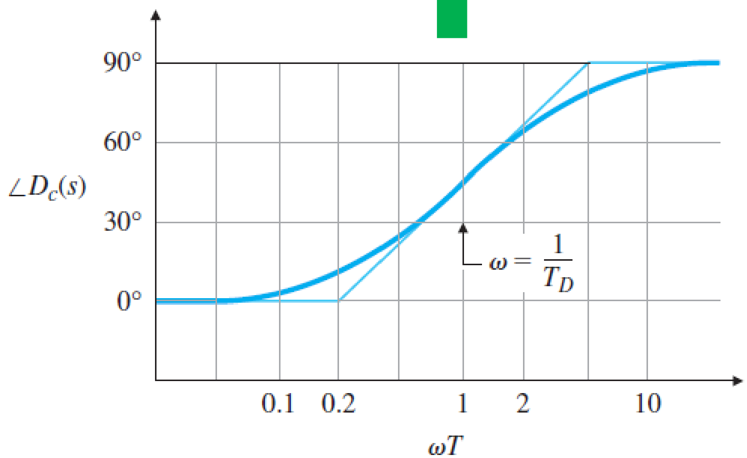
$$\Rightarrow s(s+1) + K(s+2) = 0$$



- PD Control or PD Compensation $D_c(s) = (T_D s + 1)$







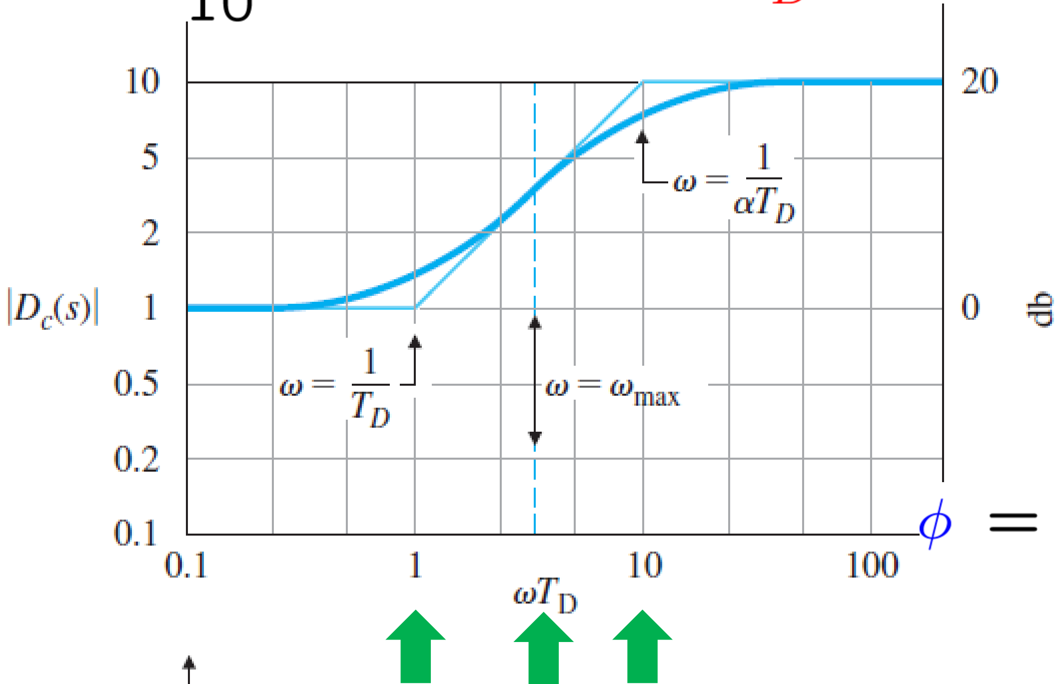
- Increase in phase
- +1 slope above $\omega = 1/T_D$
- Gain increased with frequency
  - Undesirable
  - Amplify high-frequency noise

- To alleviate the high-frequency amplification of the PD compensation,
- A first-order pole is added in the denominator at frequencies substantially higher than the break point of PD compensator.
- Thus, the phase increase (or lead) still occurs,
- But, the amplification at high frequencies is limited.
- The resulting lead compensation has a transfer function of:

$$D_c(s) = \frac{T_D\, s + 1}{\alpha\, T_D\, s + 1}, \quad \alpha < 1$$

Lead Compensation

$$\alpha = \frac{1}{10} \qquad D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}, \quad \alpha < 1$$

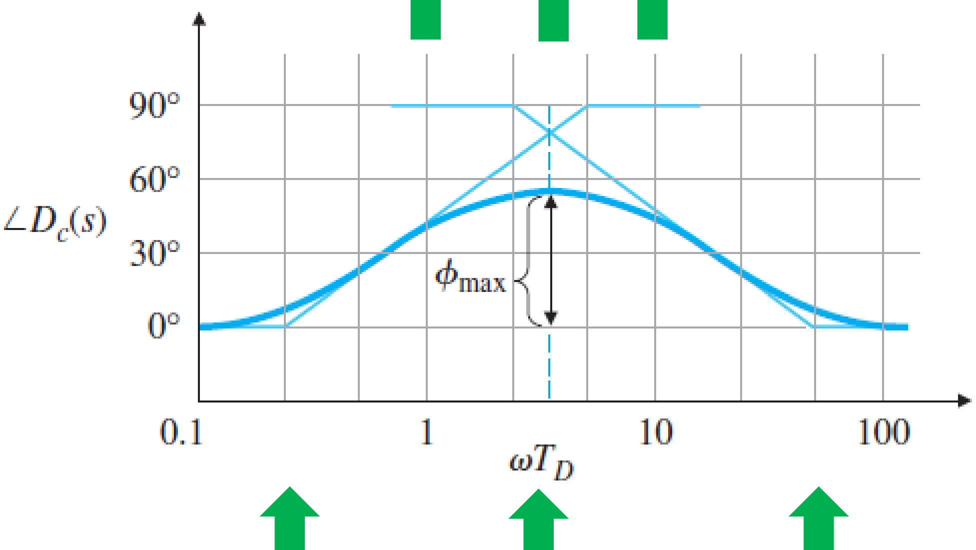


- Phase lead
- Much less amplification

  at high frequencies

- Phase Contribution

$$\phi = \tan^{-1}(T_D w) - \tan^{-1}(\alpha T_D w)$$



- Max Phase

$$w_{max} = \frac{1}{T_D\sqrt{\alpha}}$$

$$\sin\phi_{max} = \frac{1-\alpha}{1+\alpha}$$

$$\alpha = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}}$$

- The maximum phase occurs at a frequency that lie midway between the two break-point frequencies (sometimes called corner frequencies) on a logarithmic scale

$$\log w_{\max} = \log \frac{\frac{1}{\sqrt{T_D}}}{\sqrt{\alpha T_D}} = \log \frac{1}{\sqrt{T_D}} + \log \frac{1}{\sqrt{\alpha T_D}}$$

$$D_c(s) = \frac{s + z}{s + p} \qquad = \frac{1}{2} \left[\log \left(\frac{1}{T_D} \right) + \log \left(\frac{1}{\alpha T_D} \right) \right]$$

$$\Rightarrow w_{\max} = \sqrt{|z| \; |p|}$$

$$z = \frac{-1}{T_D}$$

$$\log w_{\max} = \frac{1}{2} \left(\log |z| + \log |p| \right)$$

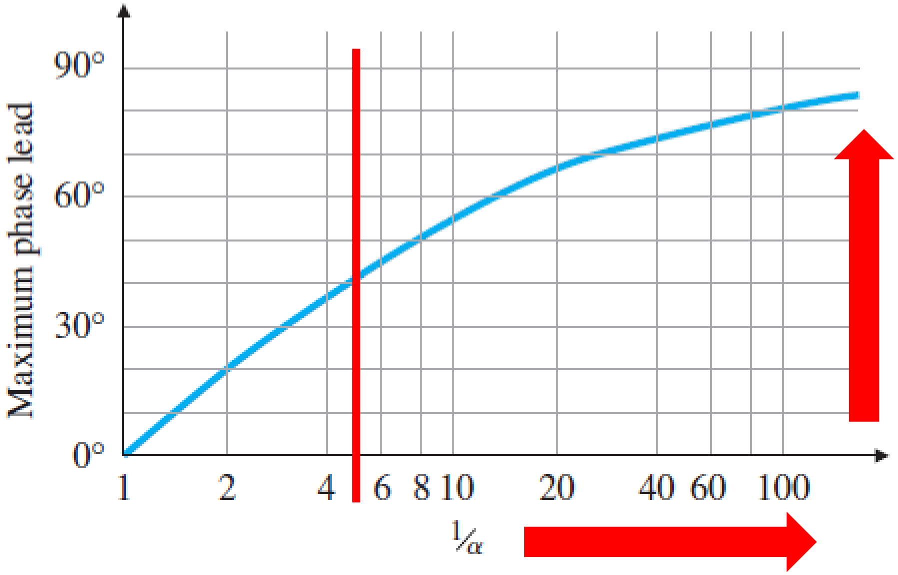
$$p = \frac{-1}{\alpha T_D}$$

- For example, a lead compensator:

a zero at $s = -2 \quad (T_D = 0.5)$ $\quad \Rightarrow \; w_{\text{max}} = \sqrt{|z|\;|p|}$

a pole at $s = -10 \quad (\alpha T_D = 0.1)$ $\quad \alpha = \frac{1}{5}$ $\quad = \sqrt{2 \times 10}$

$= 4.47 \text{ rad/sec}$



$\Rightarrow \; \phi_{\text{max}} = 40^o$

- Larger 1/α
- Produce higher amplification at higher frequencies

- Select 1/α → a good compromise between acceptable PM & noise sensitivity at high frequencies

$\Rightarrow \; \phi_{\text{max}} = 70^o$

- Double-lead compensation for greater phase lead:

$$D_c(s) = \left(\frac{T_D\, s + 1}{\alpha\, T_D\, s + 1} \right)^2$$

- Example 6.15: Lead Compensation for a DC Motor

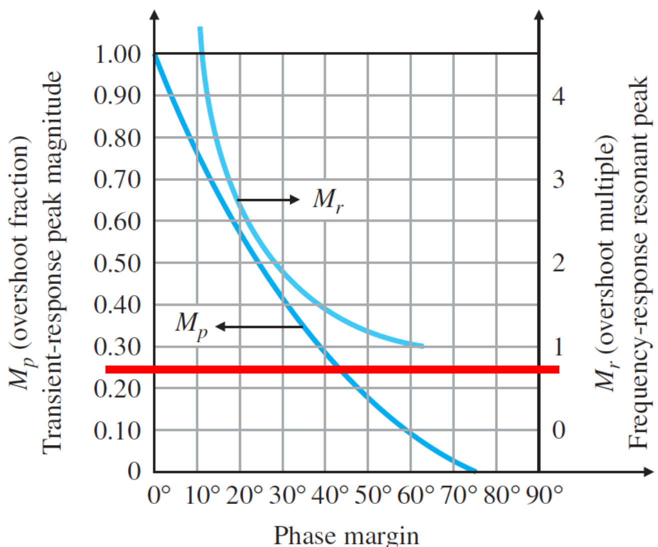
$$G(s) = \frac{1}{s(s+1)}$$

- Steady-state error <= 0.1 for a unit-ramp input
- Overshoot $M_p < 25\%$

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{1 + K\, D_c(s)\, G} \right] R(s) \qquad R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \left[\frac{1}{s + K\, D_c(s) \left(\frac{1}{s+1} \right)} \right] = \frac{1}{K\, D_c(0)}$$

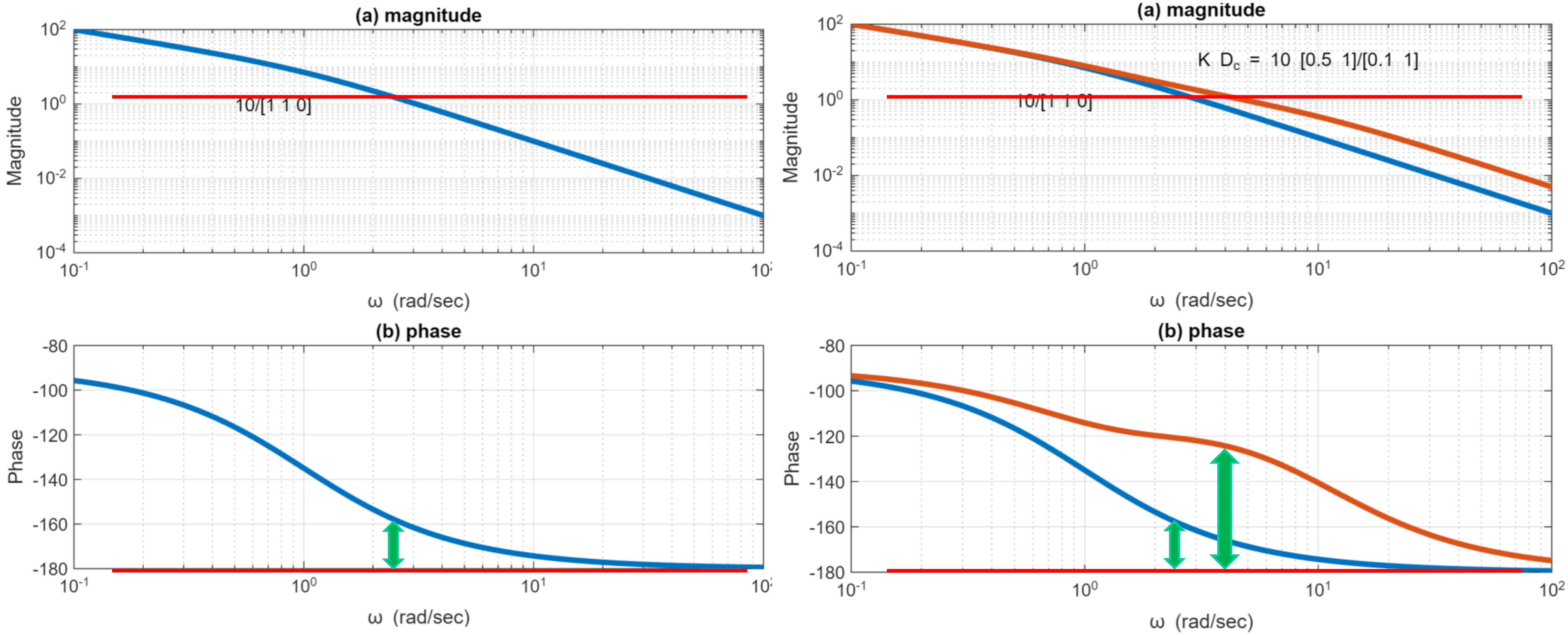
$$\Rightarrow |K\, D_c(0)| \geq 10$$



$$\Rightarrow \text{PM} \geq 45^o$$

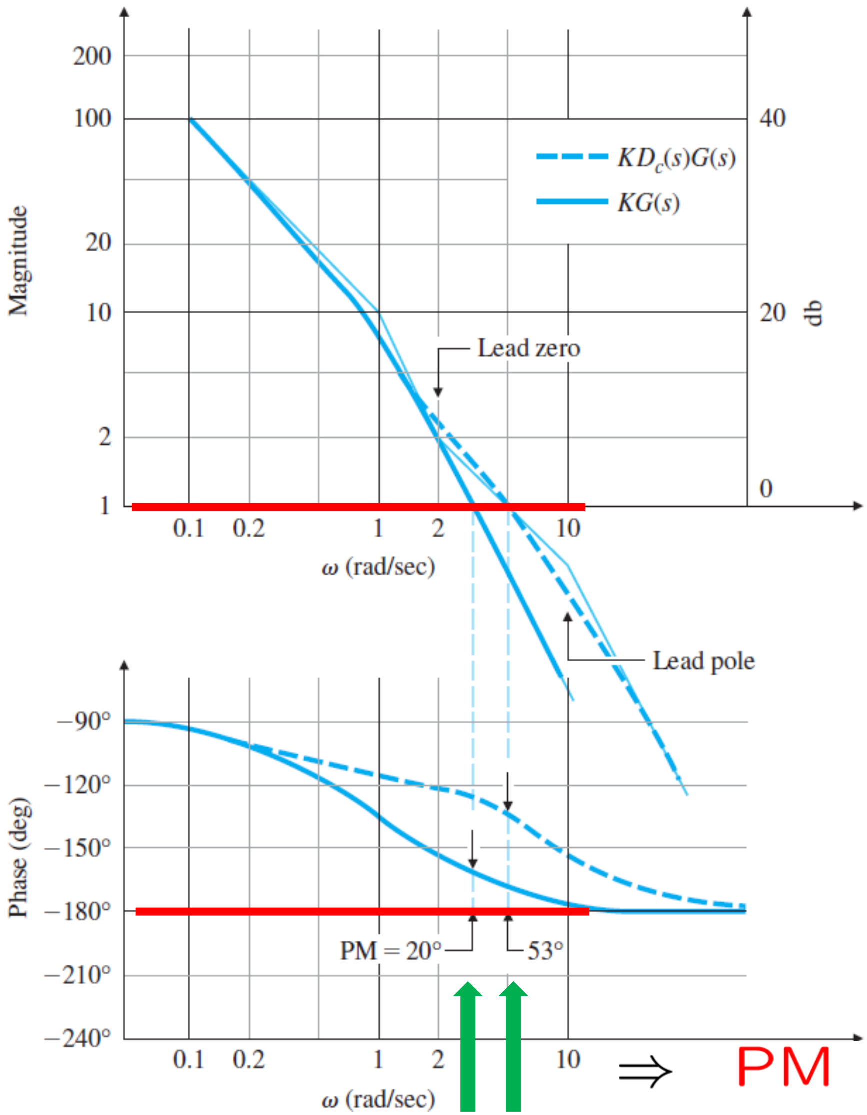
Example 6.15: Lead Compensation for a DC Motor

$K\ G(s)$ $\qquad$ $K\ D_c(s)\ G(s)$



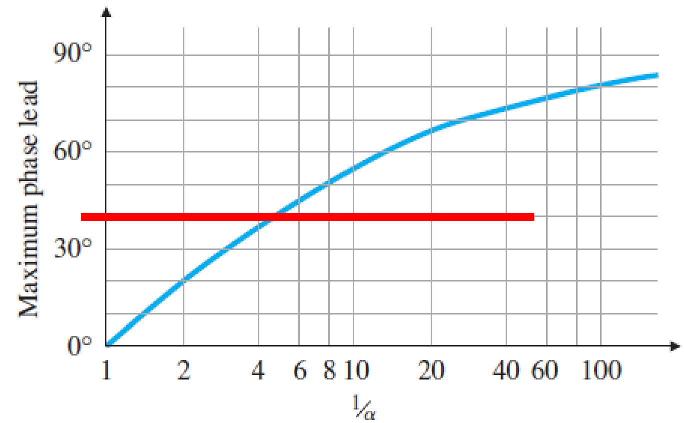
▪ Example 6.15: Lead Compensation for a DC Motor

$K\ G(s)$ $\qquad$ $K\ D_c(s)\ G(s)$



$\Rightarrow$ PM $\geq 25^o$, at $w_c = 3$

$\Rightarrow$ Phase lead $= 40^o$,



$$\Rightarrow \frac{1}{\alpha} = 5$$

a zero at $w = 2$ rad/sec

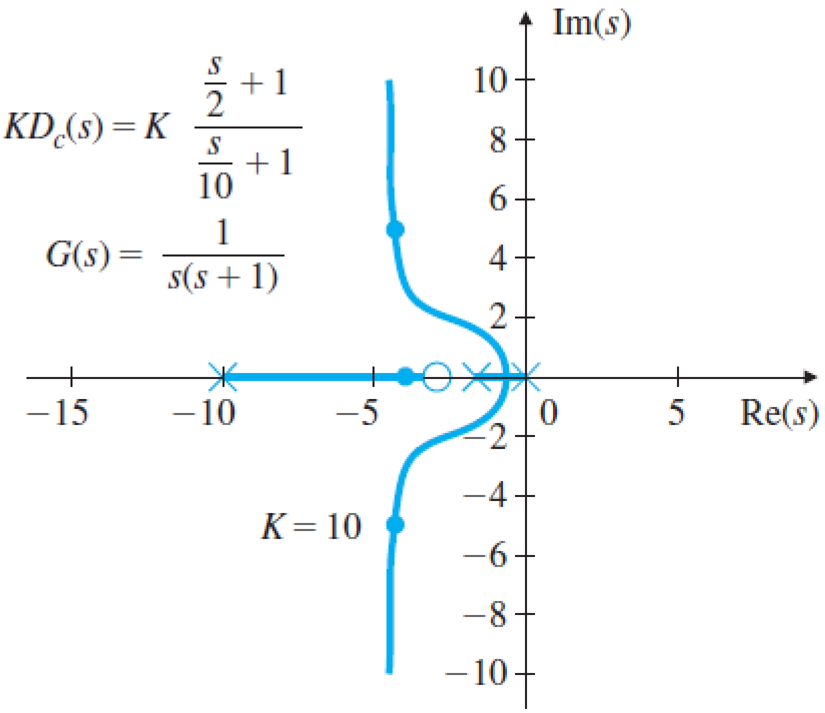
a pole at $w = 10$ rad/sec

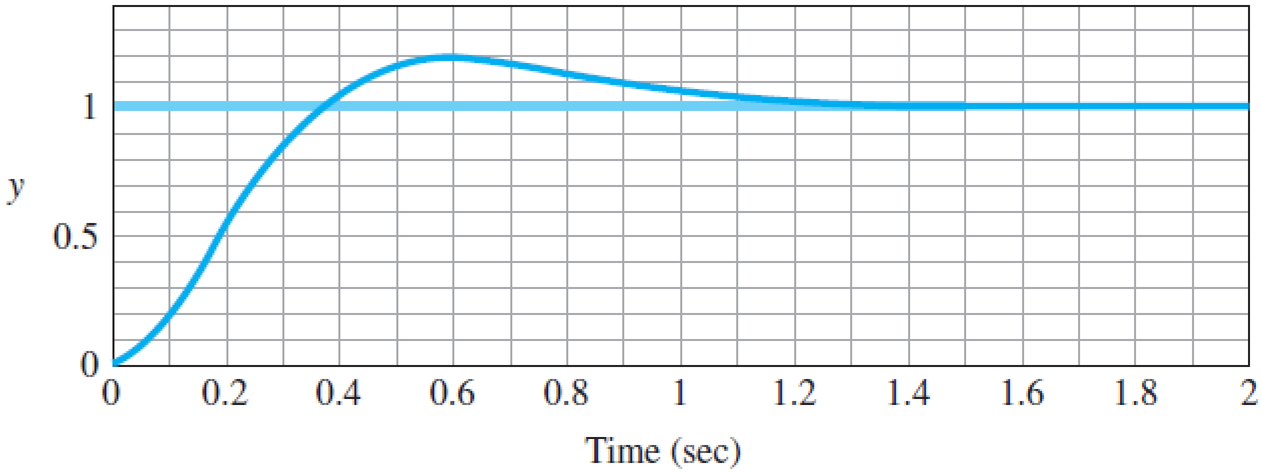
$$K\ D_c(s) = 10\ \frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$\Rightarrow$ PM $= 53^o$, at $w_c = 5$

- Example 6.15: Lead Compensation for a DC Motor

$$K\ D_c(s) = 10\,\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$





Design Parameters for Lead Compensation

1. Crossover frequency ω_c,

   which determines

   bandwidth ω_{BW}, rise time t_r, settling time t_s

2. Phase Margin (PM),

   which determines

   damping coefficient ζ, overshoot M_p

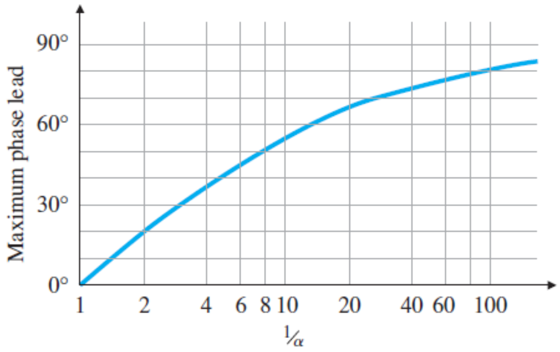
3. The low-frequency gain,

   which determines

   the steady-state error characteristics

1. Determine gain K to satisfy error or bandwidth requirements:
   a) To meet error requirements, pick K
      to satisfy error constants (K_P, K_v, K_a), so that e_{ss} is met.
   b) To meet bandwidth requirements, pick K
      so that the OL crossover frequency
      is a factor of two below the desired CL bandwidth.

2. Evaluate the PM of the uncompensated system
   using the value of K obtained from Step 1
3. Allow for extra margin (about 10^o) and
   determine the needed phase lead ϕ_{max}



4. Determine α

5. Pick ω_{max} to be the crossover frequency;

   a zero at $1/T_D = w_{max}\sqrt{\alpha}$

   a pole at $1/(\alpha T_D) = w_{max}/\sqrt{\alpha}$

6. Draw the compensated frequency response and check PM

7. Iterate on the design.
   - Adjust compensator parameters (poles, zeros, gain) until all specification are met.
   - Add an additional lead compensator if necessary.

- Example 6.16: Lead Compensation for Temperature Control System

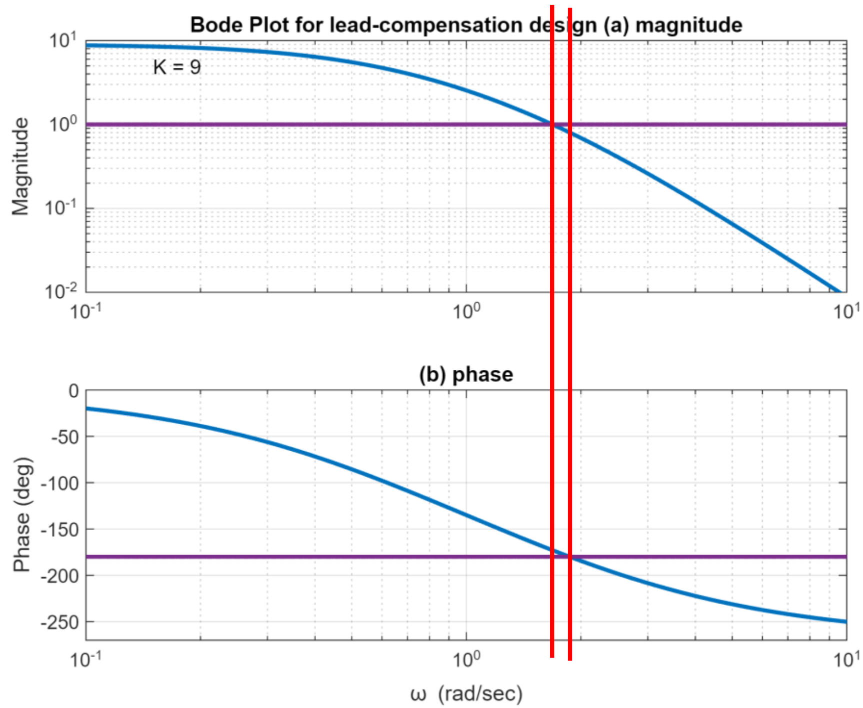
$$K\,G(s) = \frac{K}{(\frac{s}{0.5}+1)(\frac{s}{1}+1)(\frac{s}{2}+1)}$$

- $K_p = 9$
- $\text{PM} > 25^o$

1. Determine gain K:

$$K_p = \lim_{s\to 0} K\,G(s) = K = 9$$

2. Bode plot of un-uncompensated system: KG(s), with K = 9

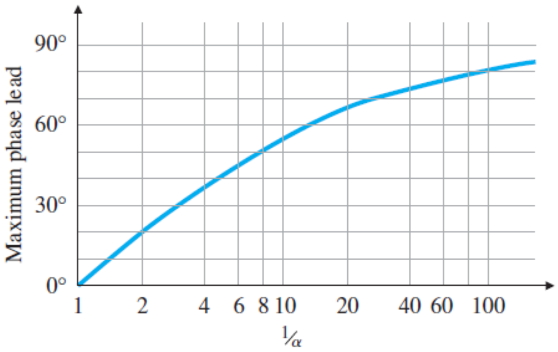


→ GM = 1.25, PM = 7.14, Wcg = 1.87, Wcp = 1.68

- Example 6.16: Lead Compensation for Temperature Control System

3. Allow for 10^o of extra margin → $25^o + 10^o - 7^o = 28^o$

4. Pick α → 1/α = 3



5. Zero & Pole

a zero at 1 $T_D = 1$

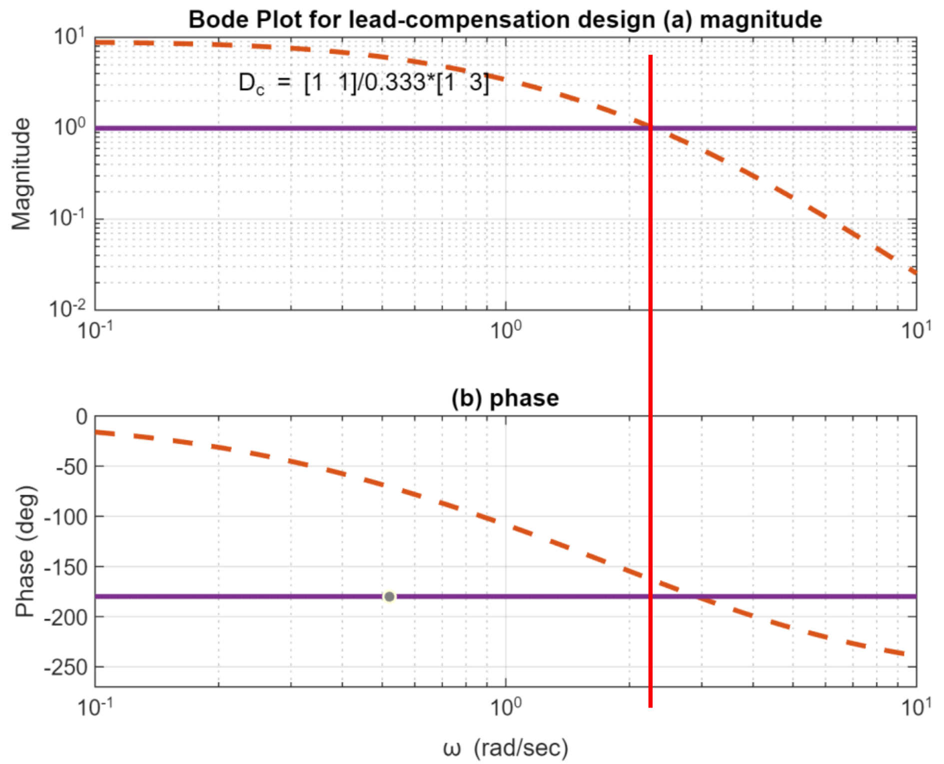
a pole at 3 $\alpha T_D = 1/3$

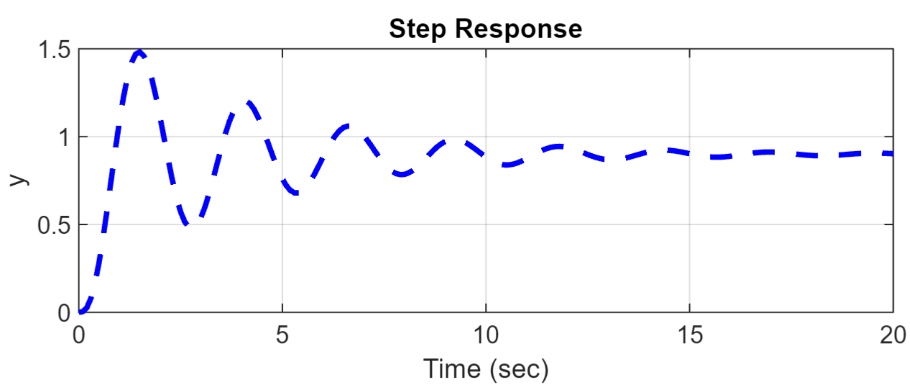
$$D_1(s) = \frac{(\frac{s}{1}+1)}{(\frac{s}{3}+1)} = \frac{1}{0.333}\left(\frac{s+1}{s+3}\right)$$

- Example 6.16: Lead Compensation for Temperature Control System

$$D_1(s) = \frac{(\frac{s}{1}+1)}{(\frac{s}{3}+1)} = \frac{1}{0.333}\left(\frac{s+1}{s+3}\right)$$

- $\text{PM} = 16^o$





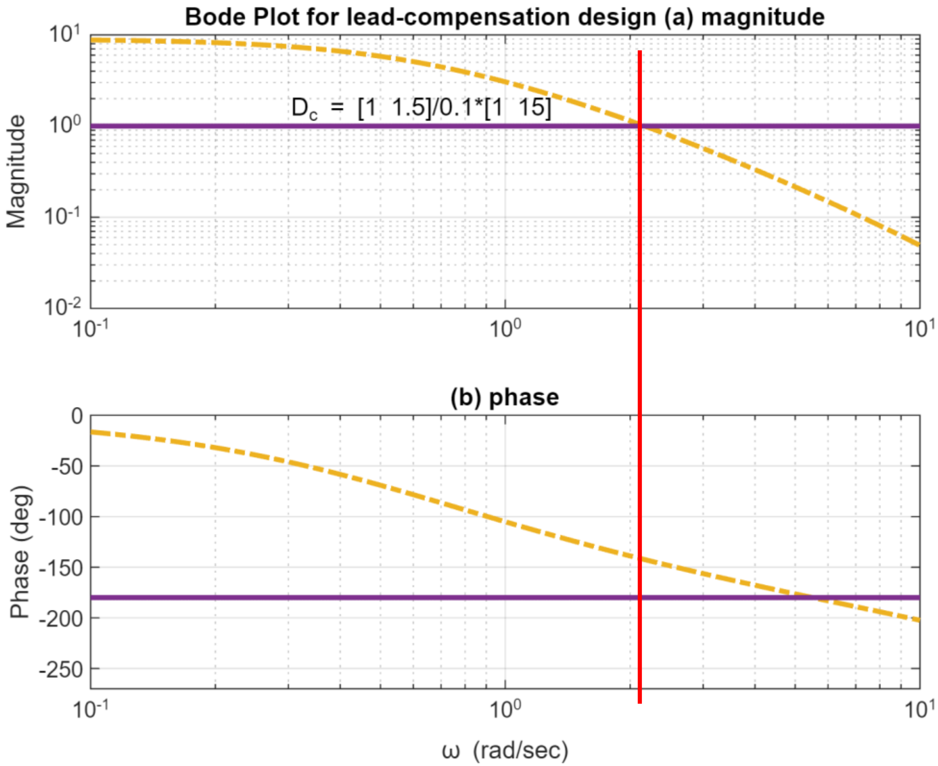
Example 6.16: Lead Compensation for Temperature Control System

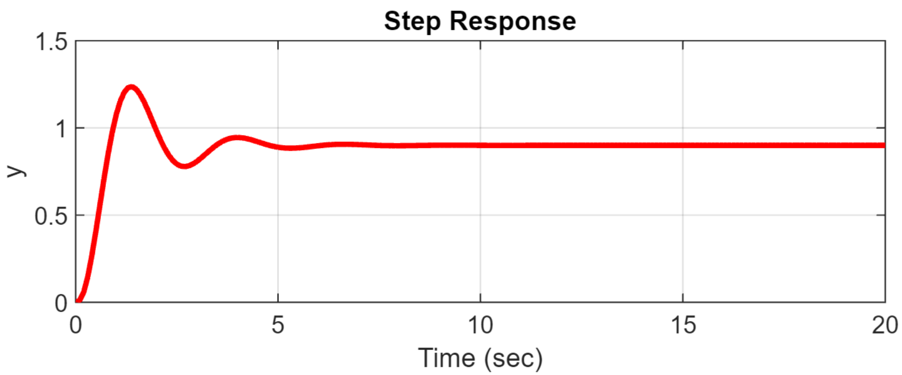
7. Move zero:

a zero at $s = -1.5$ $\alpha = 1/10$

$$D_2(s) = \frac{(\frac{s}{1.5}+1)}{(\frac{s}{15}+1)} = \frac{1}{0.1}\left(\frac{s+1.5}{s+15}\right)$$

- PM $= 38^o$





- Example 6.16: Lead Compensation for Temperature Control System

KG(s), with K = 9

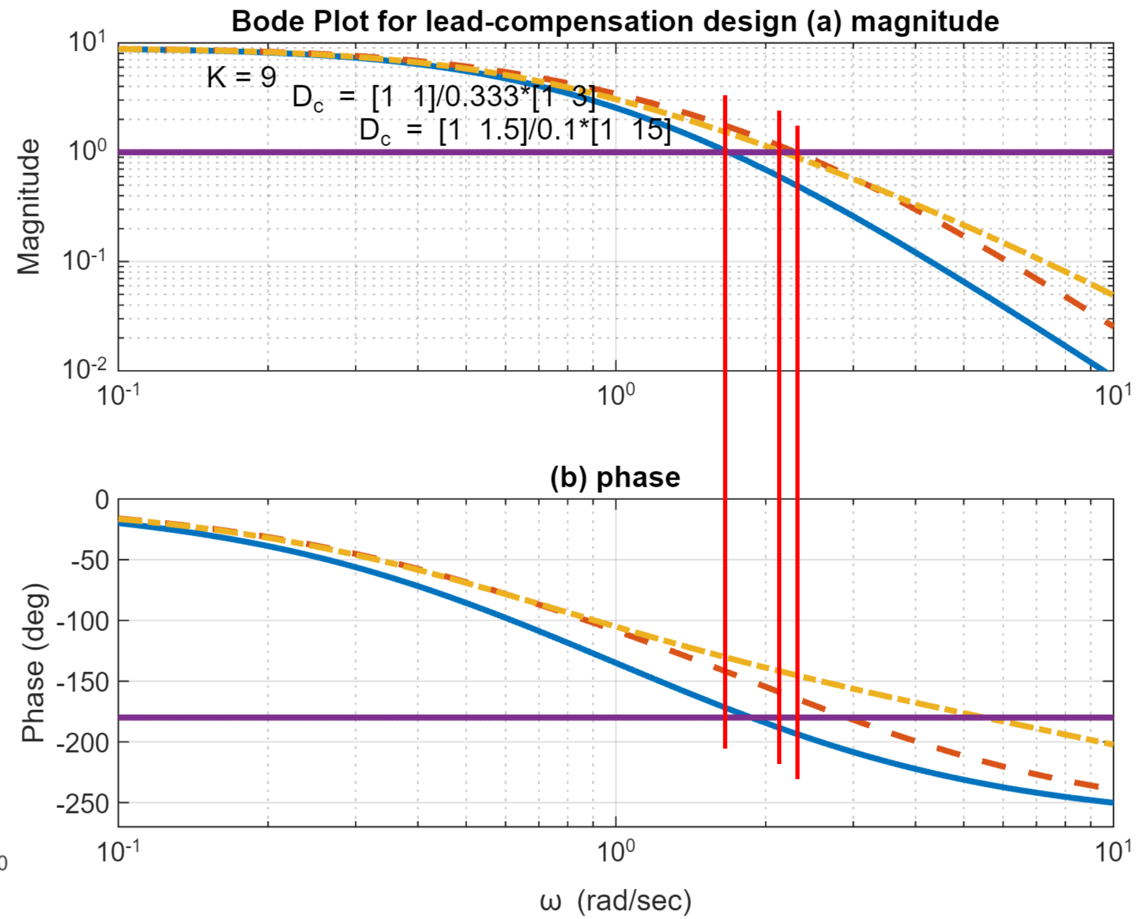
$$D_1(s) = \frac{(\frac{s}{1}+1)}{(\frac{s}{3}+1)}$$

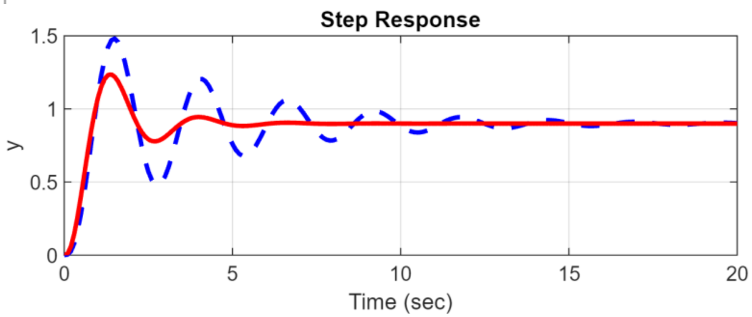
$$D_2(s) = \frac{(\frac{s}{1.5}+1)}{(\frac{s}{15}+1)}$$

- $\mathrm{PM} = 7.14^o$
- $\mathrm{PM} = 16^o$
- $\mathrm{PM} = 38^o$

→ GM, PM, Wcg, Wcp

GM	PM	Wcg	Wcp
1.25	7.12	1.87	1.68
1.62	15.92	2.91	2.30
5.65	37.56	5.51	2.17





- Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

$$K\,G(s) = K \frac{10}{s\,(\frac{s}{2.5}+1)\,(\frac{s}{6}+1)}$$

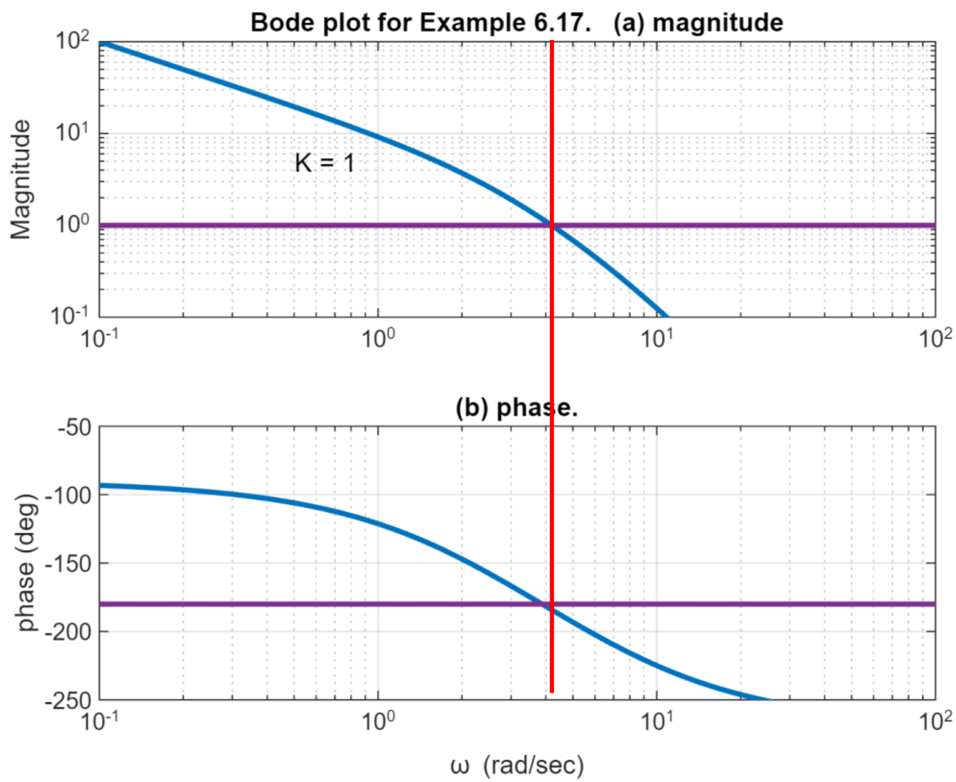
- $K_v = 10$
- $\text{PM} = 45^o$

1. Determine gain K:

$$K_v = \lim_{s\to 0} s\,K\,G(s) = K \times 10 = 10 \Rightarrow K = 1$$

2. Bode plot of KG(s), K = 1

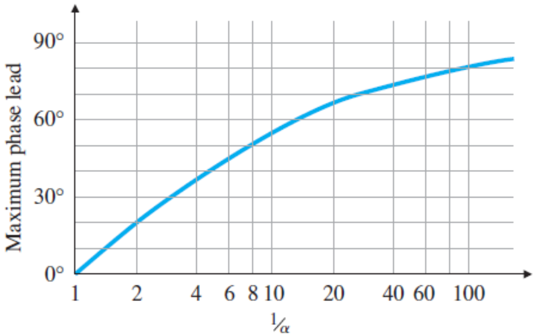
→ PM ~= - 4, Wcp ~= 4



- Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

3. Allow for 5^o of extra margin

→ $45^o + 5^o - (-4^o) = 54^o$



4. Pick α → 1/α = 10

5. Zero & Pole

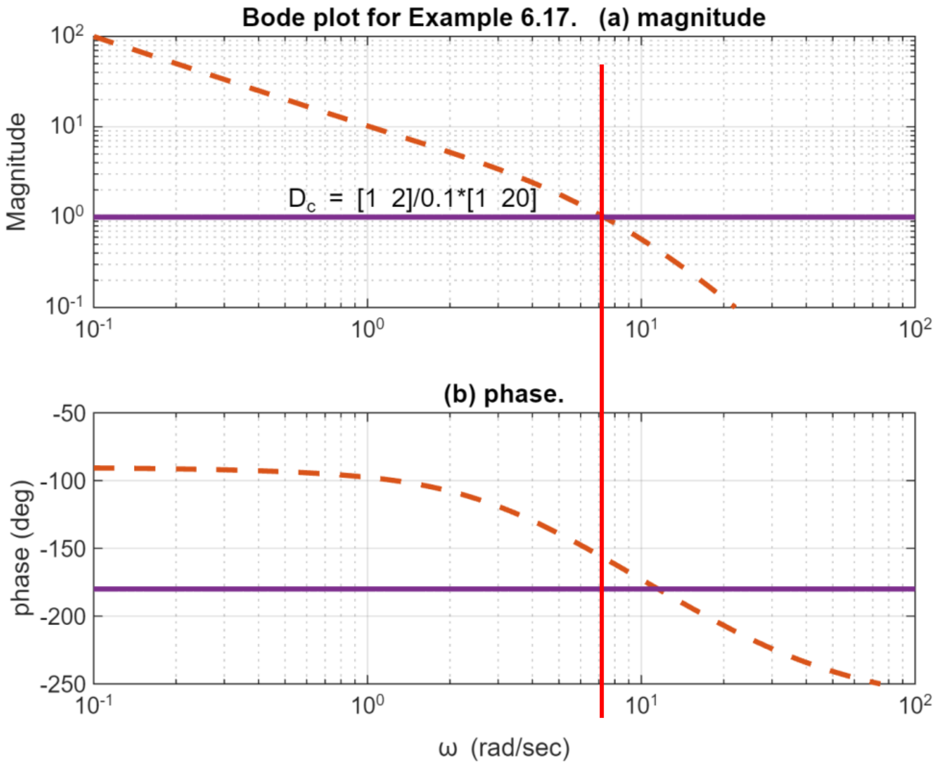
a zero at 2

a pole at 20

$$D_1(s) = \frac{(\frac{s}{2}+1)}{(\frac{s}{20}+1)}$$

$$= \frac{1}{0.1}\left(\frac{s+2}{s+20}\right)$$

→ PM ~= 23, Wcp ~= 7

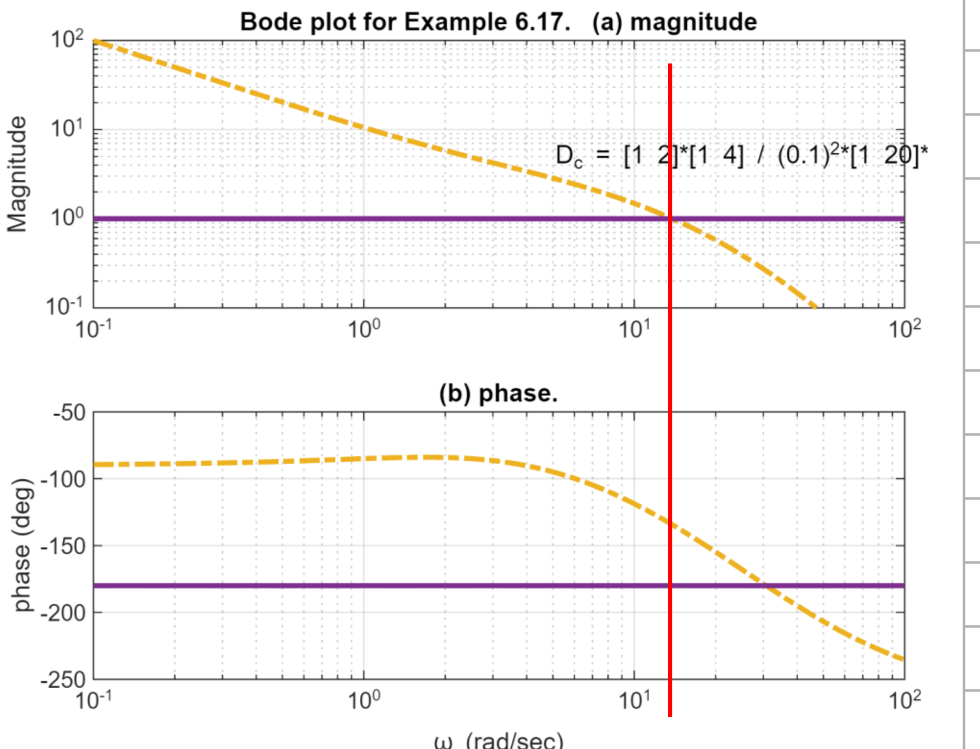


- Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

7. A double-lead compensator:

$$D_2(s) = \frac{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{\left(\frac{s}{20}+1\right)\left(\frac{s}{40}+1\right)} = \frac{1}{(0.1)^2}\frac{(s+2)(s+4)}{(s+20)(s+40)}$$

- PM $= 46^o$



▪ Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

KG(s), K = 1 $\quad D_1(s) = \dfrac{\left(\frac{s}{2}+1\right)}{\left(\frac{s}{20}+1\right)} \quad D_2(s) = \dfrac{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{\left(\frac{s}{20}+1\right)\left(\frac{s}{40}+1\right)}$

→ PM ~= - 4 → PM ~= 23 → PM ~= 46

→ GM, PM, Wcg, Wcp

GM	PM	Wcg	Wcp
0.85	- 4.16	3.87	4.19
2.32	22.90	11.51	7.31
3.85	45.58	30.77	13.83