Fall 2022 (111-1)

控制系統
Control Systems

Unit 6G
Bode's Gain-Phase Relationship

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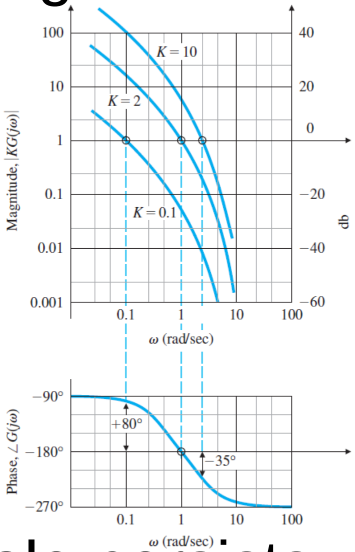
Sep 2022 – Dec 2022

Bode's Theorem

- One of Bode's important contributions is the following theorem:



- For any stable minimum-phase system

  (no RHP zeros/poles),

- The phase of G(jw) is uniquely related to

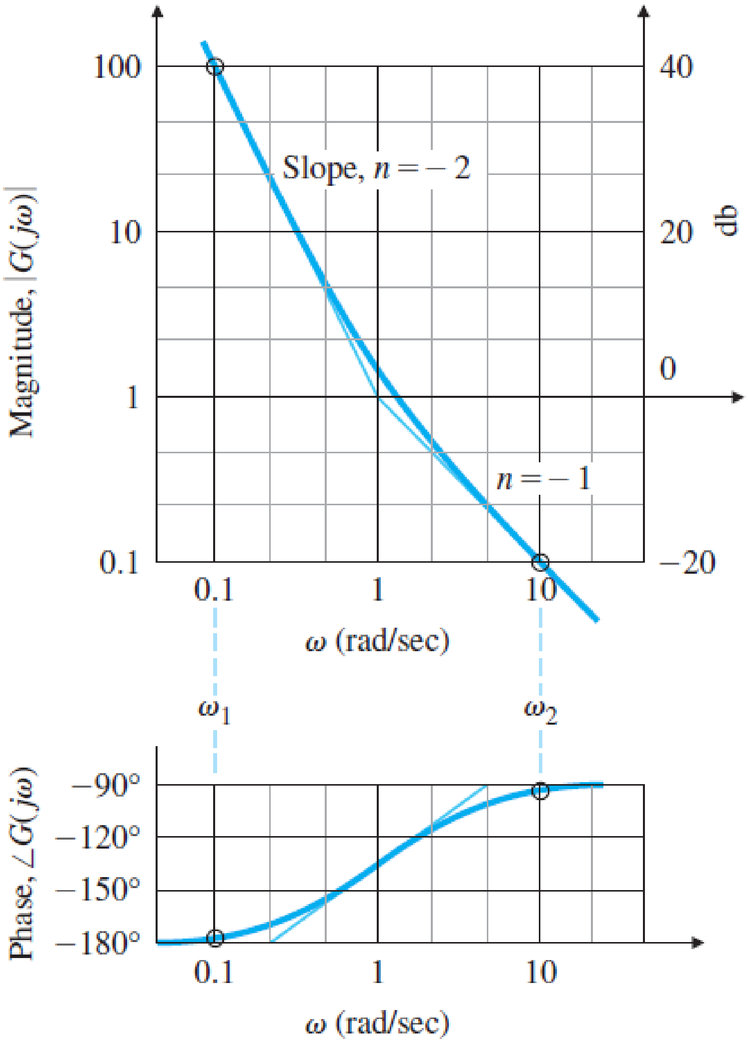
  the magnitude of G(jw)

- When the slope of |G(jw)| versus ω on a log-log scale persists at a constant value for approximately a decade of frequency, the relationship is particularly simple and is given by:

$$\angle G(jw) \approx n \times 90^o$$

- n: the slope of |G(jw)| in units of decade of amplitude per decade of frequency

Bode's Theorem



- Slope:
  - At ω_1= 0.1, (n = -2)
  - At ω_2= 10, (n = -1)
- Phase:
  - At ω_1= 0.1, -180^o
  - At ω_2= 10, -90^o

Bode's Theorem

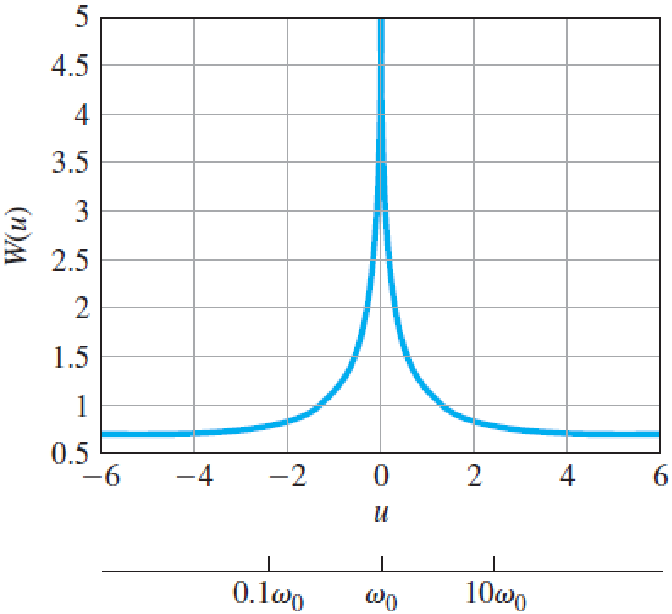
- An exact statement of the Bode Gain-Phase Theorem is:

$$\angle G(jw) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \left(\frac{dM}{du} \right) W(u)\, du \quad \text{in radians}$$

- Where
  - M = log magnitude = ln $|G(jw)|$
  - u = normalized frequency = $\ln(\omega / \omega_0)$
  - dM/du ~= slope n
  - $W(u)$ = weighting function

    = ln(coth|u|/2)

$$W(u) \approx \frac{\pi^2}{2} \delta(u)$$



Bode's Theorem

- But, usually use $\angle G(jw) \approx n \times 90^o$
- When | KG(jw) | = 1 $\angle G(jw) \approx -90^o$ if $n = -1$

  $\angle G(jw) \approx -180^o$ if $n = -2$
- For stability, we want:

$$\angle G(jw) > -180^o \quad \text{for PM to be} > 0$$

- Therefore, adjust the |KG(jw)| curve
- So that it has a slope of –1 at the crossover frequency ω_c
- If the slope = -1 for a decade above/below ω_c, then PM ~= 90^o
- However, to ensure a reasonable PM,

  it is usually necessary only to insist that

  a -1 slope persist for a decade in frequency centered at ω_c

Bode's Theorem

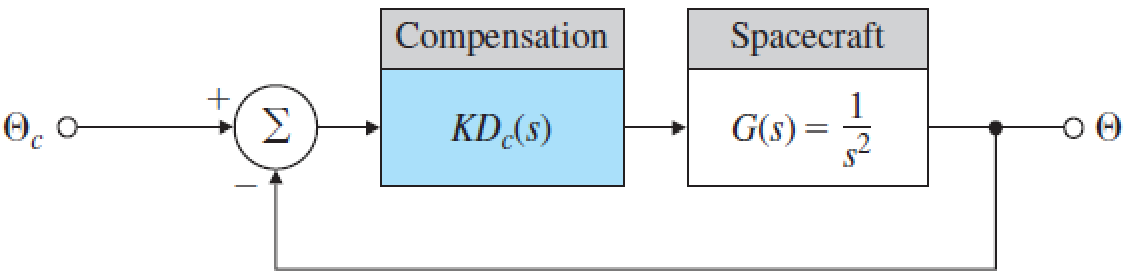
- A very simple design criterion:
- Adjust the slope of the magnitude curve |KG(jw)|
- So that it crosses over magnitude 1 with a slope of -1

  for a decade around ω_c
- This criterion will usually be sufficient

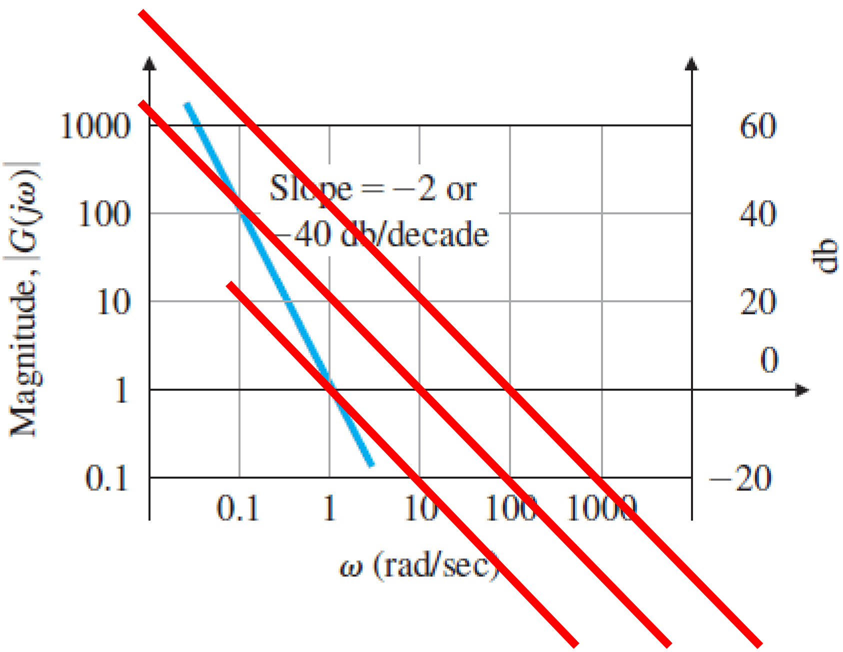
  to provide an acceptable PM and adequate system damping.
- To achieve the desired speed of response,
- the system gain is adjusted
- so that the crossover point is at a frequency

  that will yield the desired bandwidth or speed of response.
- Natural Freq ω_n ~= Bandwidth ω_{BW} ~= Crossover Freq ω_c

- Example 6.14: Use of Simple Design Criterion for Spacecraft Attitude Control



$$K\,D_c(s) \;=\; K\,(\,T_D\,s + 1\,)$$



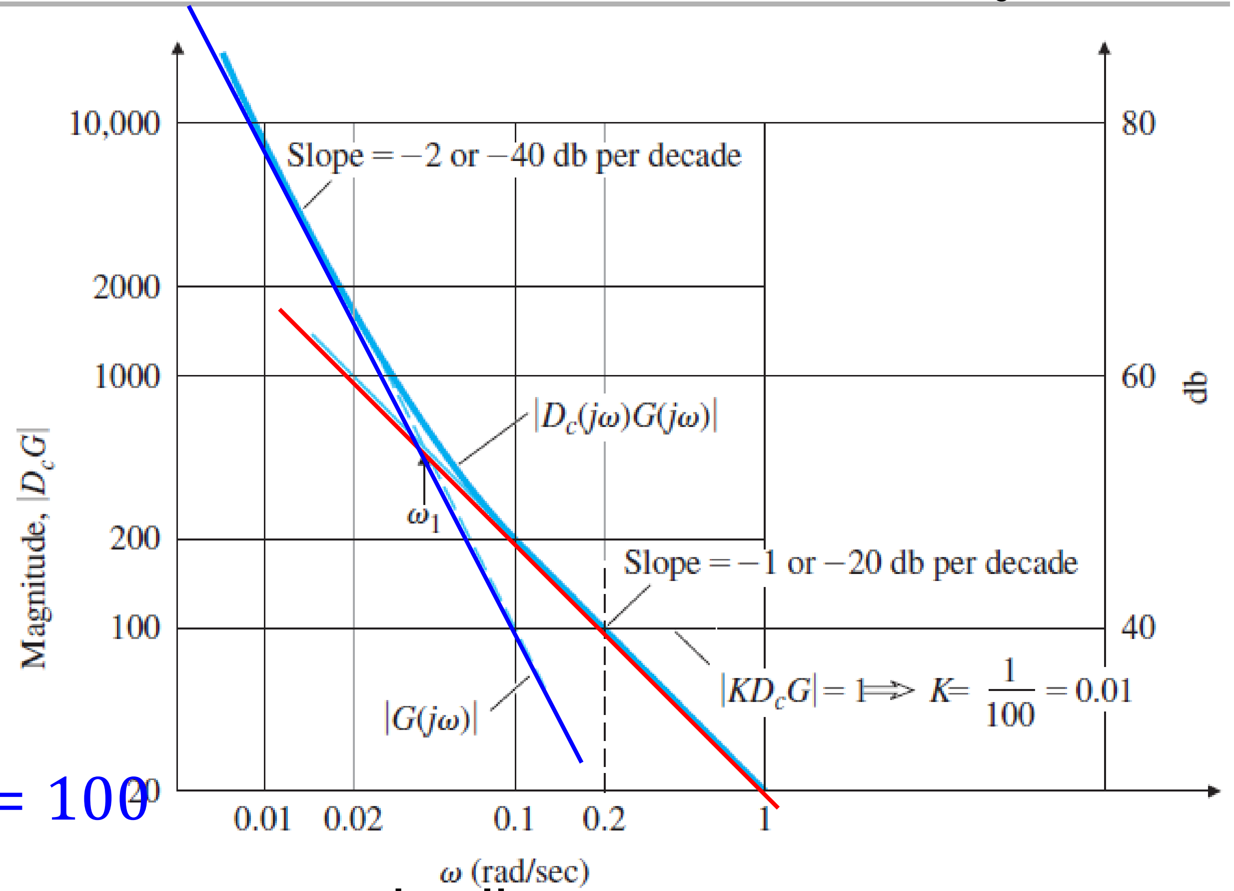
- Adjust K
- to provide desired bandwidth
- Adjust break point $\omega_1 = 1/T_D$
- to provide the -1 slope

  at the crossover frequency

0.2 rad/sec

Example

1. Plot |G(jw)|
2. Modify the plot to include $|D_c(jw)|$ with $\omega_1 = 0.05$, $T_D = 20$ → slope=-1 at 0.2

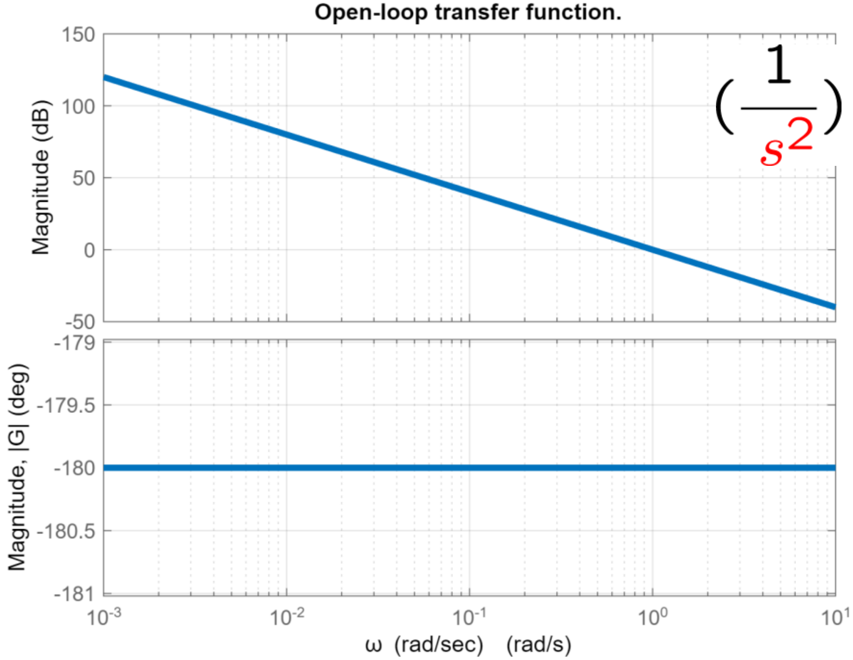


3. Determine $|D_c\ G\ | = 100$ where the $|D_c\ G\ |$ curve crosses the line $\omega = 0.2$
4. Compute

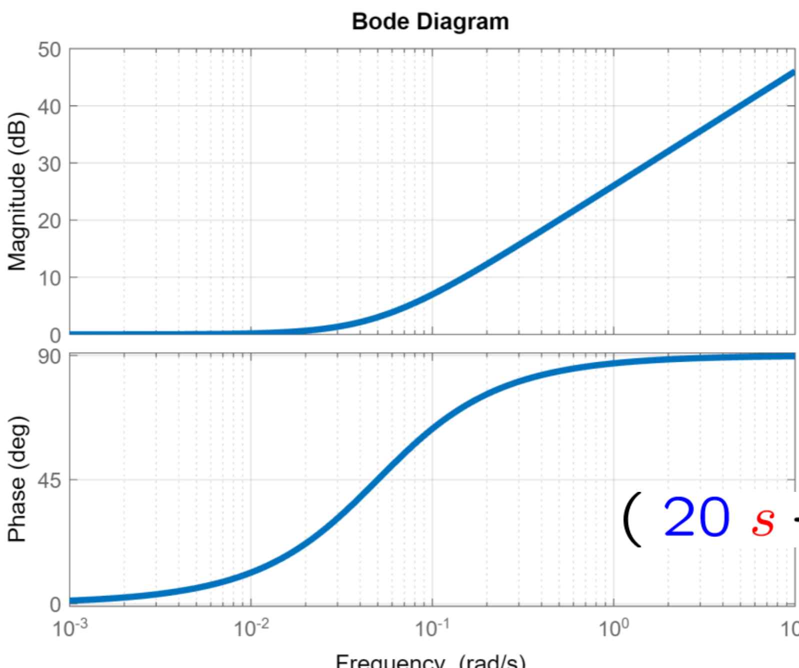
$$K = \frac{1}{|D_cG|_{w=0.2}} = \frac{1}{100} = 0.01$$

$$\Rightarrow K\ D_c(s) = 0.01\ (\ 20\ s + 1\ )$$

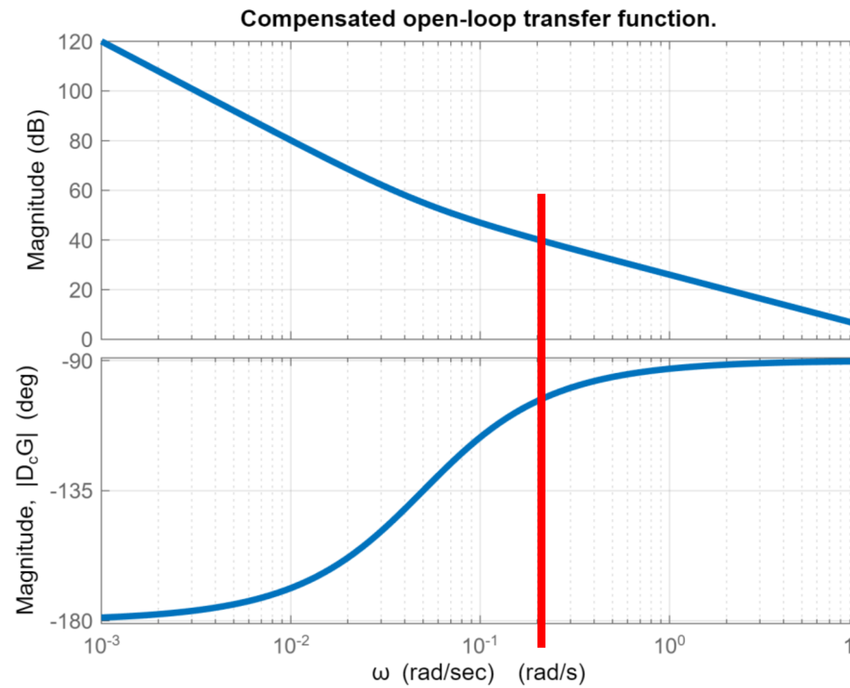
Example



$$\left(\frac{1}{s^2}\right)$$

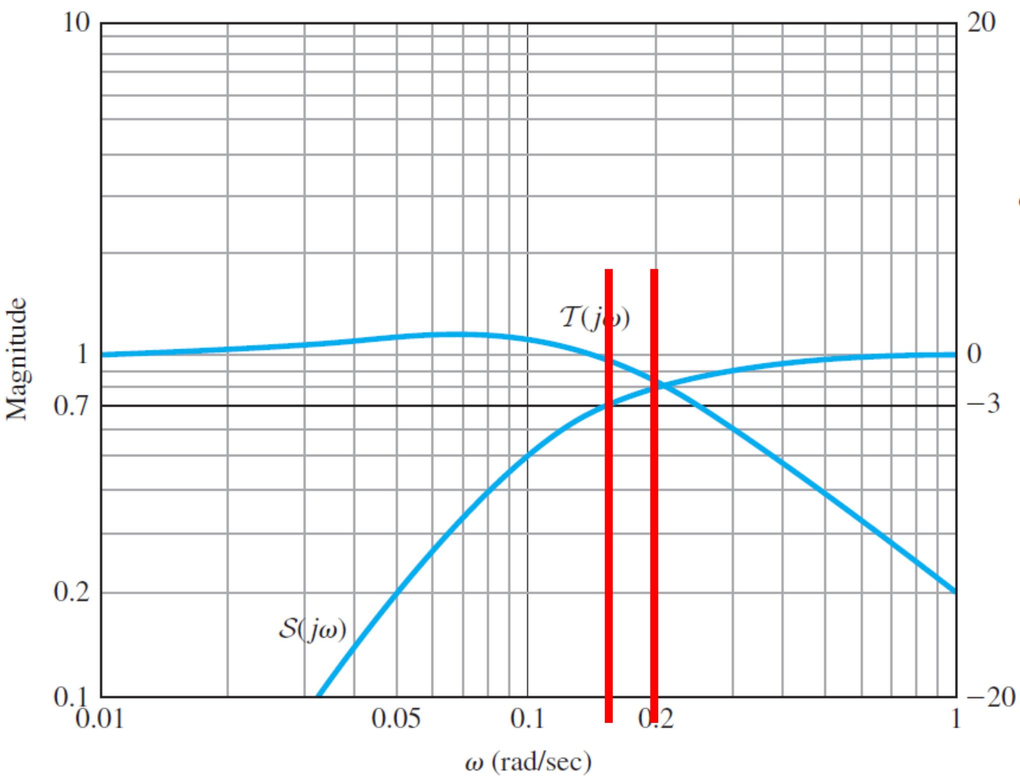


$$(\,20\,s + 1\,)$$



$$\frac{(20\,s + 1\,)}{s^2}$$

- The closed-loop frequency-response magnitude *T(jw)* and the sensitivity function *S(jw)*
- Desired Bandwidth = 0.2 rad/sec
- Disturbance Rejection (-3db) at 0.15 rad/sec



- Overshoot = 14%