Fall 2022 (111-1)

控制系統
Control Systems

Unit 6C
Non-Minimum Phase and Steady-State Errors

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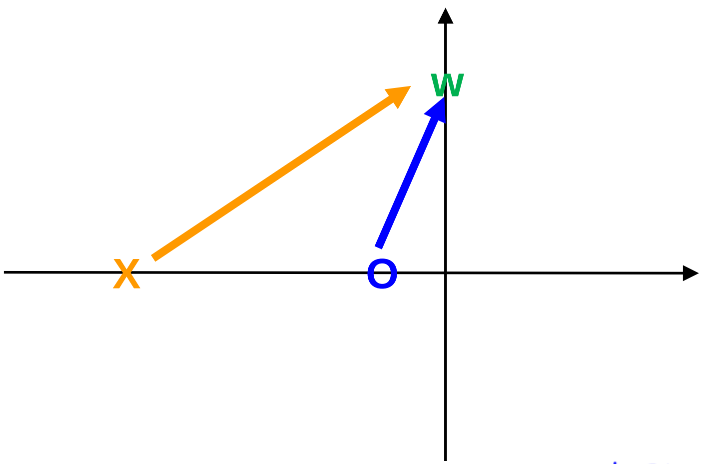
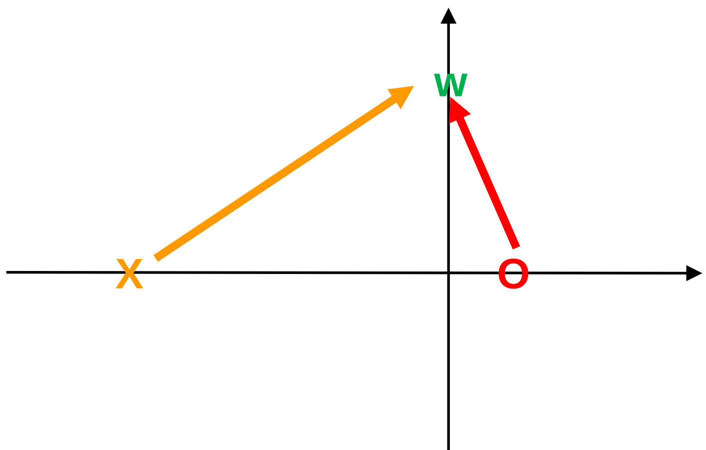
▪ Non-Minimum-Phase Systems

$$G_1(s) \;=\; 10\,\frac{s+1}{s\;+\;10} \qquad\qquad G_2(s) \;=\; 10\,\frac{s-1}{s\;+\;10}$$





Magnitude: $|G_1(s)| \;=\; |G_2(s)|$

Phase: $\angle G_1(s), \angle G_2(s)$: different

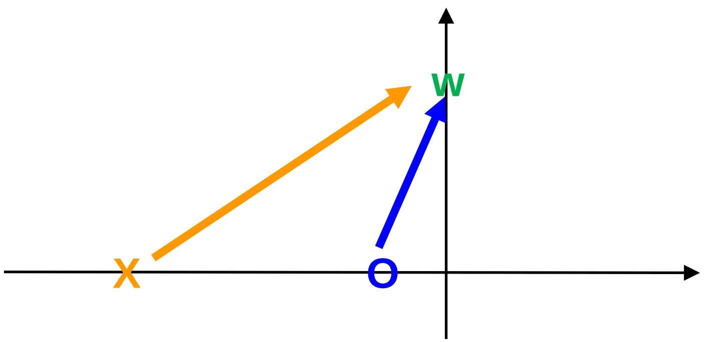
For this example,

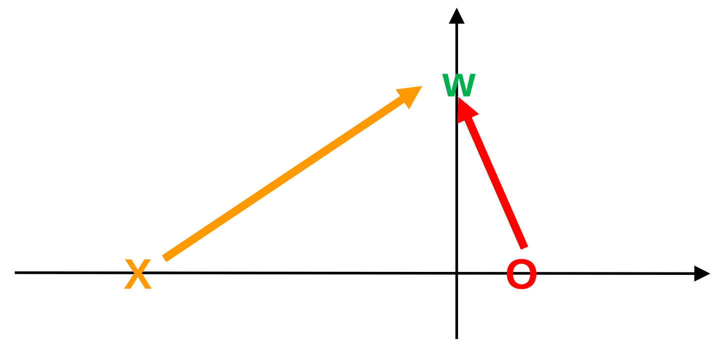
$\rightarrow$ small w : $|\angle G_1(s) - \angle G_2(s)| = 180^o$

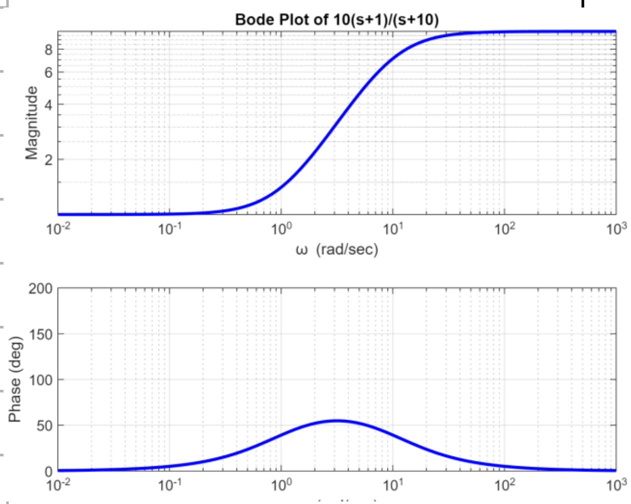
$\rightarrow$ large w : $|\angle G_1(s) - \angle G_2(s)| = 0^o$

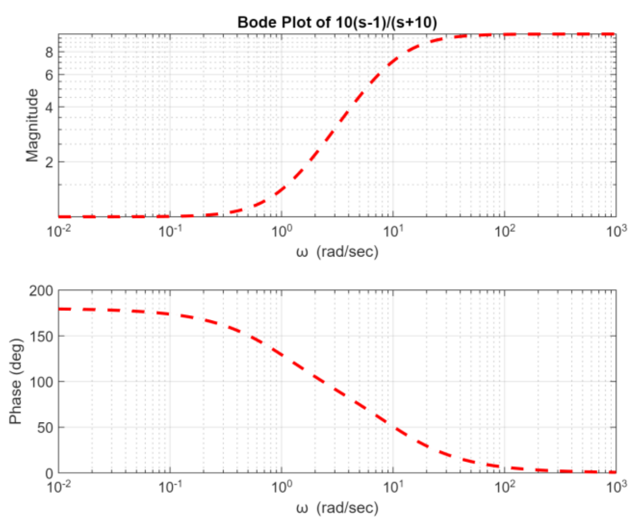
Non-Minimum-Phase Systems

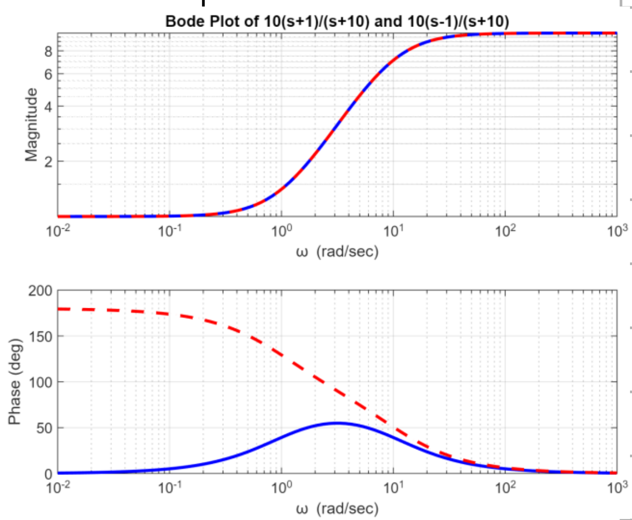
$$G_1(s) = 10\,\frac{s+1}{s+10} \qquad G_2(s) = 10\,\frac{s-1}{s+10}$$











→ small w : $|\angle G_1(s) - \angle G_2(s)| = 180^o$

→ large w : $|\angle G_1(s) - \angle G_2(s)| = 0^o$

Steady-State Errors

- In Section 4.2,

Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

  - As the gain of the open-loop transfer function increases,
  - Steady-State Error of a feedback system decreases.

- In Section 6.1.1,

$$K\,G(jw) = K_0\,(jw)^n\,\frac{(jw\tau_1+1)\,(jw\tau_2+1)\cdots}{(jw\tau_a+1)\,(jw\tau_b+1)\cdots}$$

  - At very low frequencies, OL TF is approximated by

$$K\,G(jw) \approx K_0\,(jw)^n$$

  - Larger the magnitude on low-frequency asymptote,
  - Lower steady-state errors

Steady-State Errors

$$K\ G(jw) \approx K_0\ (jw)^n$$

Errors as a Function of System Type			
Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

- For n = 0, a Type 0 system,

$$e_{ss} = \frac{1}{1 + K_p}$$

- For n = -1, a Type 1 system,
  - The low-frequency asymptote has a slope of -1.
  - The gain = K_o/ω and the velocity-error constant is:

$$K_v = K_o$$

  - For a unity-feedback system with a unit-ramp input,
  - The steady-state error is:

$$e_{ss} = \frac{1}{K_v}$$

- The easiest way of determining K_v in a Type 1 system
- is to read the magnitude of the low-frequency asymptote

  at ω = 1 rad/sec,
- because this asymptote is $A(\omega) = K_v / \omega$.

- In some cases, the lowest-frequency break point

  will be below ω = 1 rad/sec;
- Therefore, the asymptote needs to extend to ω = 1 rad/sec

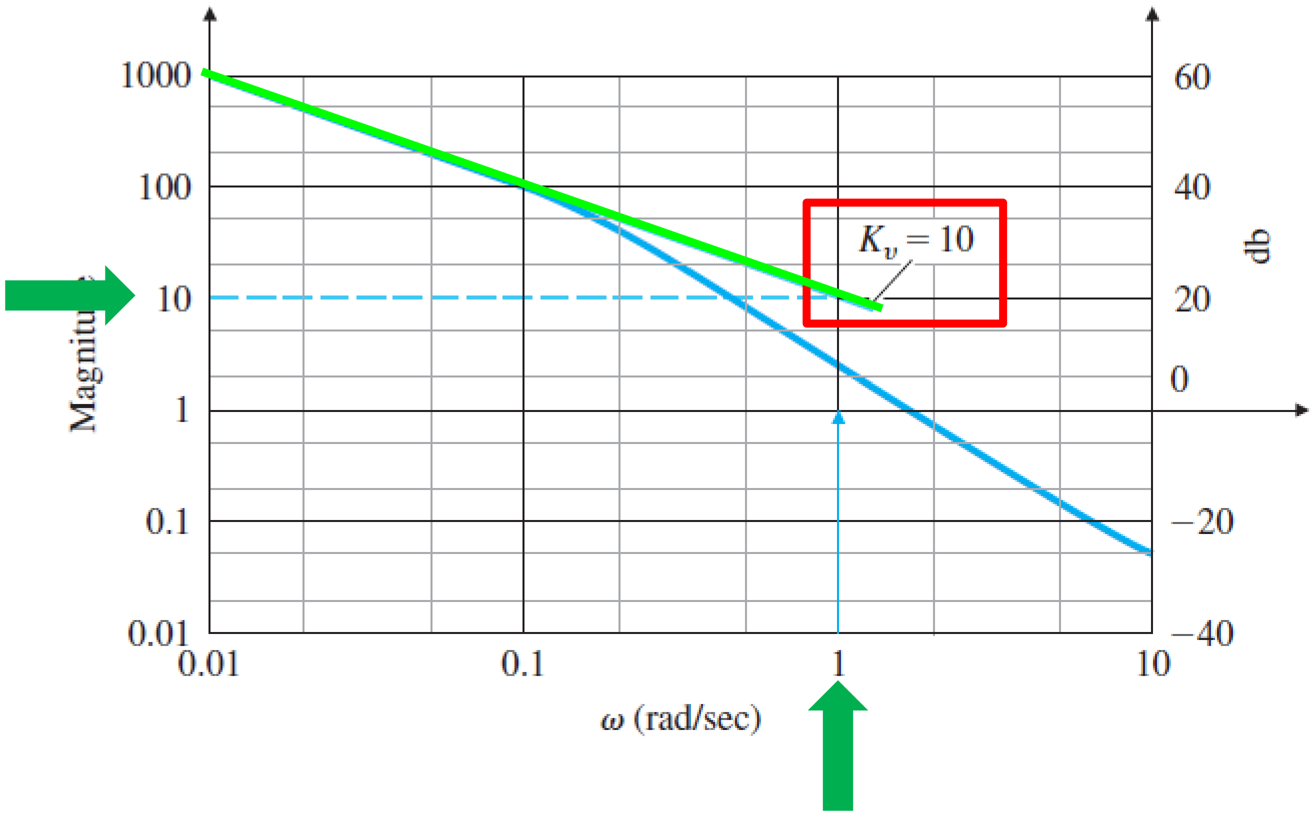
  in order to read K_v directly.

▪ Example 6.7: Computation of K_v

$$KG(s) = \frac{10}{s(s+1)}$$

- slope = -1
- Type 1 system

$$e_{ss} = \frac{1}{K_v} = 0.1$$



- Or, by A(ω) = K_v / ω → 1000 = K_v / 0.01 → K_v = 10
- → 100 = K_v / 0.1 → K_v = 10