Fall 2022 (111-1)

控制系統
Control Systems

Unit 6B
Bode Plot Techniques

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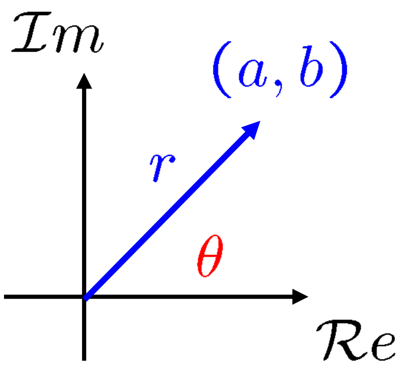
Bode Plot Techniques

- The hand plotting was developed by H.W. Bode at Bell Laboratories between 1932-1942.
- Now, most control system designers use computer programs to illustrate the Bode plot.
- However, it is still important to develop good intuition so that you can quickly identify erroneous computer result and perform sanity check and determine approximate result by hand
- The idea in Bode's method is to plot magnitude curves using a logarithmic scale and phase curves using a linear scale

Magnitude & Phase Representation:



$$a + jb \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = re^{j\theta}$$

$$|a + jb| = \sqrt{a^2 + b^2} \qquad \left|\frac{1}{a+jb}\right| = \frac{1}{\sqrt{a^2+b^2}}$$

$$\angle a + jb = \tan^{-1}(\tfrac{b}{a}) \qquad \angle\frac{1}{a+jb} = -\tan^{-1}(\tfrac{b}{a})$$

$$X(jw) = \mathcal{R}e\{X(jw)\} + j\ \mathcal{I}m\{X(jw)\} = |X(jw)|e^{j\angle X(jw)}$$

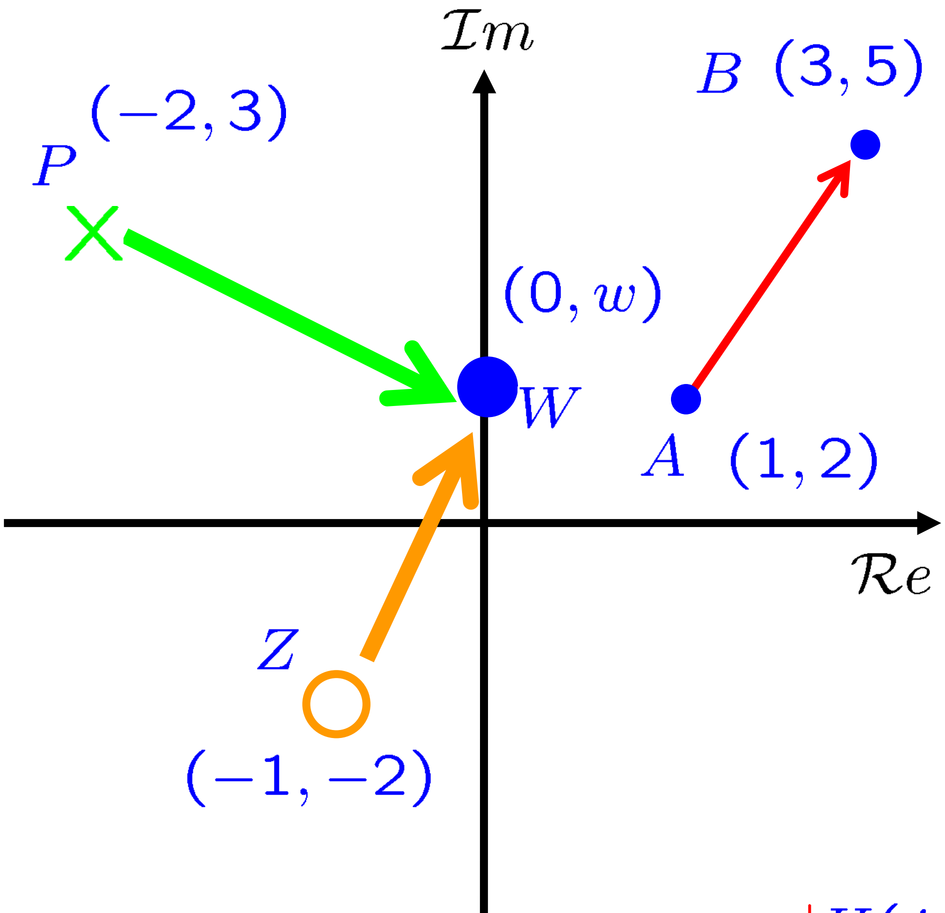
$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} = \left|X(e^{jw})\right|e^{j\angle X(e^{jw})}$$

$|X(jw)|$ or $|X(e^{jw})|$: magnitude

$\angle X(jw)$ or $\angle X(e^{jw})$: phase angle

Magnitude-Phase Representation in Complex Plane (s-plane)

- In s-plane:



$$\overrightarrow{AB} = (3+5j)-(1+2j) = 2+3j$$

$$\overrightarrow{AB} = (3,5)-(1,2) = (2,3)$$

$$|\overrightarrow{AB}| = \sqrt{2^2+3^2}$$

$$\angle\overrightarrow{AB} = \tan^{-1}\frac{(5-2)}{(3-1)}$$

$$H(s) = \frac{s-(-1-2j)}{s-(-2+3j)}$$

$$|H(jw)| = \frac{|jw-(-1-2j)|}{|jw-(-2+3j)|} = \frac{|\overrightarrow{ZW}|}{|\overrightarrow{PW}|}$$

$$\angle H(jw) = \angle\overrightarrow{ZW} - \angle\overrightarrow{PW}$$

- For example,

$$G(jw) = \frac{\vec{s_1}\ \vec{s_2}}{\vec{s_3}\ \vec{s_4}\ \vec{s_5}} = \frac{(r_1 e^{j\theta_1})(r_2 e^{j\theta_2})}{(r_3 e^{j\theta_3})(r_4 e^{j\theta_4})(r_5 e^{j\theta_5})}$$

$$= \frac{r_1 r_2}{r_3 r_4 r_5} e^{j(\theta_1+\theta_2-\theta_3-\theta_4-\theta_5)}$$

$$|G(jw)| = \frac{r_1 r_2}{r_3 r_4 r_5} \qquad \angle G(jw) = (\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$\log_{10}|G(jw)| = \log_{10} r_1 + \log_{10} r_2 - \log_{10} r_3 - \log_{10} r_4 - \log_{10} r_5$$

- Power db:

$$|G(jw)|_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$$

- Voltage db:

$$|G(jw)|_{\text{db}} = 20 \log_{10} \frac{V_2}{V_1}$$

Advantages of Using Bode Plot Techniques

- Advantages of working with Frequency Response

  in terms of Bode Plots

1. Dynamic compensator design

   can be based entirely on Bode plots.

2. Bode plots can be determined experimentally.

3. Bode plots of systems in series (or tandem) simply add,

   which is quite convenient.

4. The use of a log scale permits

   a much wider range of frequencies

   to be displayed on a single plot

   than is possible with linear scales.

Bode Form of the Transfer Function

- The open-loop transfer function:

$$K\ G(s) = K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$K\ G(jw) = K_0\ (jw)^n \frac{(jw\tau_1+1)\ (jw\tau_2+1)\cdots}{(jw\tau_a+1)\ (jw\tau_b+1)\cdots}$$

- For example,

$$K\ G(jw) = K_0 \frac{(jw\tau_1+1)}{(jw)^2\ (jw\tau_a+1)}$$

$$\angle\ K\ G(jw) = \angle\ K_0 + \angle\ (jw\tau_1+1) - \angle (jw)^2 - \angle (jw\tau_a+1)$$

$$\log|K\ G(jw)| = \log|K_0| + \log|(jw\tau_1+1)| - \log|(jw)^2| - \log|(jw\tau_a+1)|$$

$$|K\ G(jw)|_{\text{db}} = 20\log|K_0| + 20\log|(jw\tau_1+1)| - 20\log|(jw)^2| - 20\log|(jw\tau_a+1)|$$

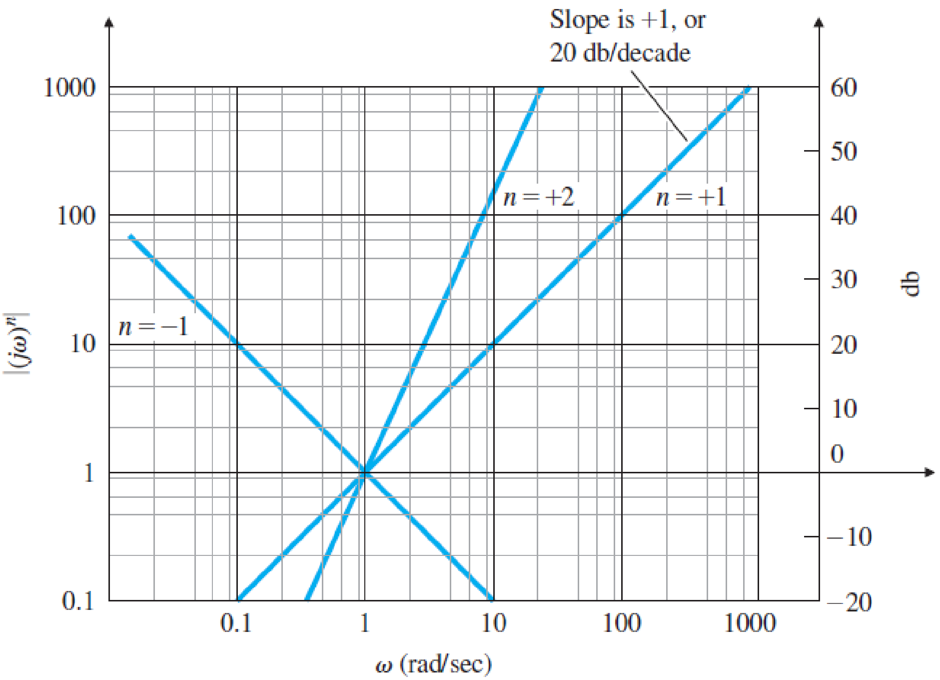
- Class 1: Singularities at the origin $K_0 \, (jw)^n$
- Class 2: First-order term $(jw\tau + 1)^{\pm 1}$
- Class 3: Second-order term $\left[\left(\frac{jw}{w_n}\right)^2 + 2\,\zeta\,\frac{jw}{w_n} + 1 \right]^{\pm 1}$

- Class 1: $K_0 \, (jw)^n$

$$\log K_0 |(jw)^n|$$

$$= \log K_0 + n \log |jw|$$

$$\angle K_0 \, (jw)^n = n \times 90^0$$



Classes of Terms of Transfer Function

- Class 2: Magnitude $(j w \tau + 1)^{\pm 1}$

a) For $\omega\tau << 1$, $j\omega\tau + 1 \cong 1$

b) For $\omega\tau >> 1$, $j\omega\tau + 1 \cong j\omega\tau$

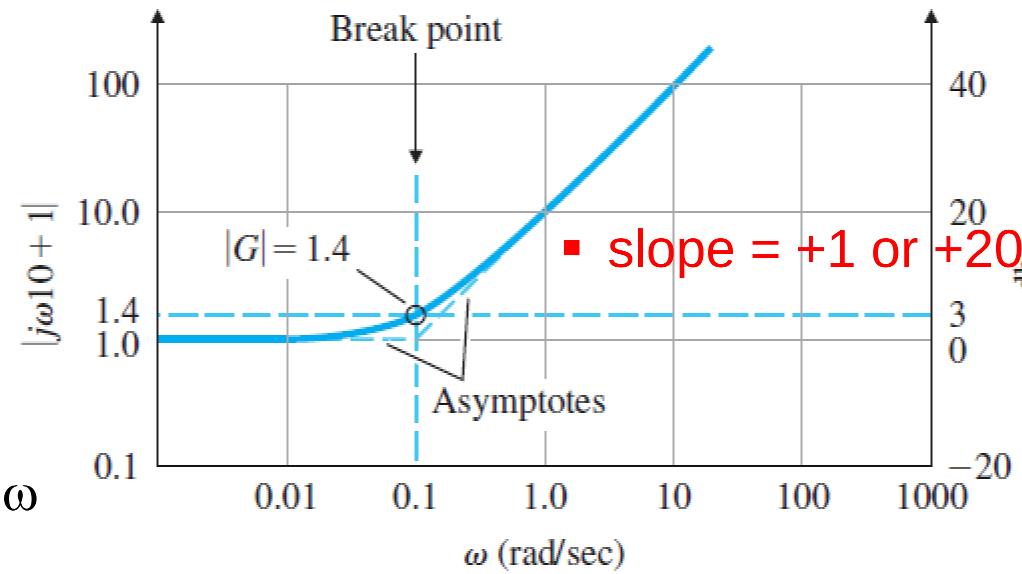
$$? \; G(s) = \frac{1}{10\,s + 1}$$

- $\omega = 1/\tau$: Break Point

- For example,

$$G(s) = 10\,s + 1$$

$$G(jw) = j(10w) + 1$$



- slope = +1 or +20db

a) For $10\omega << 1$, $j10\omega + 1 \cong 1$

b) For $10\omega >> 1$, $j10\omega + 1 \cong j10\omega$

- $\omega = 1/10$: Break Point

$$|G(j0.1)| = |j(1) + 1| = 1.414 = +3 \text{ db}$$

Classes of Terms of Transfer Function

- Class 2: Phase

$$(jw\tau + 1)^{\pm 1}$$

a) For $\omega\tau << 1$, $\angle 1 = 0^o$

b) For $\omega\tau >> 1$, $\angle j\omega\tau = 90^o$

c) For $\omega\tau \cong 1$, $\angle (j\omega\tau+1) \cong 45^o$

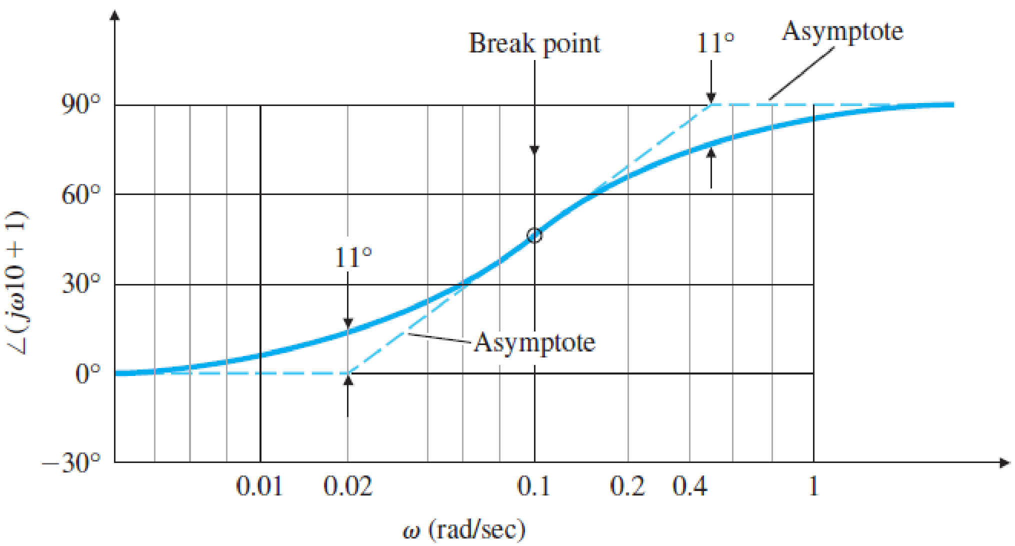
- $\omega = 1/\tau$: Break Point

- For example,

$$G(s) = 10\,s + 1$$

$$G(jw) = j(10w) + 1$$

- $\omega = 1/10$: Break Point



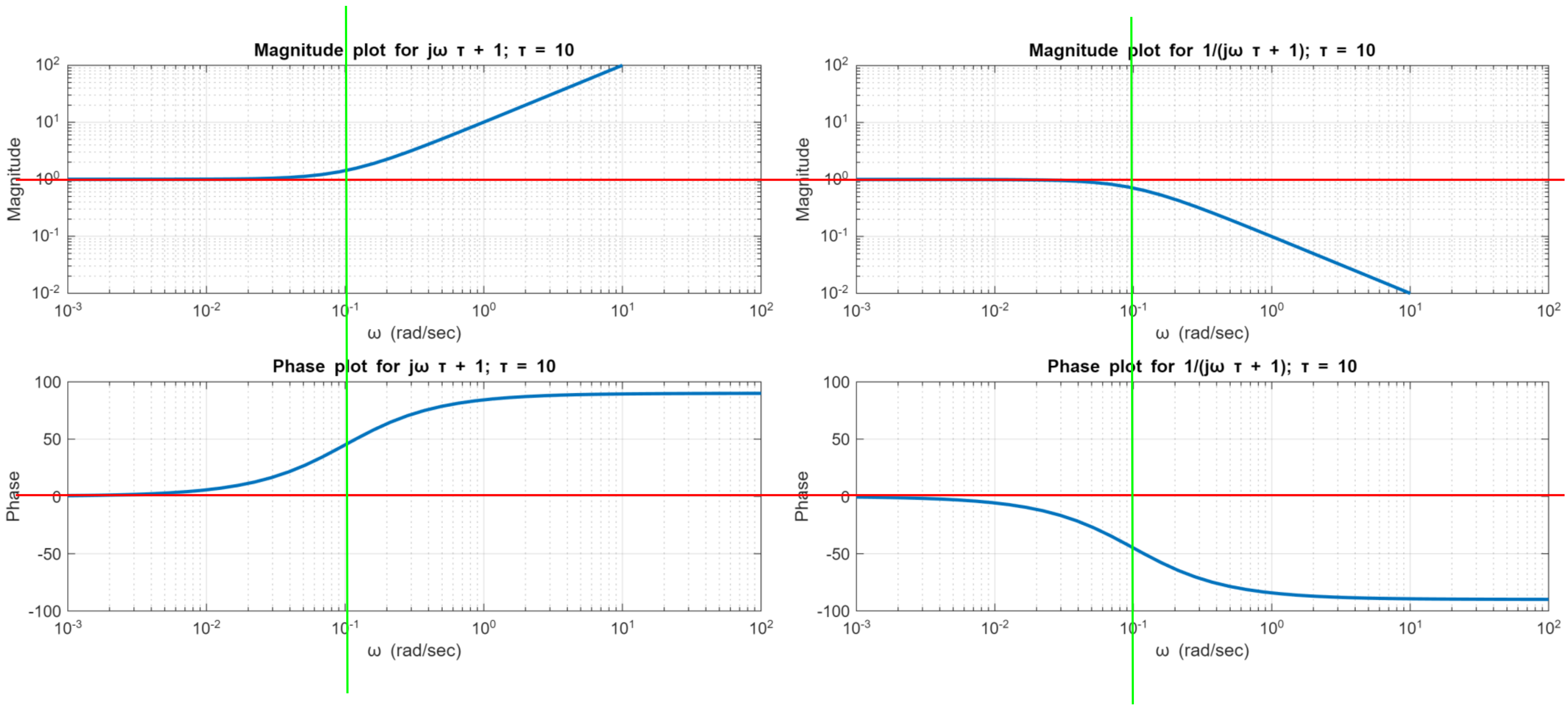
Classes of Terms of Transfer Function

- Class 2:

$$(j\omega\tau + 1)^{\pm 1}$$

$$G(s) = 10\,s + 1$$

$$G(s) = \frac{1}{10\,s + 1}$$



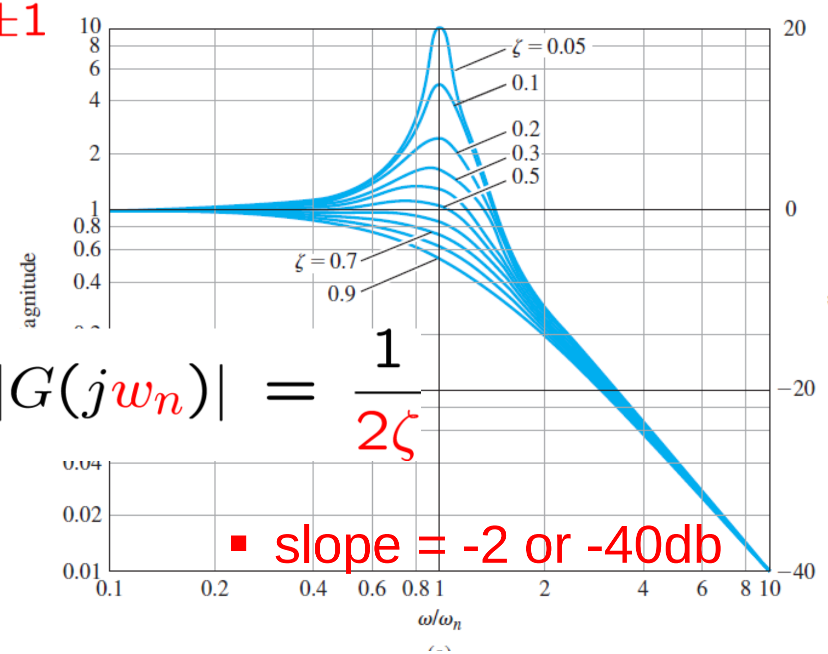
Classes of Terms of Transfer Function

- Class 3: $\left[\left(\frac{jw}{w_n} \right)^2 + 2\,\zeta\, \frac{jw}{w_n} + 1 \right]^{\pm 1}$

$$|\, G(s = jw) \,|$$

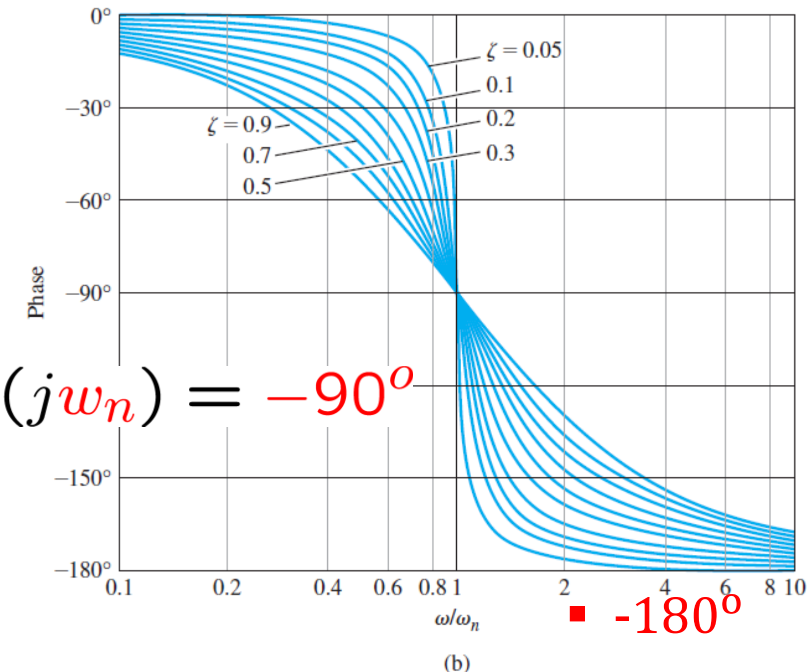
$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$



$$|G(jw_n)| = \frac{1}{2\zeta}$$

- slope = -2 or -40db



$$\angle G(jw_n) = -90^o$$

- -180^o

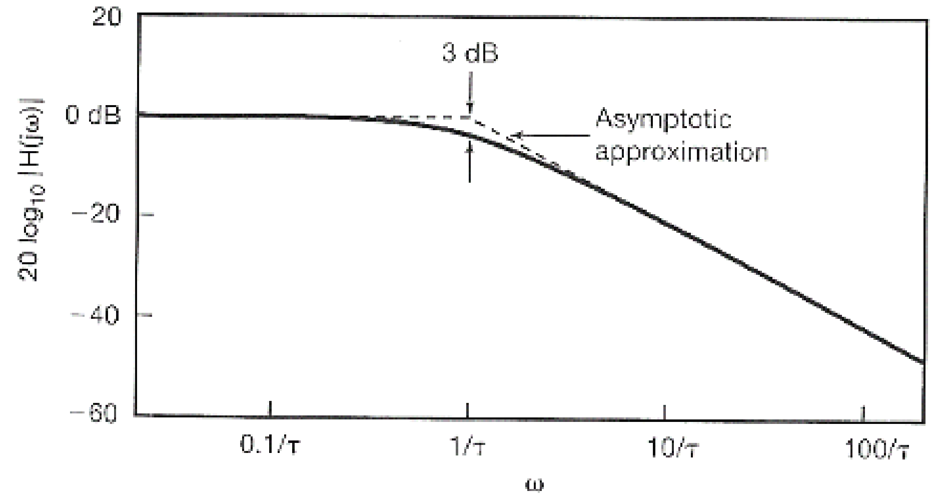
- First-Order CT Systems:

Signals & Systems

$$H(jw) = \frac{1}{jw\tau + 1}$$

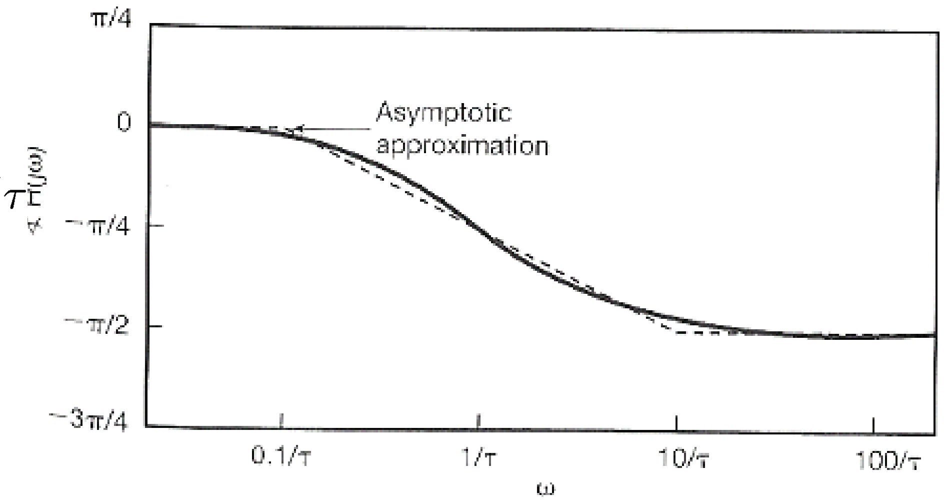
$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << 1/\tau \\ -10\log_{10}(2) \approx -3dB & w = 1/\tau \\ -20\log_{10}(w\tau) & w >> 1/\tau \\ \quad = -20\log_{10}(w) - 20\log_{10}(\tau) & \end{cases}$$



$$\measuredangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1/\tau \\ -(\pi/4)[\log_{10}(w\tau) + 1] & 0.1/\tau \le w \le 10/\tau \\ \quad = -(\pi/4)\left[\log_{10}(w) + \log_{10}(\tau) + 1\right] & \\ -\pi/4 & w = 1/\tau \\ -\pi/2 & w \ge 10/\tau \end{cases}$$



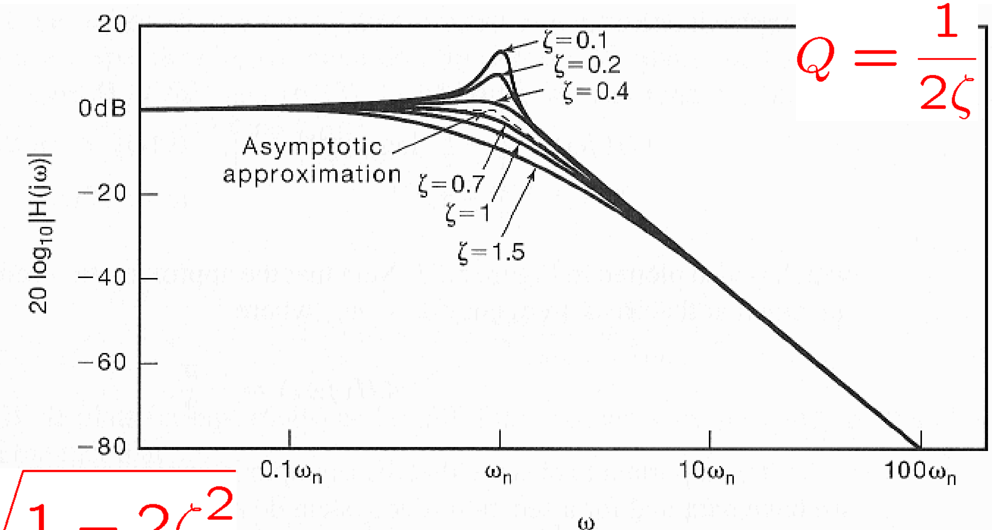
- Second-Order CT Systems: **Signals & Systems**

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

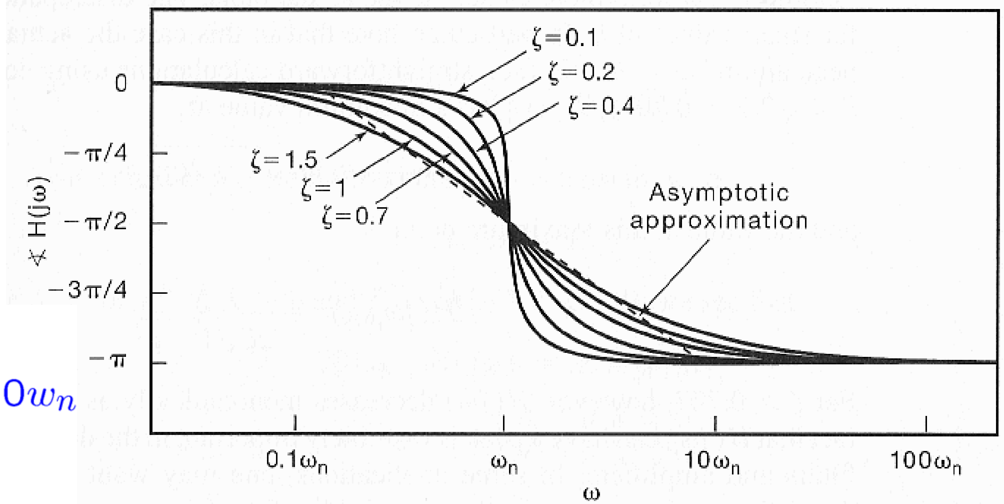
$$Q = \frac{1}{2\zeta}$$



- For $\zeta < \frac{\sqrt{2}}{2}$ $\quad w_{\max} = w_n\sqrt{1 - 2\zeta^2}$

$$\sphericalangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1w_n \\ -(\pi/2)[\log_{10}(w/w_n) + 1] & 0.1w_n \le w \le 10w_n \\ -\pi/2 & w = w_n \\ -\pi & w \ge 10w_n \end{cases}$$



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\,G(s) = \frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

(1) Break points

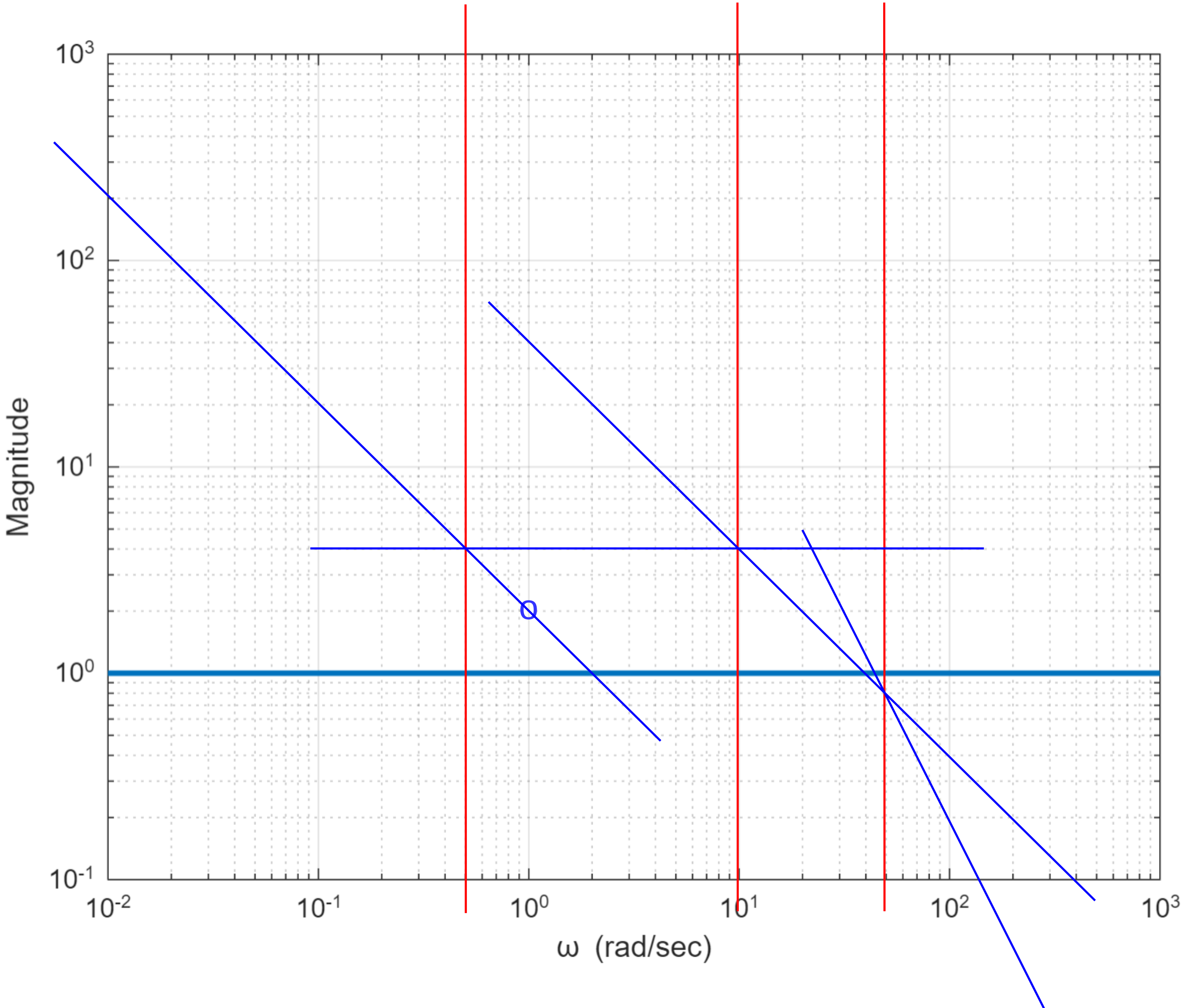
$$K\,G(jw) = \frac{2\,[\,\frac{jw}{0.5}+1\,]}{(jw)\,[\,\frac{jw}{10}+1\,]\,[\,\frac{jw}{50}+1\,]}$$

- Break points: 0.5, 10, 50

(2) Asymptotes

- Low-Frequency Asymptote: $K\,G(jw) = \frac{2}{(jw)}$ for $w < 0.1$
- ω << 0.5: slope = -1 (or -20 db/decade)
- 0.5 < ω < 10: slope = 0 (or 0 db/decade)
- 10 < ω < 50: slope = -1 (or -20 db/decade)
- 50 < ω: slope = -2 (or -40 db/decade)

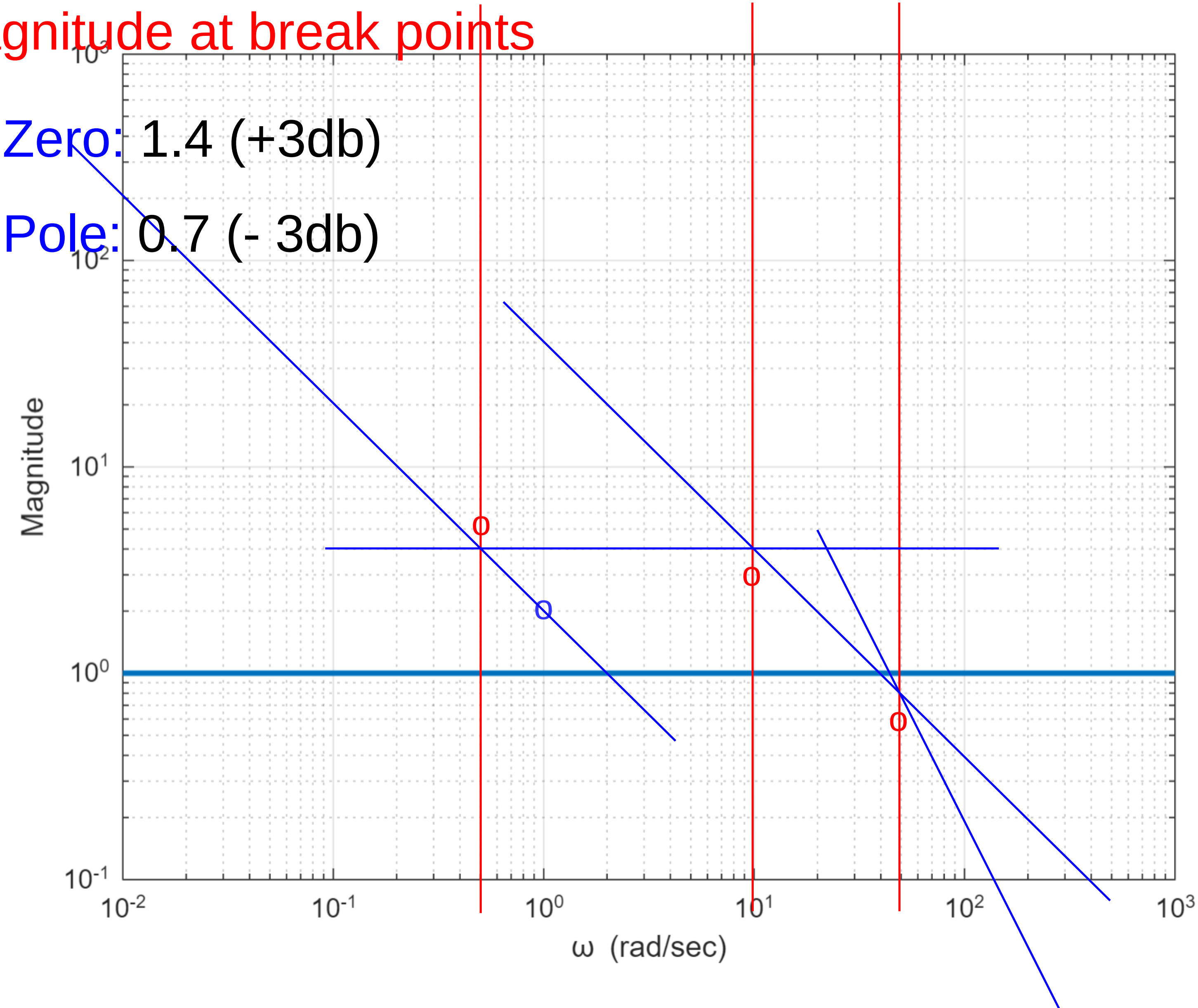
- Example 6.3: Bode Plot for Real Poles and Zeros



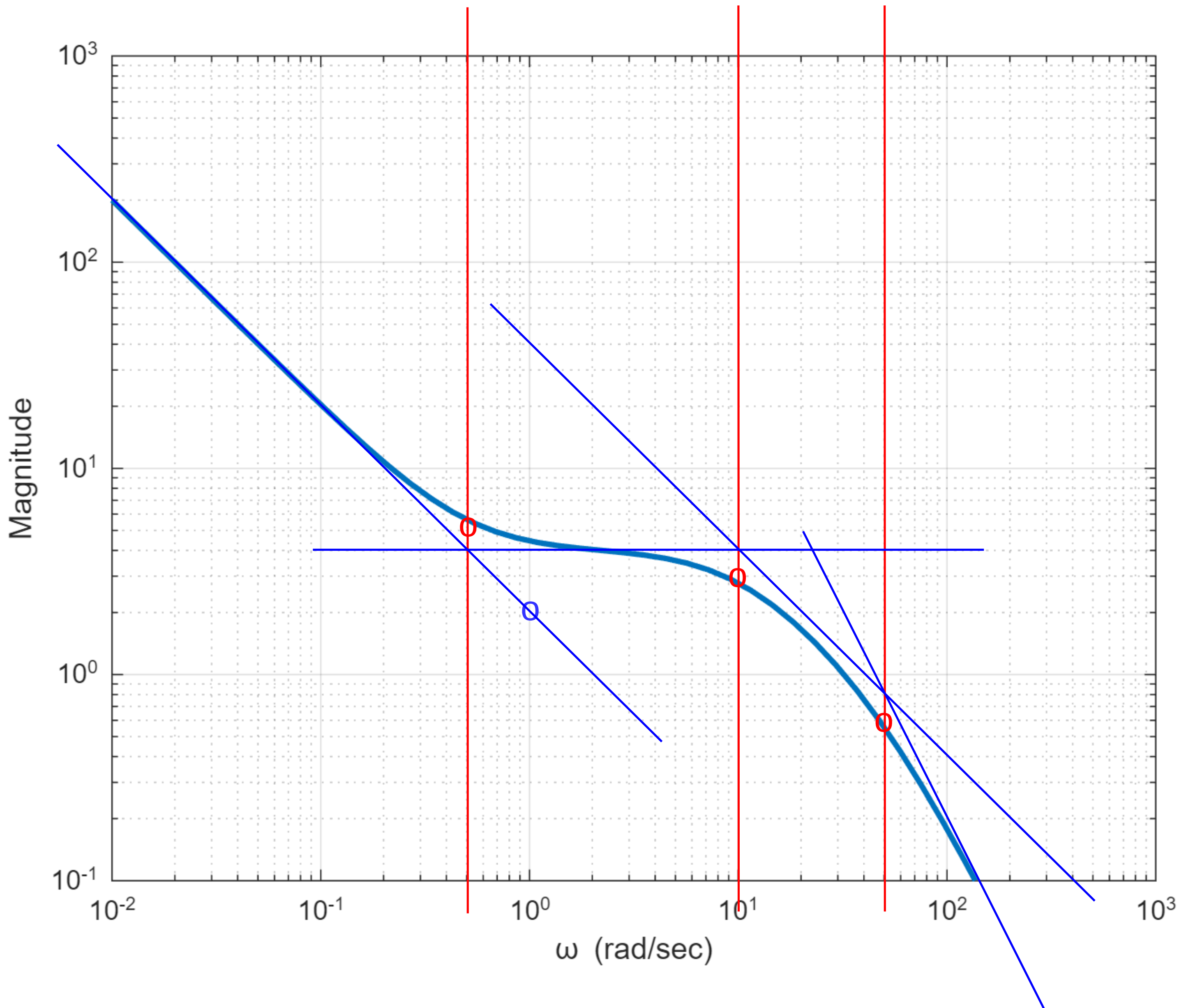
- Example 6.3: Bode Plot for Real Poles and Zeros

(3) Magnitude at break points

- By Zero: 1.4 (+3db)
- By Pole: 0.7 (- 3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\ G(s) = \frac{2000\ (s+0.5)}{s\ (s+10)\ (s+50)}$$

(1) Break points

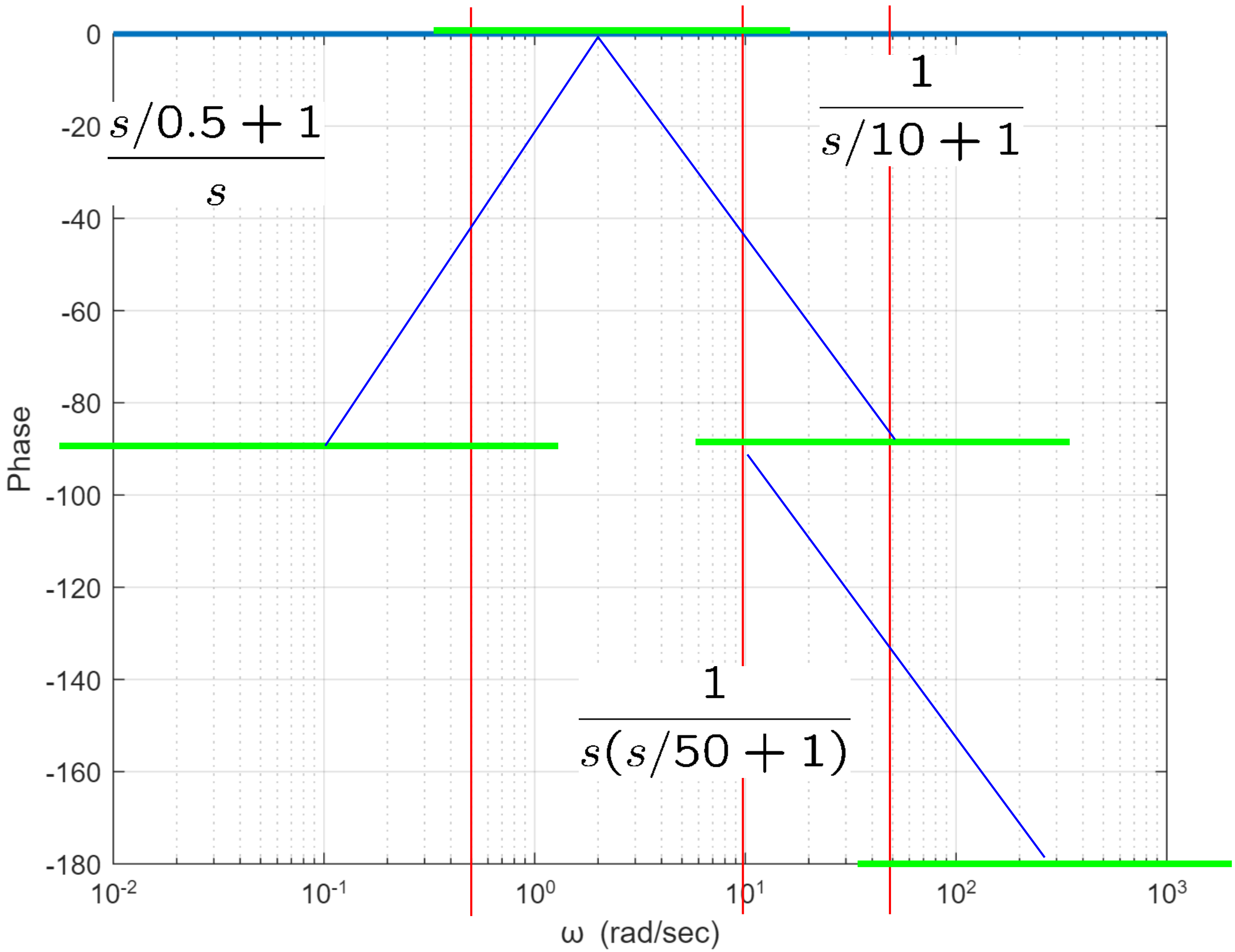
$$K\ G(jw) = \frac{2\ [\ \frac{jw}{0.5} + 1\ ]}{(jw)\ [\ \frac{jw}{10} + 1\ ]\ [\ \frac{jw}{50} + 1\ ]}$$

  - Break points: 0.5, 10, 50

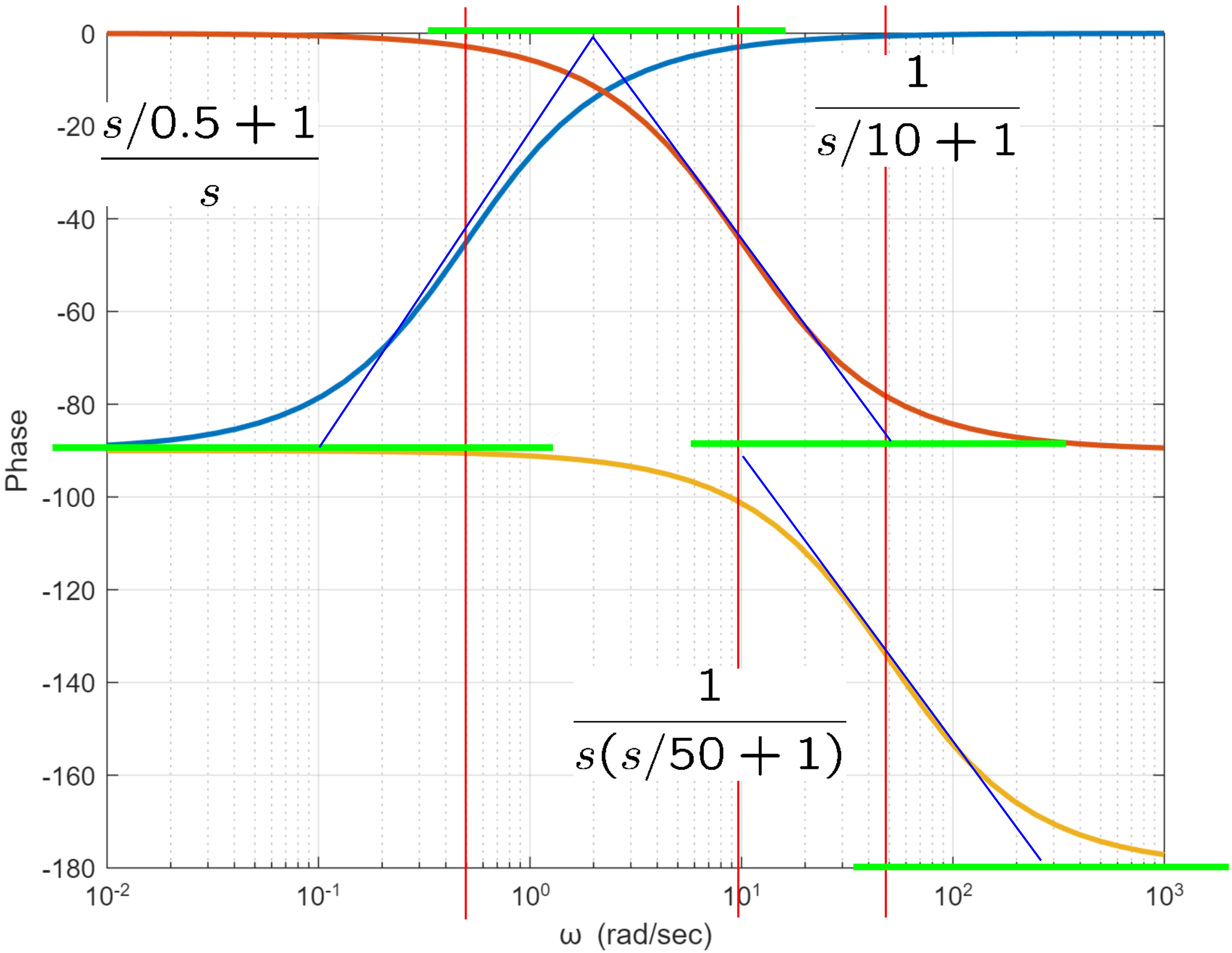
(4) Phase

  - Low-Frequency Asymptote: $K\ G(jw) = \frac{2}{(jw)}$ for $w < 0.1$
  - $\omega << 0.5$: phase = -90^o
  - $0.5 < \omega < 10$: phase = 0^o
  - $10 < \omega < 50$: phase = -90^o
  - $50 < \omega$: phase = -180^o

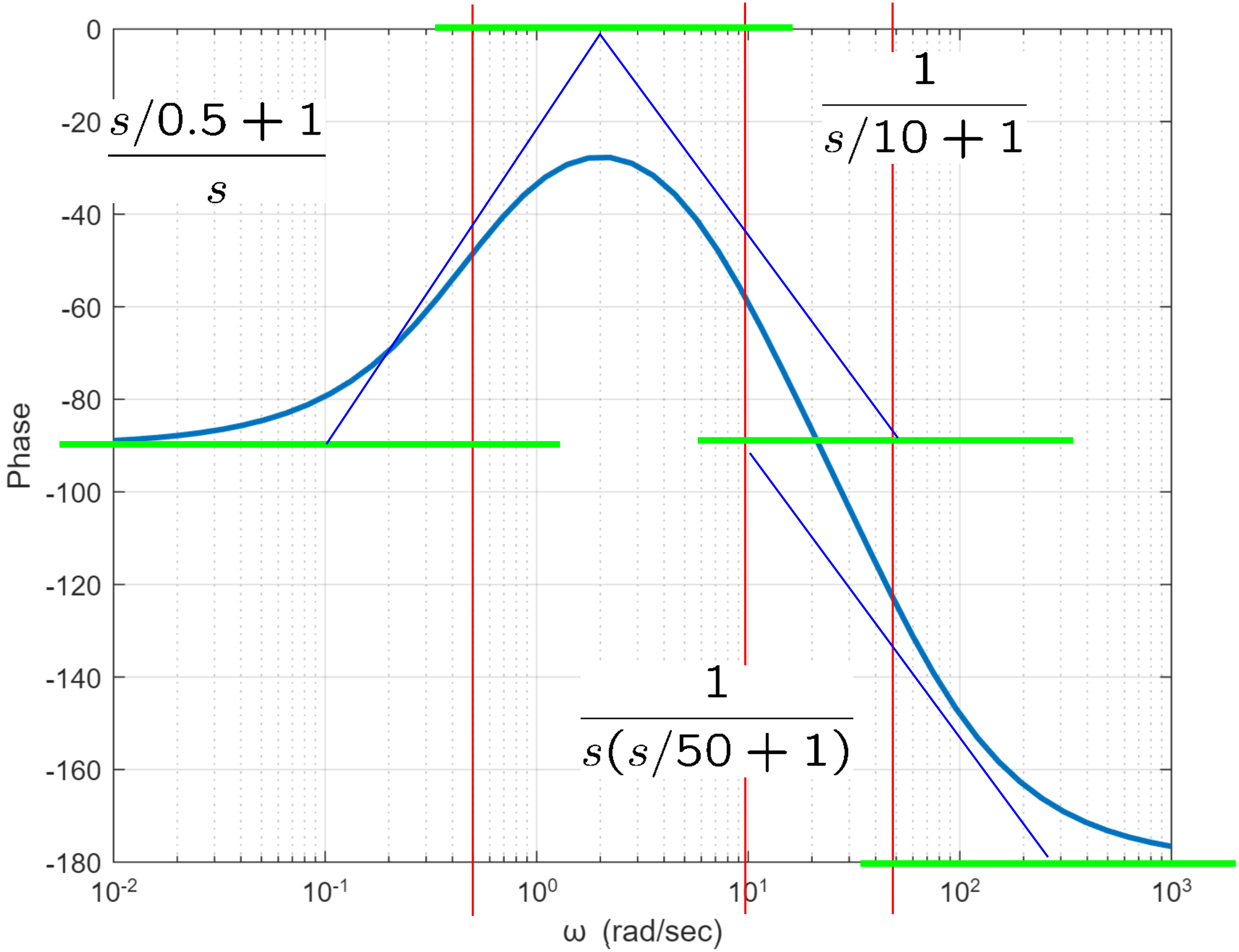
- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

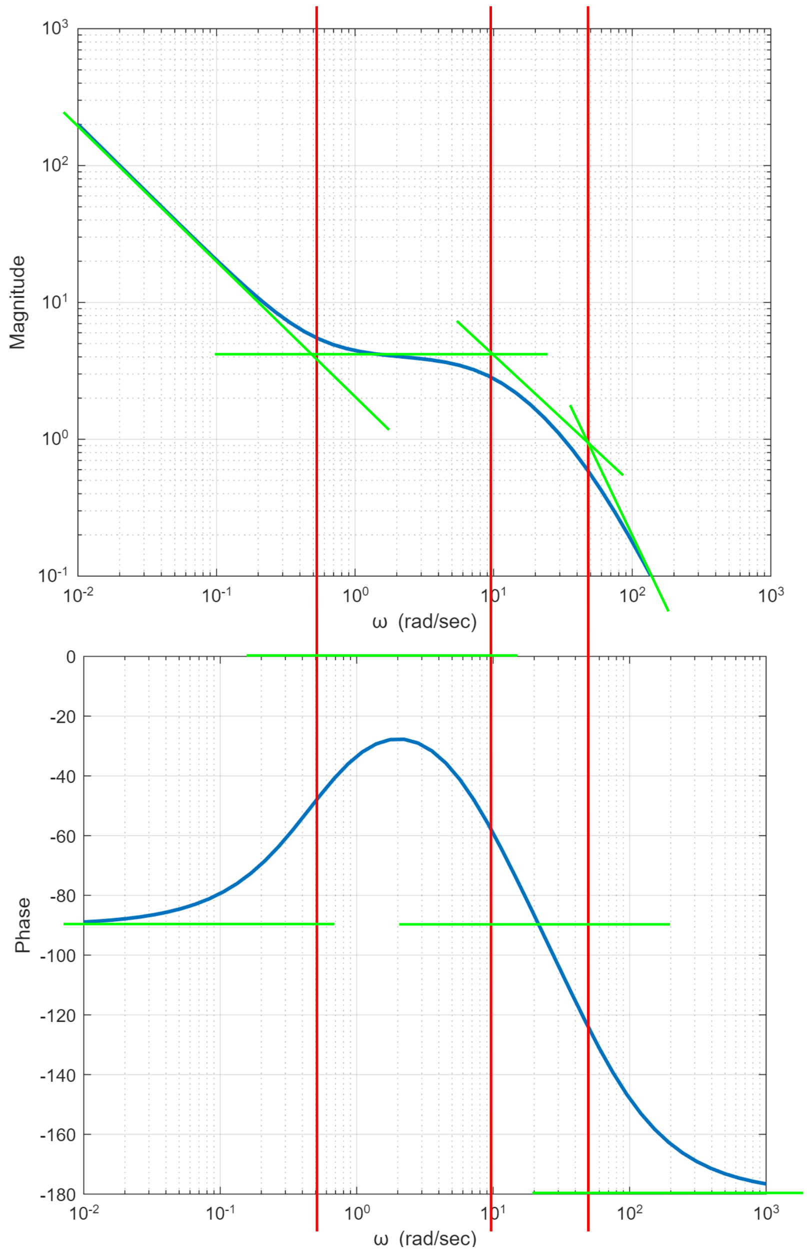


- Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

$$\frac{2\left[\,\frac{jw}{0.5}+1\,\right]}{(jw)\left[\,\frac{jw}{10}+1\,\right]\left[\,\frac{jw}{50}+1\,\right]}$$

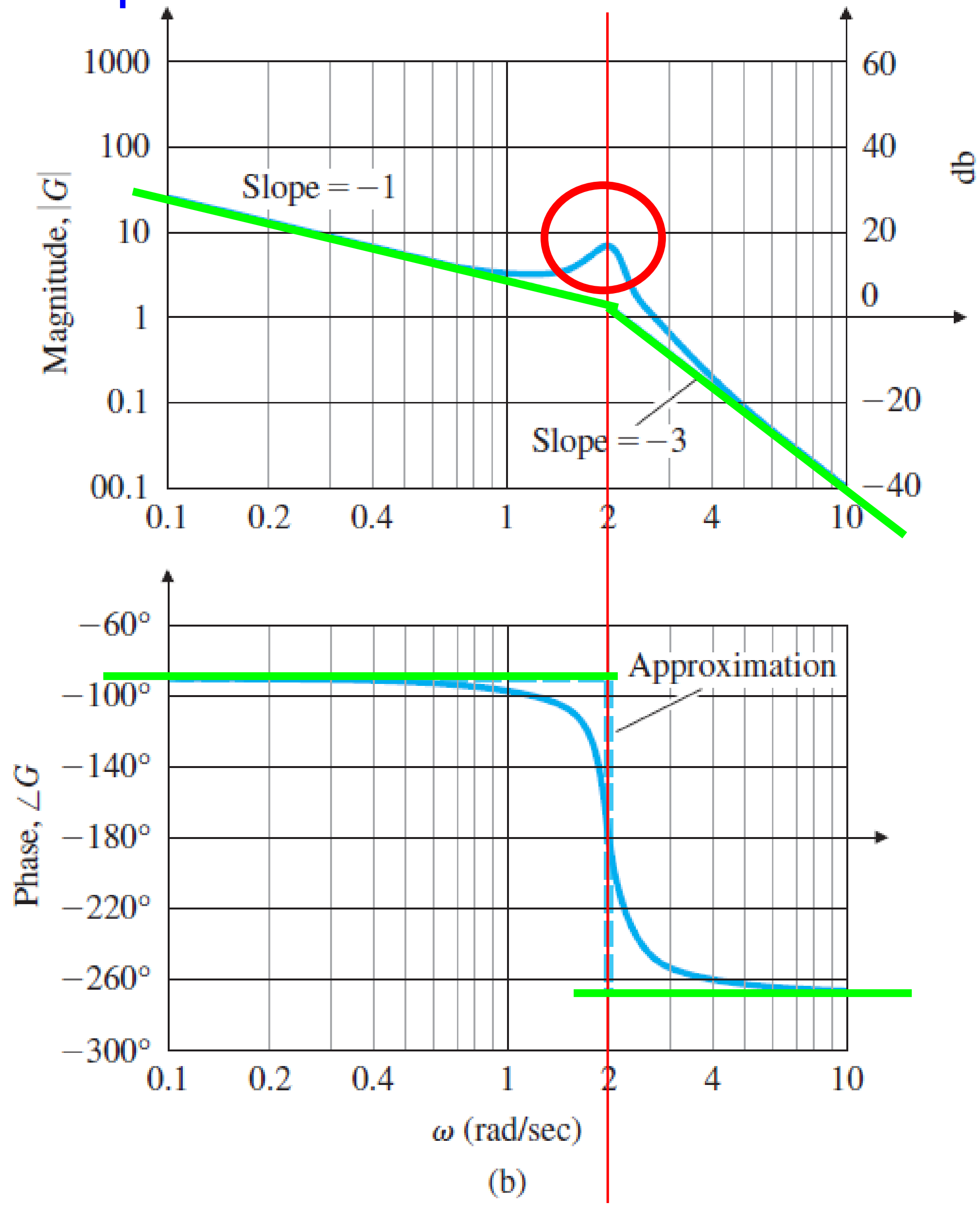
- Break points: 0.5, 10, 50



- Example 6.4: Bode Plot for Complex Poles

$$K\,G(s) = \frac{10}{s\,[s^2 + 0.4s + 4]}$$

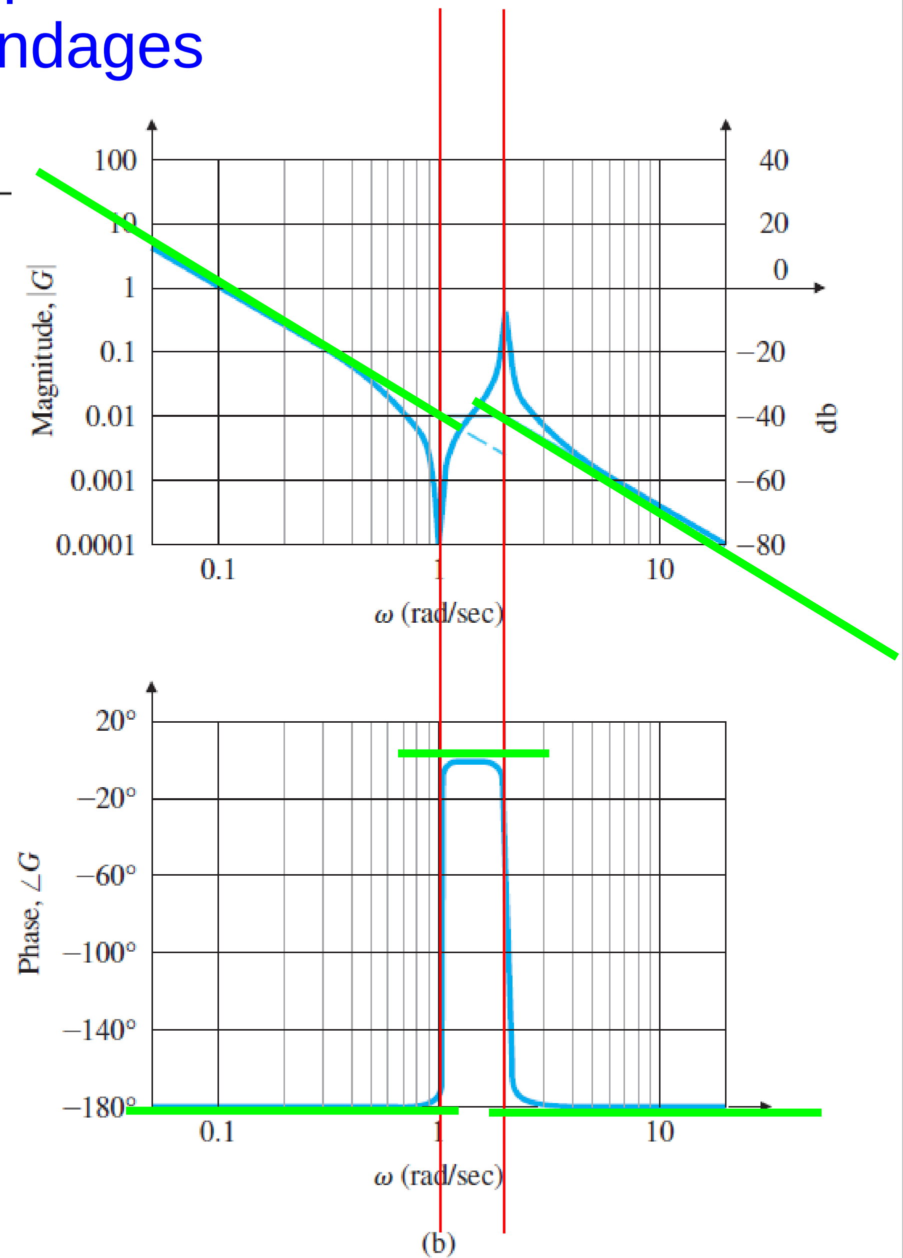
$$= \frac{10}{4}\,\frac{1}{s\,[\,\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1\,]}$$



- Break points: 2

- Example 6.5: Bode Plot for Complex Poles and Zeros: Satellite with Flexible Appendages

$$K\ G(s) = \frac{0.01\ [s^2 + 0.01s + 1]}{s^2\ [(\frac{s^2}{4}) + 0.02(\frac{s}{2}) + 1]}$$



- Break points: 1, 2

Example 6.6: Computer-Aided Bode Plot for Complex Poles and Zeros

```
num = 0.01*[1 0.01 1];
den  = conv([1 0 0],[.25 0.01 1]);

w = logspace(-2,2,1000);

[m,p] = bode( num, den, w);

subplot(2,1,1)
loglog( w, m, 'LineWidth', 2);

subplot(2,1,2)
semilogx( w, p, 'LineWidth', 2);
```