Fall 2022 (111-1)

控制系統
Control Systems

Unit 60
Bode Plot (w-Domain)

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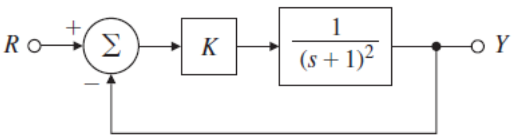
NTU-EE

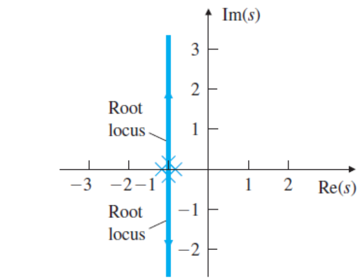
Sep 2022 – Dec 2022

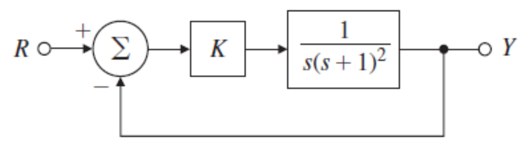
Unit 6

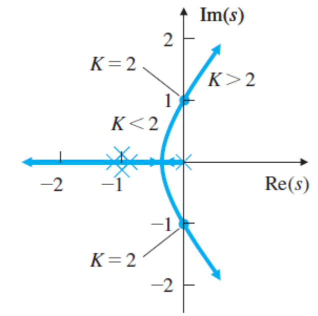
Bode Plot

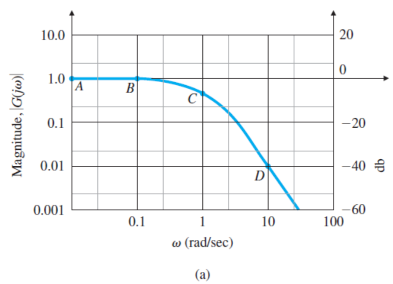
Key Ingredients

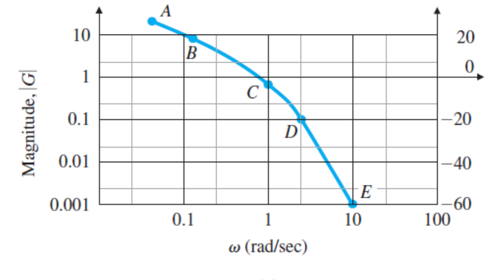


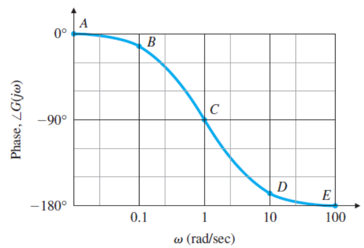


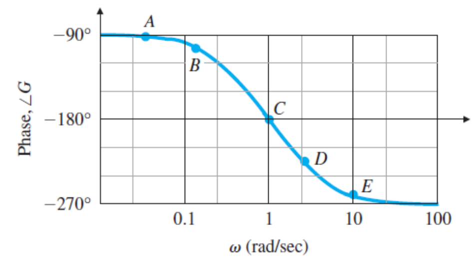


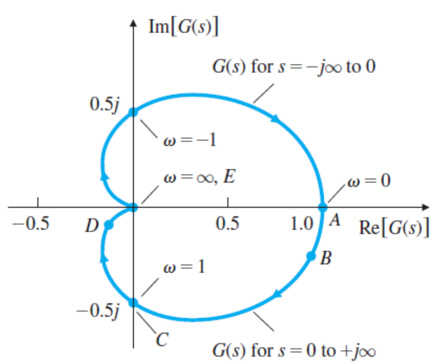


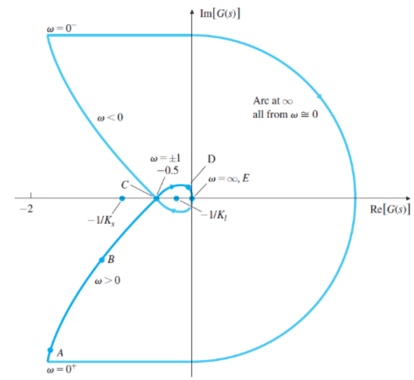












- Root Locus:
- Bode Plot:
- Nyquist Plot:

Your Jobs are To

- By Hand:
  - **Hand Writing in Exam (40%)**
- Use the Bode Plot Techniques

  to roughly sketch the magnitude and phase plots

  of any transfer function

  by identifying these critical frequency locations

- By Computer:
  - **Multiple Choice in Exam (60%)**
- Use Matlab codes

  to draw the exact Bode Plot, Nyquist Plot

  of any transfer function

- Design proper transfer function and

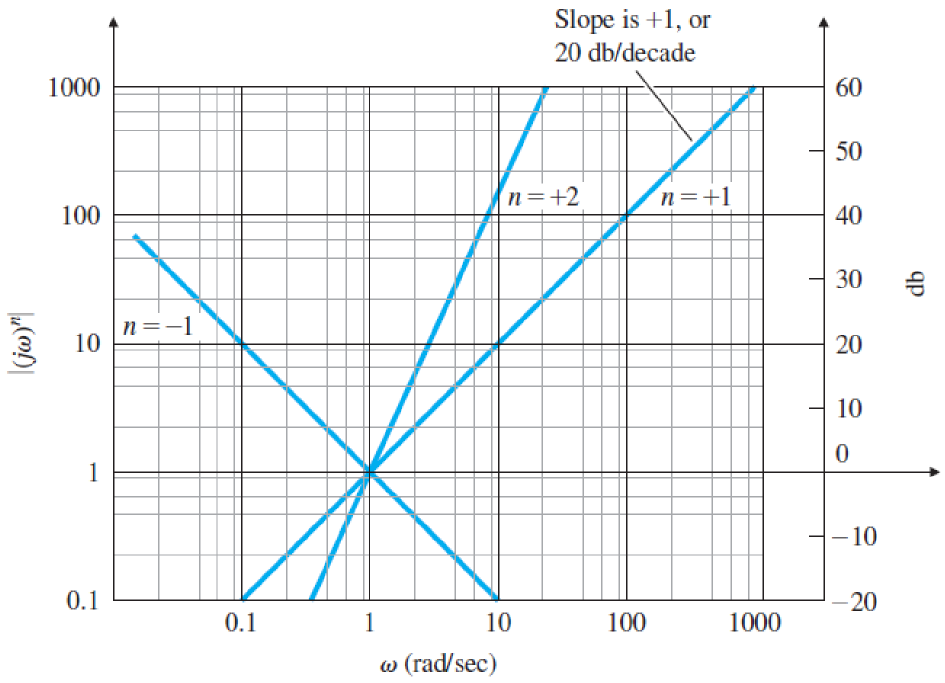
  select associated and reasonable gain value

- Class 1: Singularities at the origin $K_0 \; (jw)^n$

$$\log K_0 |(jw)^n|$$

$$= \log K_0 + n \log |jw|$$

$$\angle \; K_0 \; (jw)^n = n \times 90^0$$

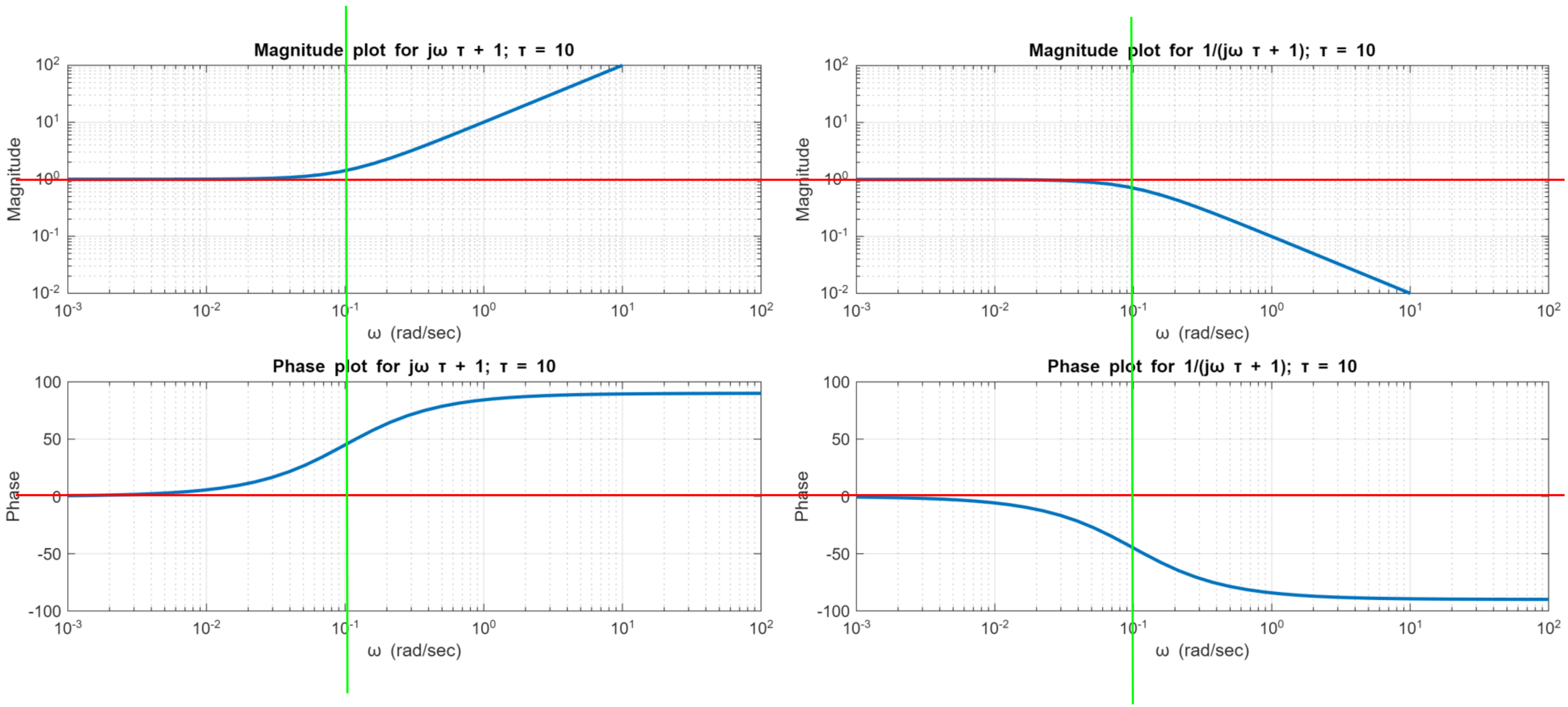


Your Jobs by Hands and Pens

- Class 2: Frist-order term

$$(jw\tau + 1)^{\pm 1}$$

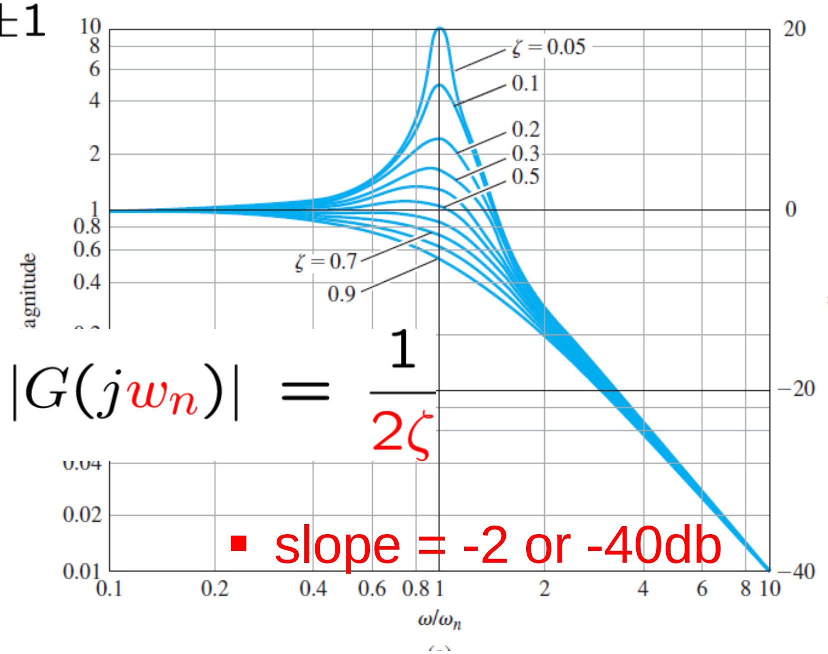
$$G(s) = 10\,s + 1 \qquad\qquad G(s) = \frac{1}{10\,s + 1}$$



Your Jobs by Hands and Pens

- Class 3: $\left[\left(\frac{jw}{w_n} \right)^2 + 2\zeta \frac{jw}{w_n} + 1 \right]^{\pm 1}$

$$|G(s = jw)|$$

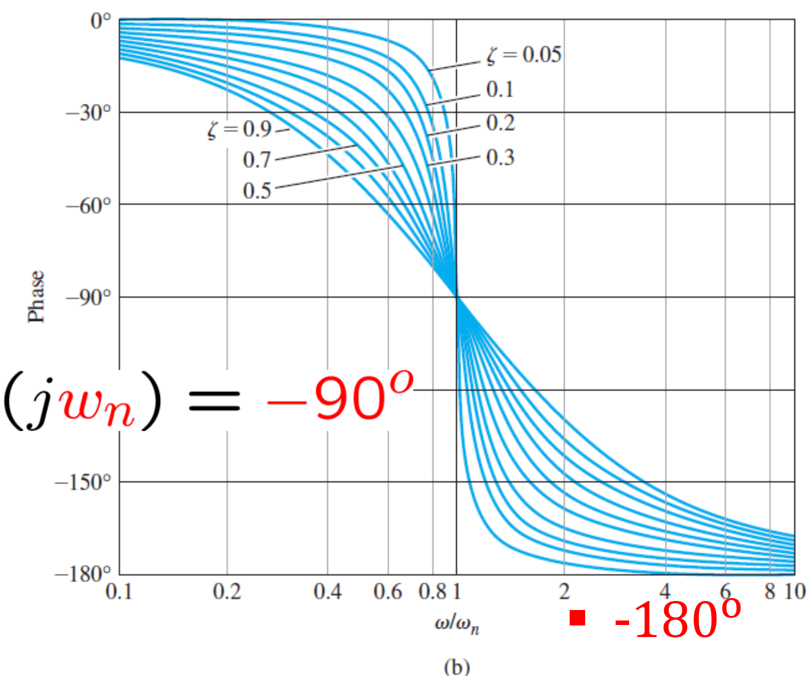


$$|G(jw_n)| = \frac{1}{2\zeta}$$

- slope = -2 or -40db

$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$



$$\angle G(jw_n) = -90^o$$

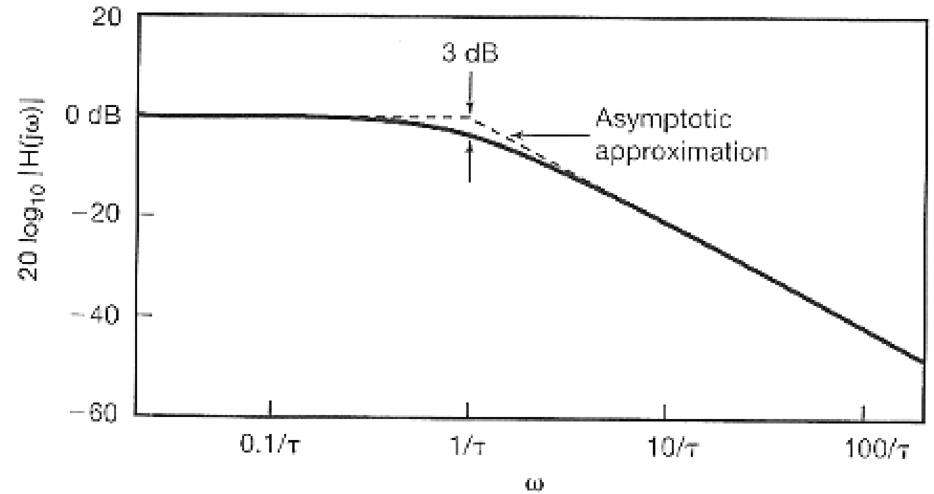
- -180^o

- First-Order CT Systems:

$$H(jw) = \frac{1}{jw\tau + 1}$$

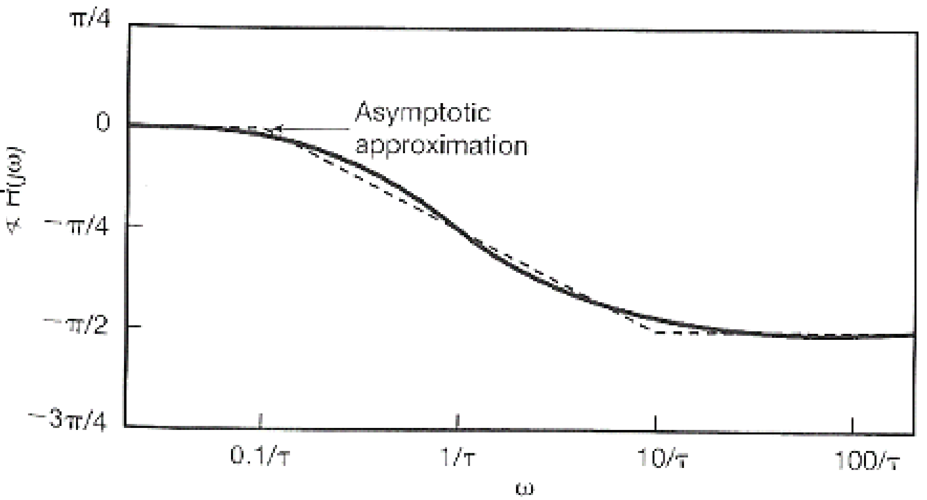
$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << 1/\tau \\ -10\log_{10}(2) \approx -3dB & w = 1/\tau \\ -20\log_{10}(w\tau) & w >> 1/\tau \\ \quad = -20\log_{10}(w) - 20\log_{10}(\tau) & \end{cases}$$



$$\measuredangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1/\tau \\ -(\pi/4)[\log_{10}(w\tau) + 1] & 0.1/\tau \le w \le 10/\tau \\ \quad = -(\pi/4)\left[\log_{10}(w) + \log_{10}(\tau) + 1\right] & \\ -\pi/4 & w = 1/\tau \\ -\pi/2 & w \ge 10/\tau \end{cases}$$

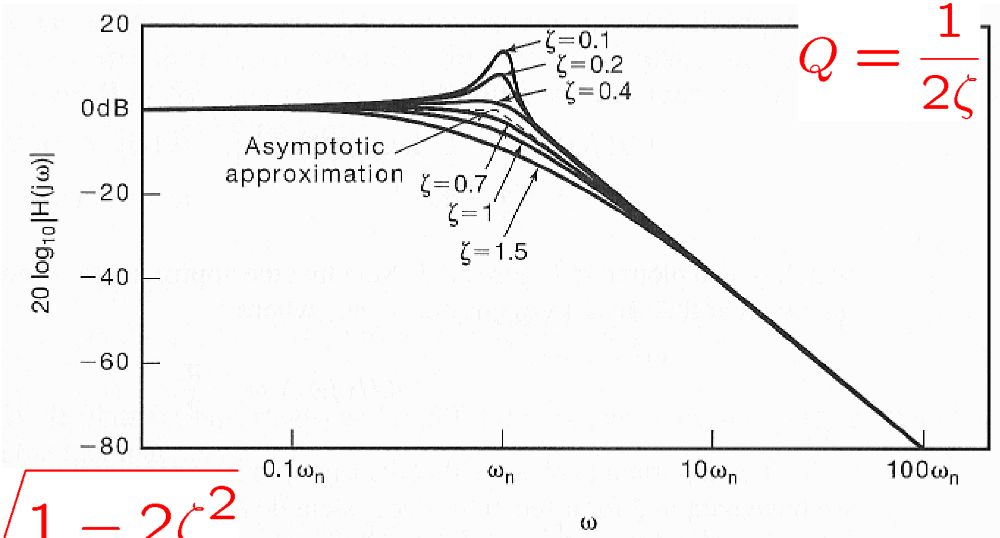


Second-Order CT Systems:

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

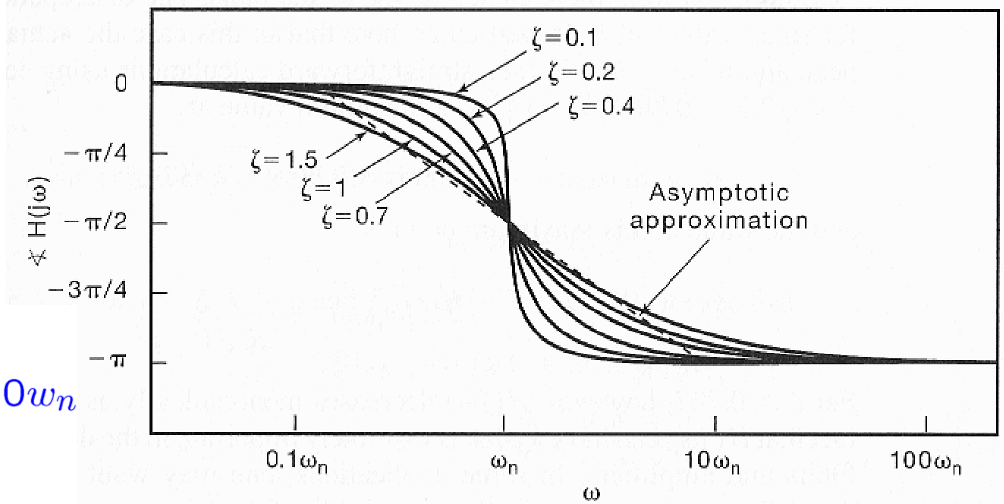
$$Q = \frac{1}{2\zeta}$$

$$20\log_{10}|H(jw)| = \begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$



- For $\zeta < \frac{\sqrt{2}}{2}$ $\quad w_{\max} = w_n\sqrt{1 - 2\zeta^2}$

$$\measuredangle H(jw) = \begin{cases} 0 & w \le 0.1 w_n \\ -(\pi/2)[\log_{10}(w/w_n) + 1] & 0.1 w_n \le w \le 10 w_n \\ -\pi/2 & w = w_n \\ -\pi & w \ge 10 w_n \end{cases}$$



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\,G(s) = \frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

(1) Break points

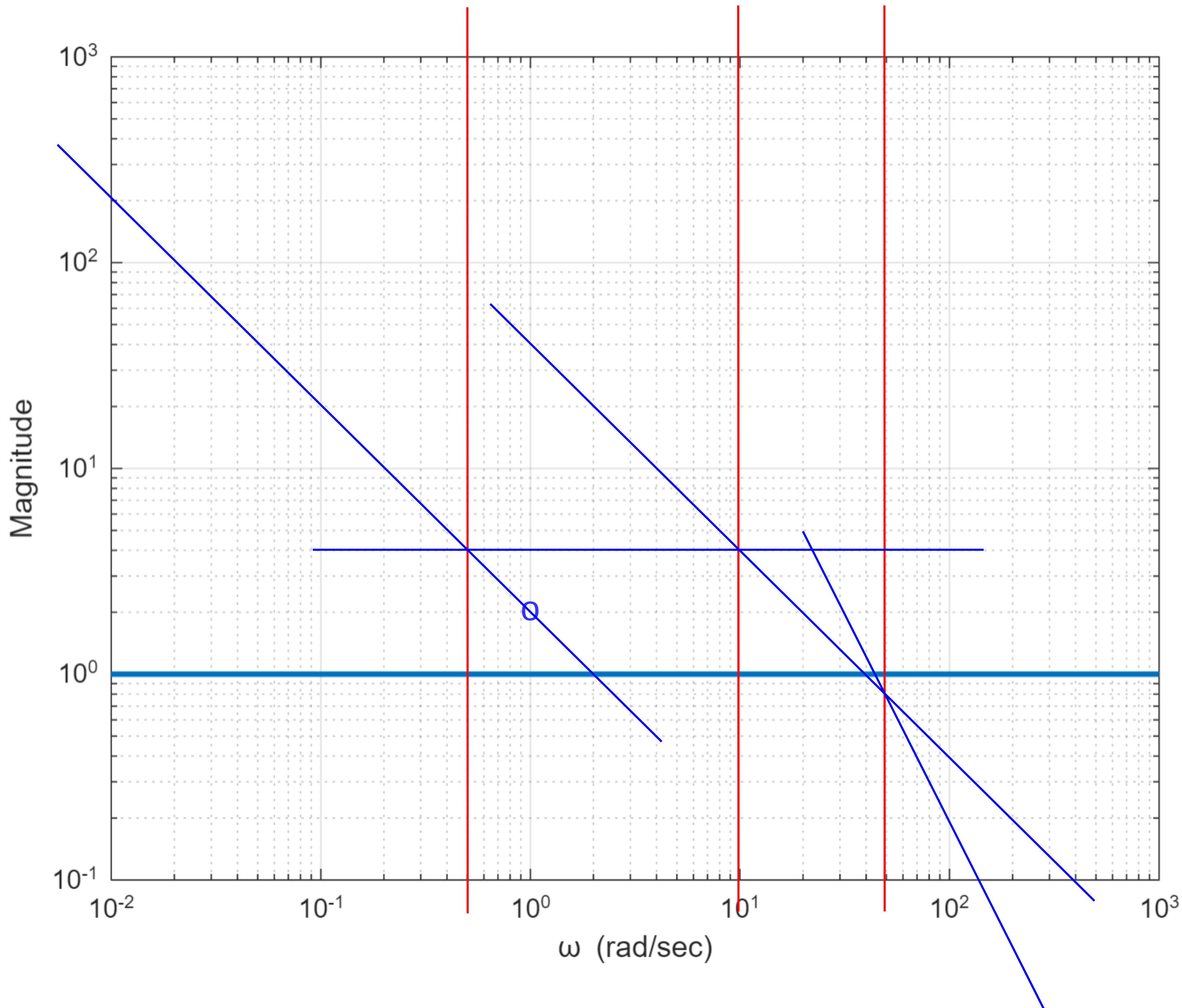
$$K\,G(jw) = \frac{2\,[\,\frac{jw}{0.5}+1\,]}{(jw)\,[\,\frac{jw}{10}+1\,]\,[\,\frac{jw}{50}+1\,]}$$

- Break points: 0.5, 10, 50

(2) Asymptotes

- Low-Frequency Asymptote: $K\,G(jw) = \frac{2}{(jw)}$ for $w < 0.1$
- ω << 0.5: slope = -1 (or -20 db/decade)
- 0.5 < ω < 10: slope = 0 (or 0 db/decade)
- 10 < ω < 50: slope = -1 (or -20 db/decade)
- 50 < ω: slope = -2 (or -40 db/decade)

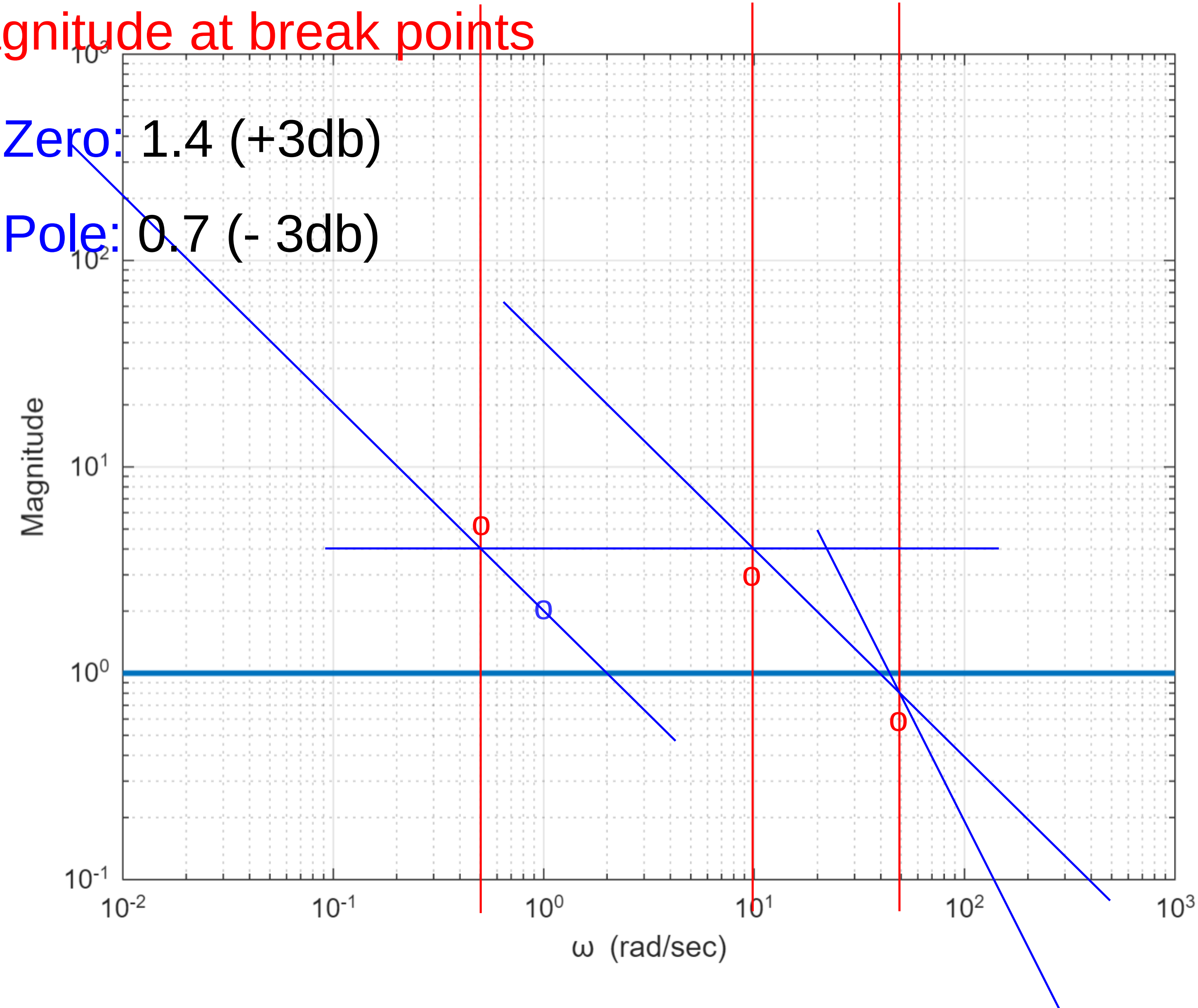
- Example 6.3: Bode Plot for Real Poles and Zeros



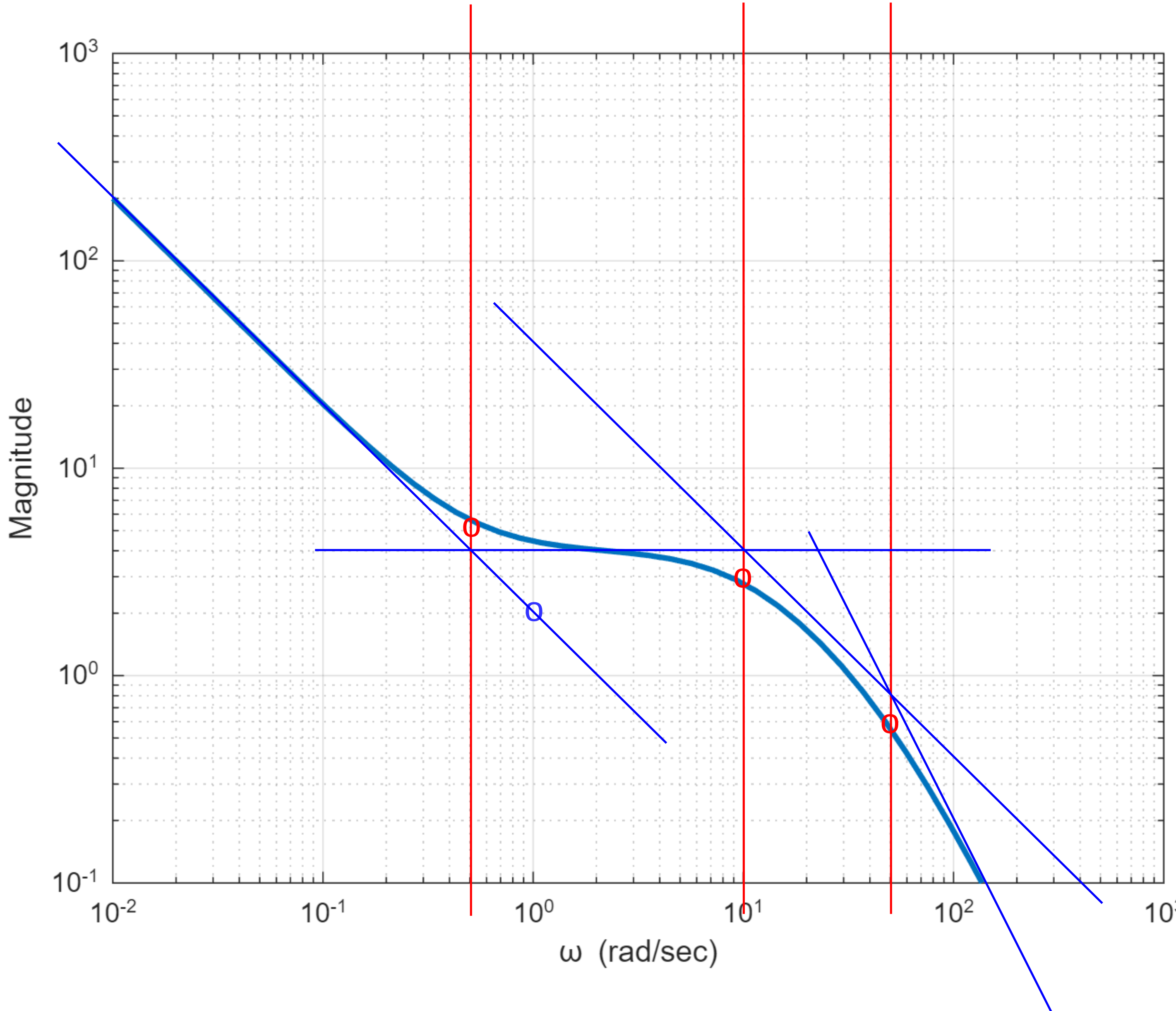
- Example 6.3: Bode Plot for Real Poles and Zeros

(3) Magnitude at break points

- By Zero: 1.4 (+3db)
- By Pole: 0.7 (- 3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\ G(s) = \frac{2000\ (s + 0.5)}{s\ (s + 10)\ (s + 50)}$$

(1) Break points

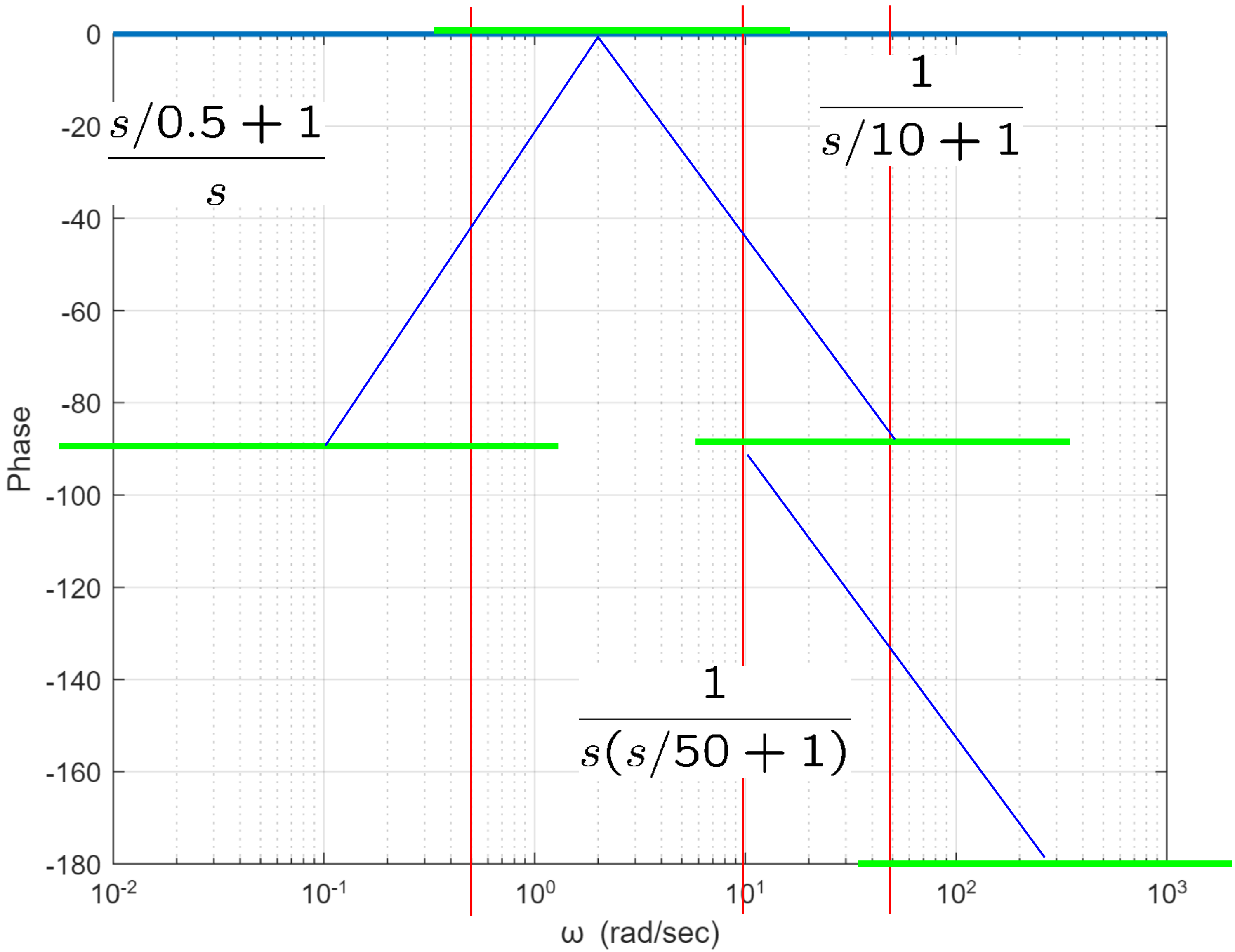
$$K\ G(jw) = \frac{2\ [\ \frac{jw}{0.5} + 1\ ]}{(jw)\ [\ \frac{jw}{10} + 1\ ]\ [\ \frac{jw}{50} + 1\ ]}$$

- Break points: 0.5, 10, 50

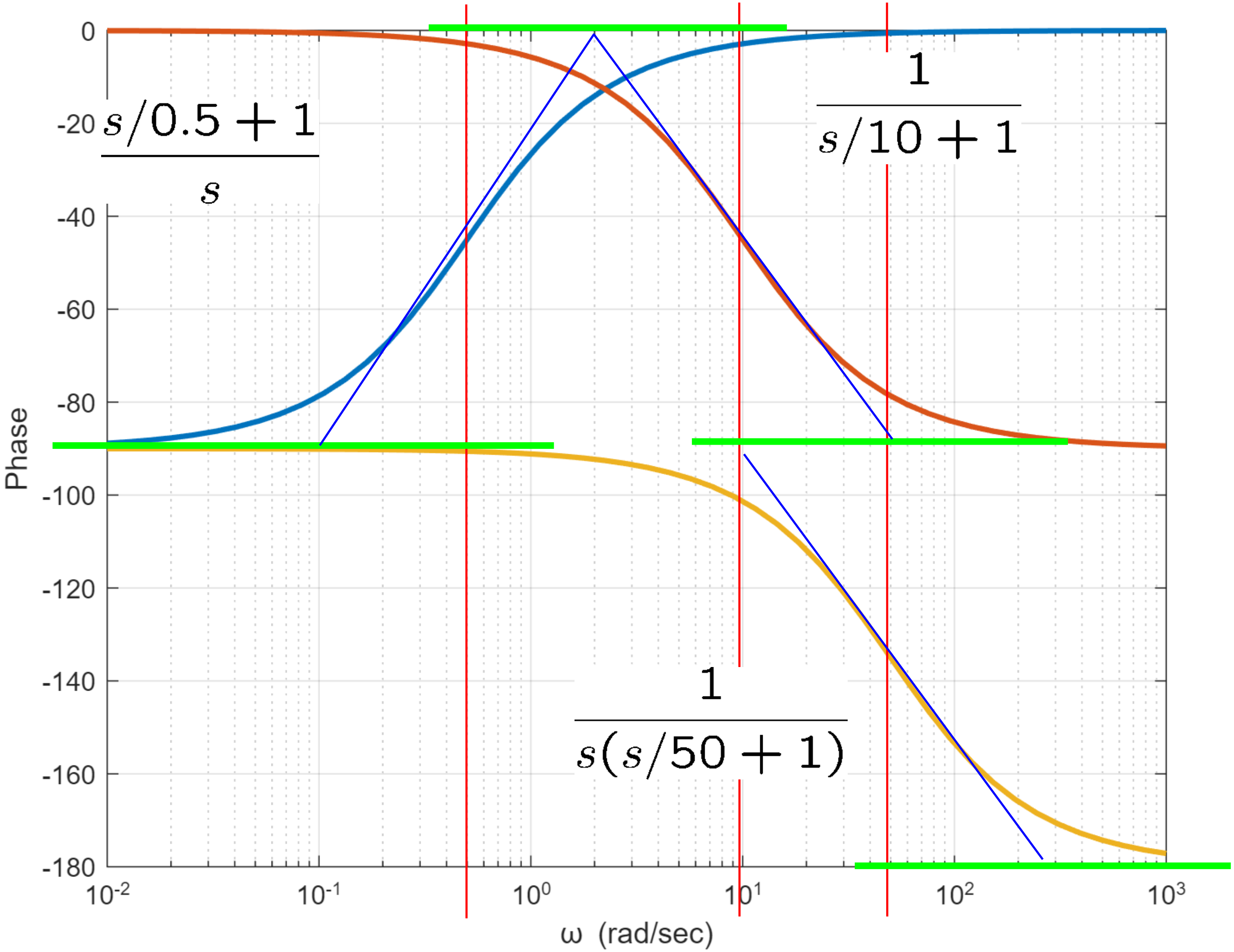
(4) Phase

- Low-Frequency Asymptote: $K\ G(jw) = \frac{2}{(jw)}$ for $w < 0.1$
- $\omega << 0.5$: phase = -90^o
- $0.5 < \omega < 10$: phase = 0^o
- $10 < \omega < 50$: phase = -90^o
- $50 < \omega$: phase = -180^o

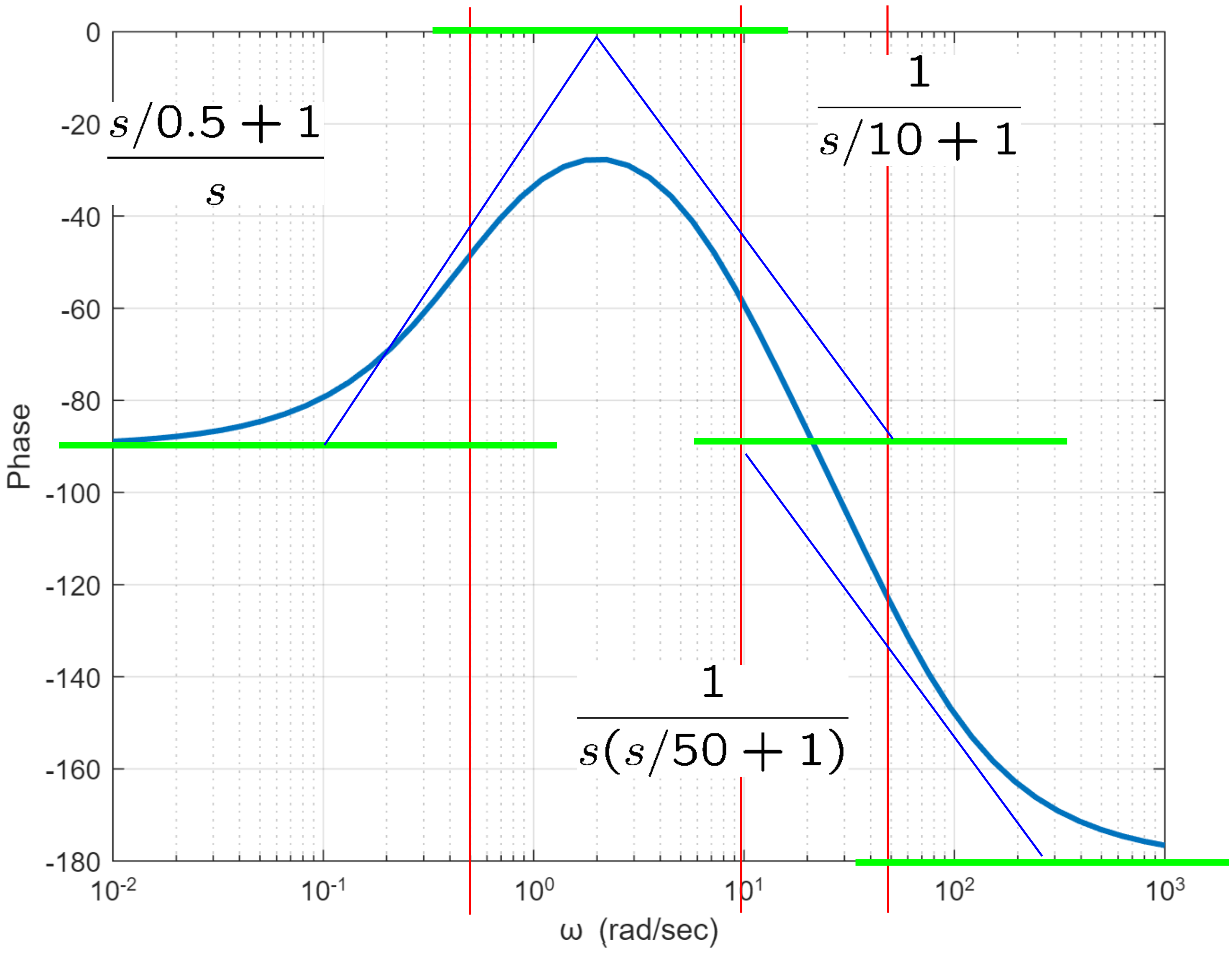
- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros



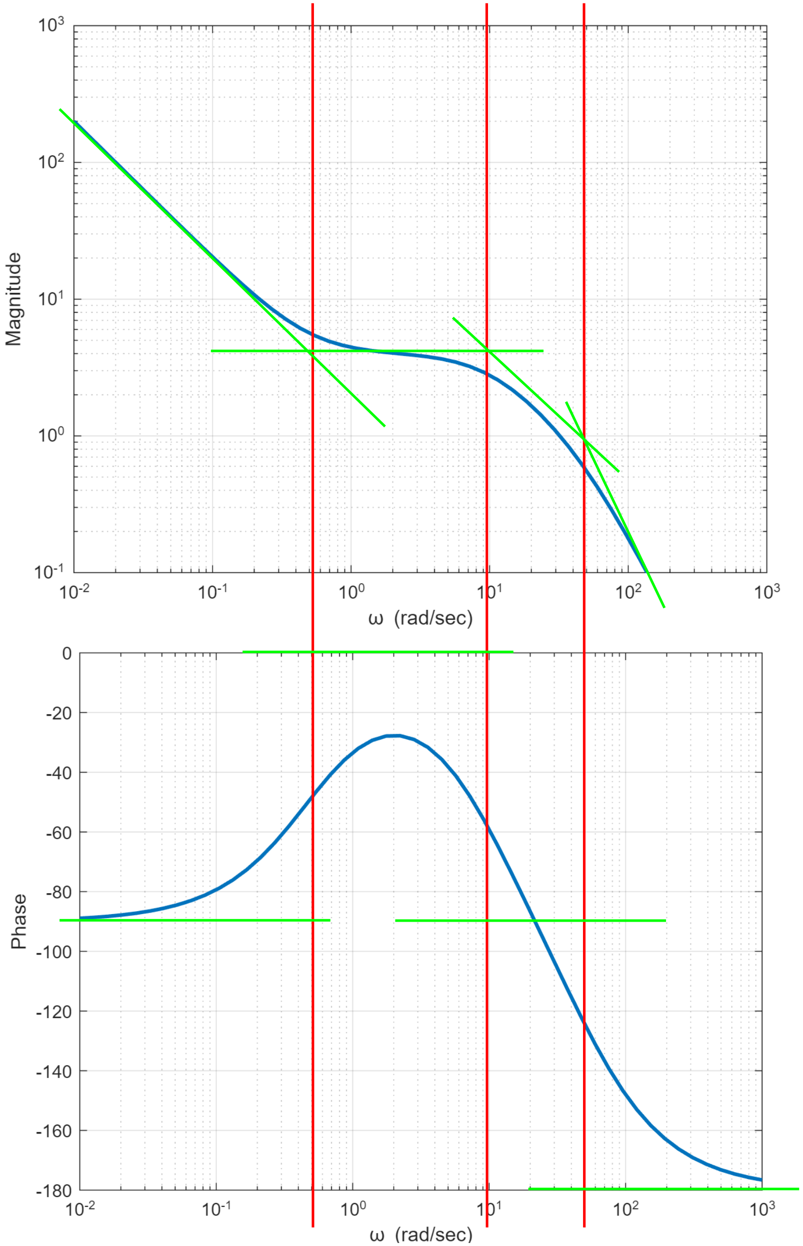
▪ Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

$$\frac{2\,[\,\frac{jw}{0.5}+1\,]}{(jw)\,[\,\frac{jw}{10}+1\,]\,[\,\frac{jw}{50}+1\,]}$$



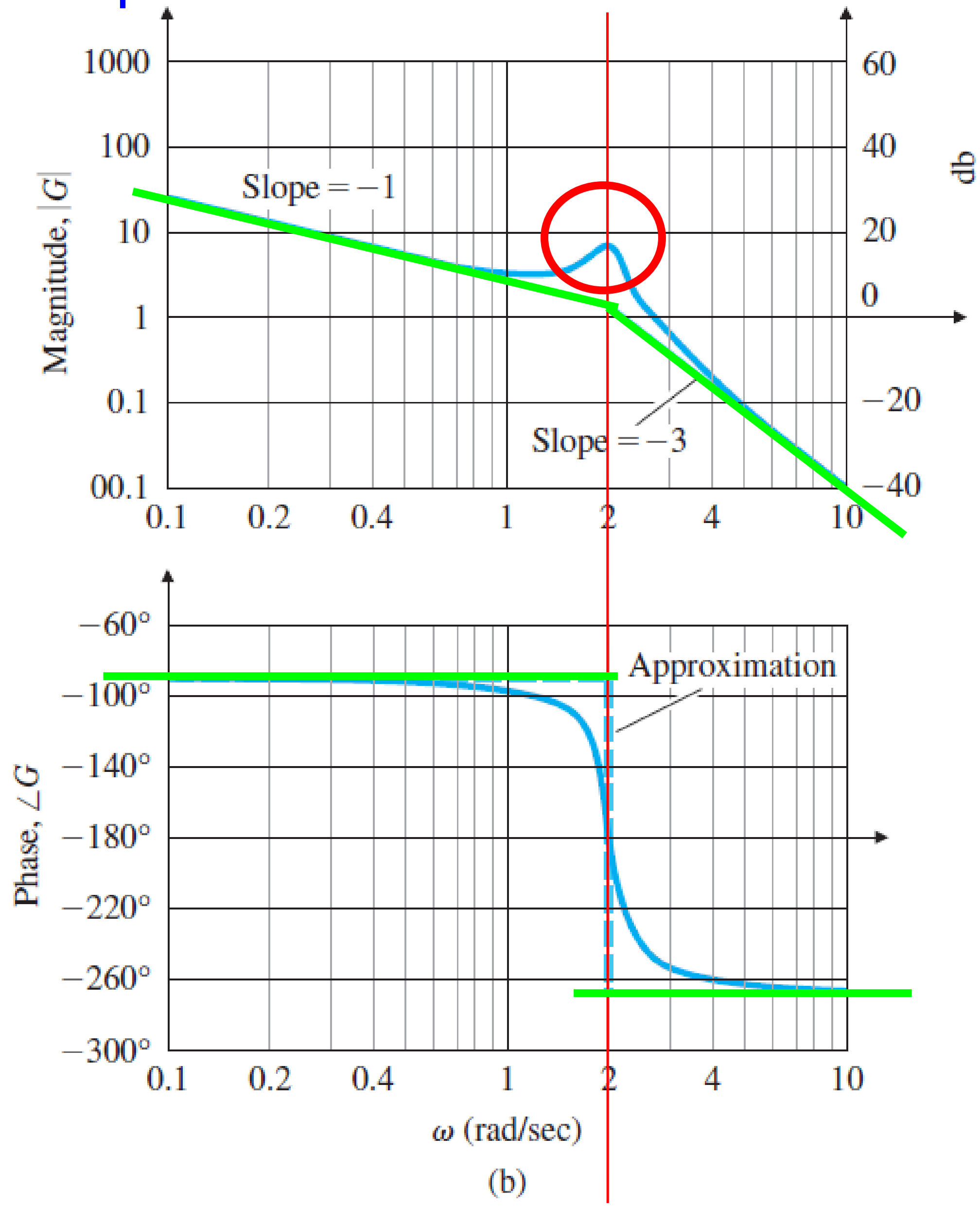
- Break points: 0.5, 10, 50

- Example 6.4: Bode Plot for Complex Poles

$$K\,G(s) = \frac{10}{s\,[s^2 + 0.4s + 4]}$$

$$= \frac{10}{4}\,\frac{1}{s\,[\,\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1\,]}$$

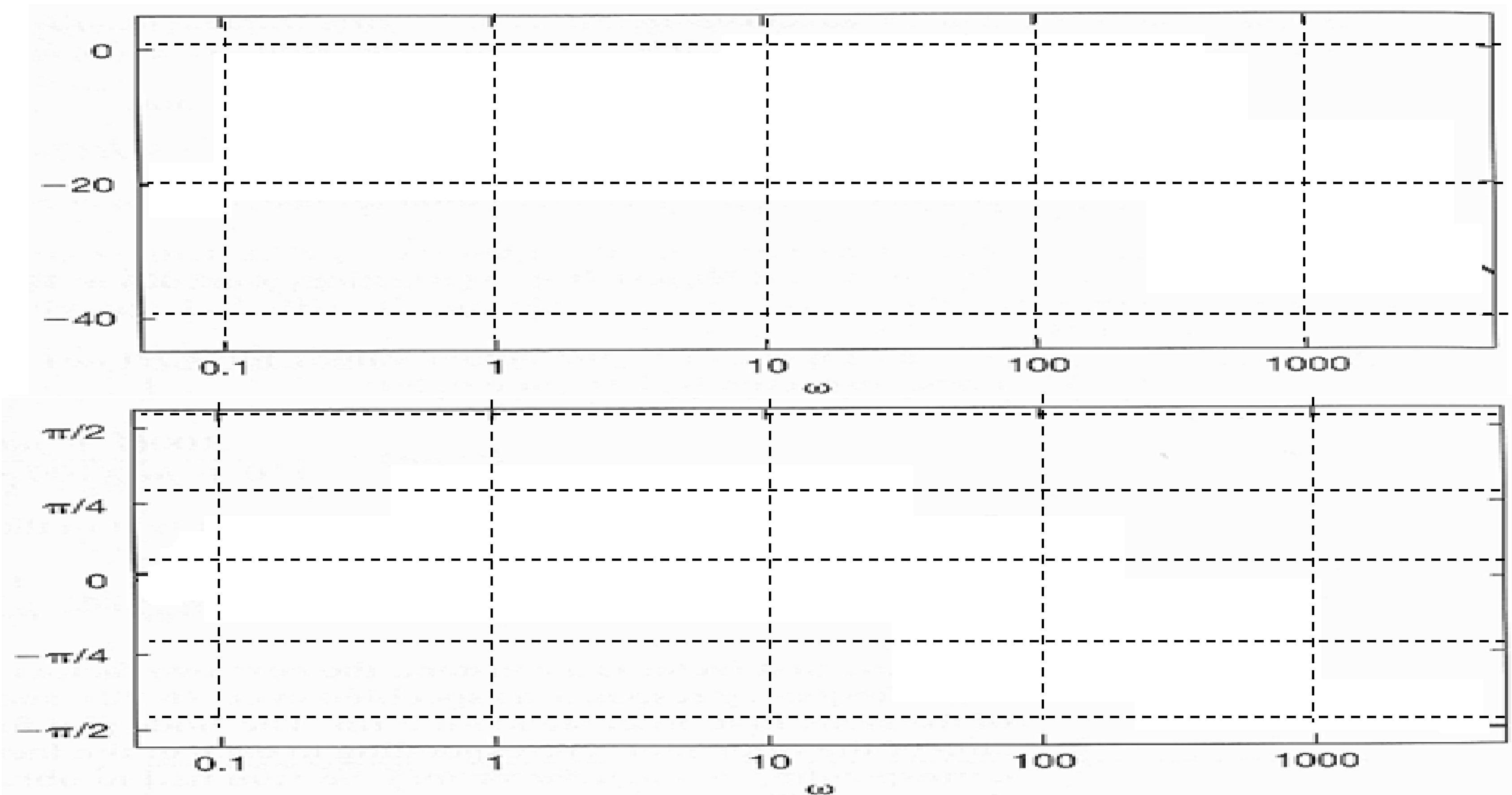
- Break points: 2



- Example 6.5:

$$H(jw) = \frac{100(1+jw)}{(10+jw)(100+jw)}$$

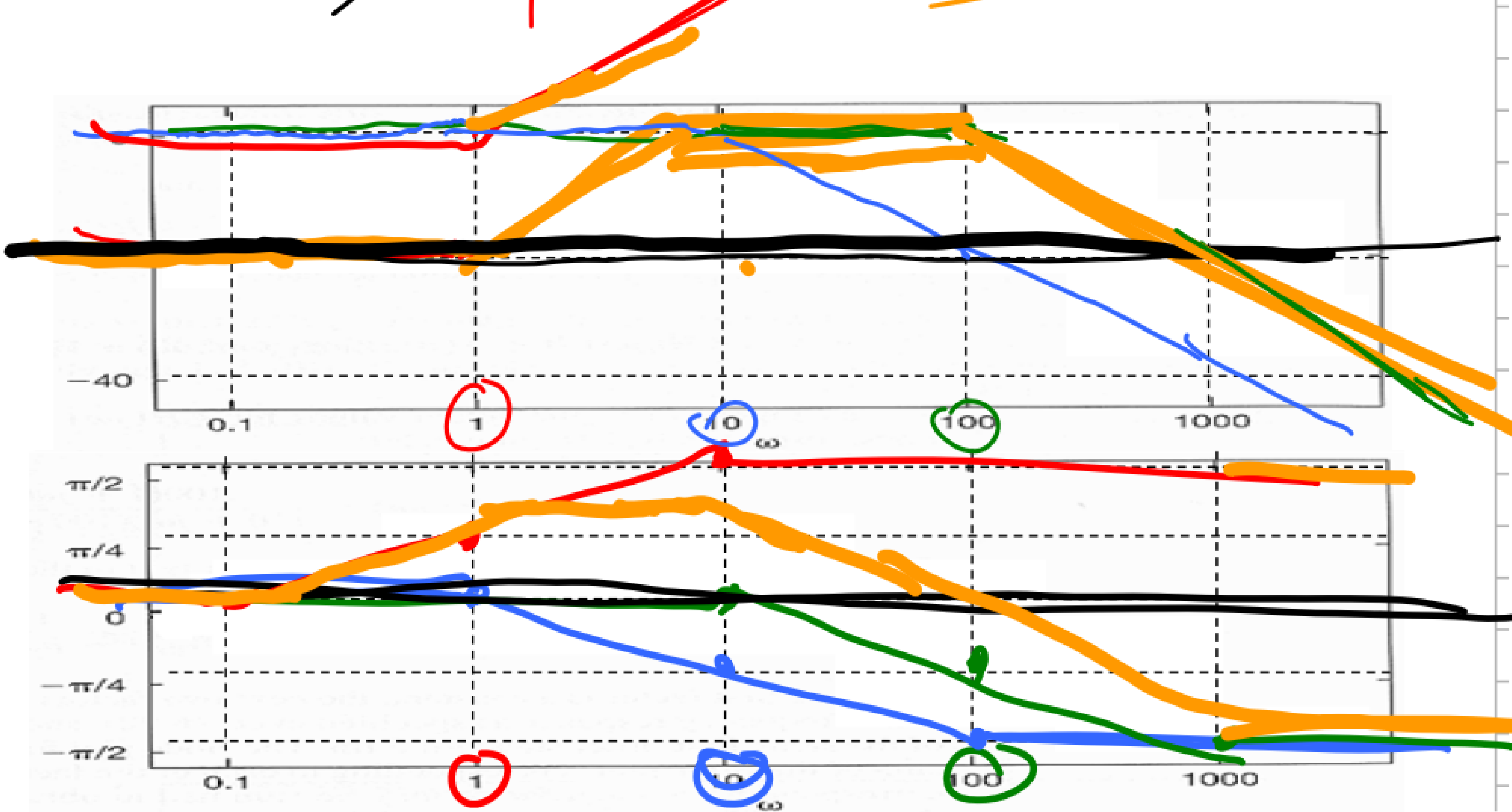
$$= \left(\frac{1}{10}\right)(1+jw)\left(\frac{1}{1+j\frac{w}{10}}\right)\left(\frac{1}{1+j\frac{w}{100}}\right)$$



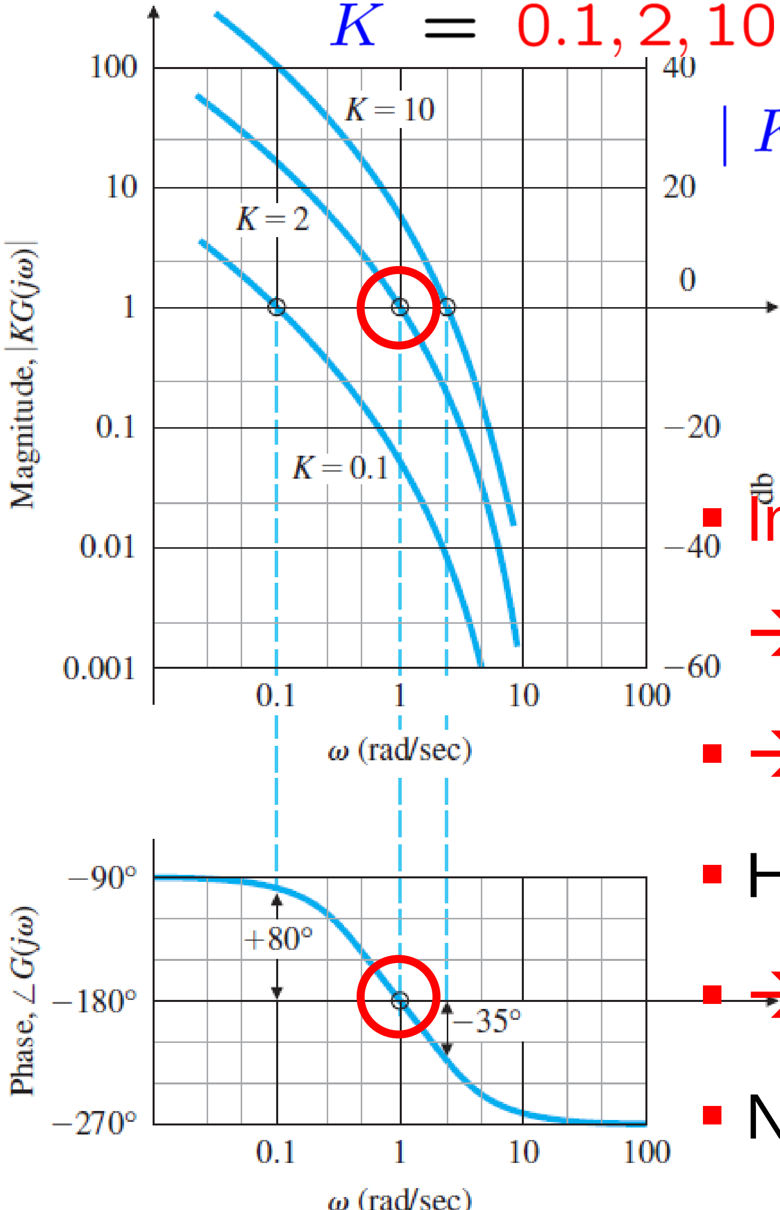
- **Example 6.5:**

$$H(jw) = \frac{100(1+jw)}{(10+jw)(100+jw)}$$

$$= \left(\frac{1}{10}\right)(1+jw)\left(\frac{1}{1+j\frac{w}{10}}\right)\left(\frac{1}{1+j\frac{w}{100}}\right)$$

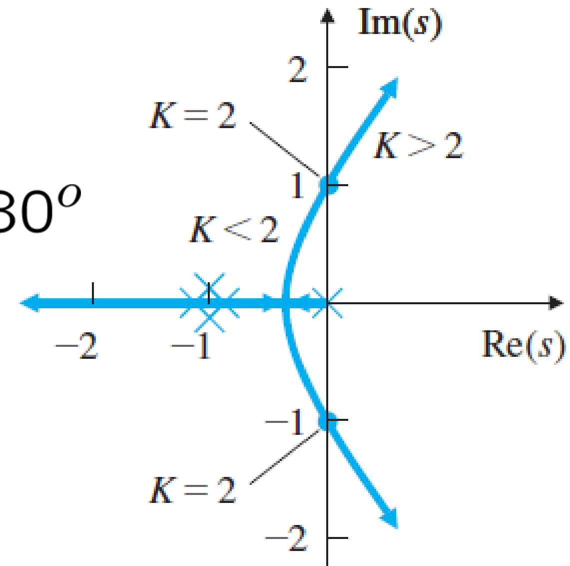


$$K = 0.1, 2, 10$$



$$| K\ G(jw) | = 1$$

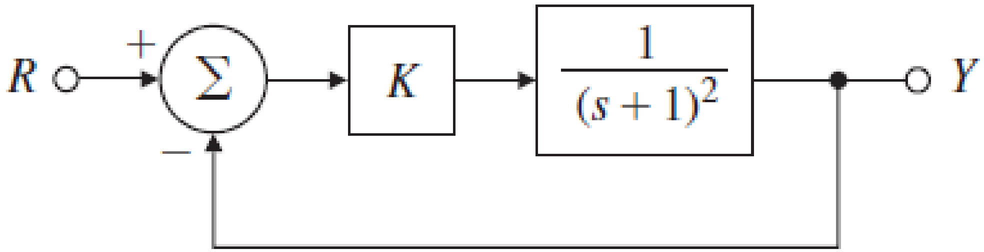
$$\angle G(jw) = 180^o$$

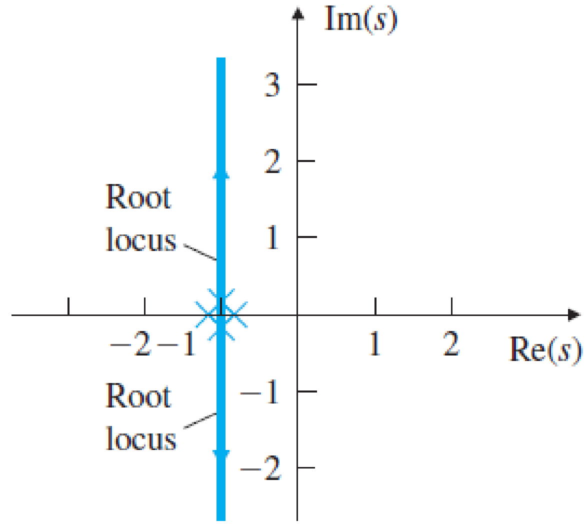


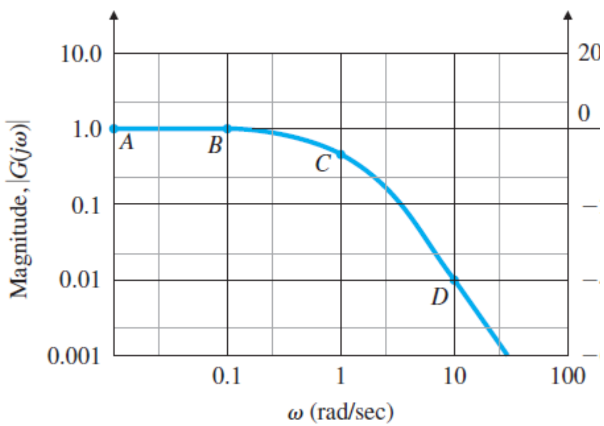
(b)

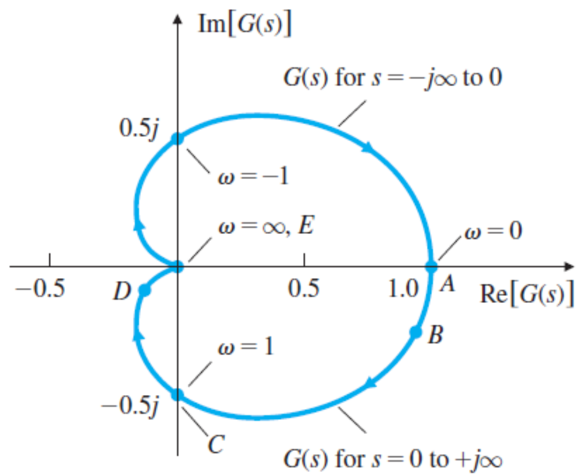
- Increasing gain
  → increase OR decrease stability?
- → | KG(jw) | < 1 !!!
- However, for some systems,
- → | KG(jw) | > 1 !!!
- Need to check their root locus
- OR, by Nyquist Stability Criterion

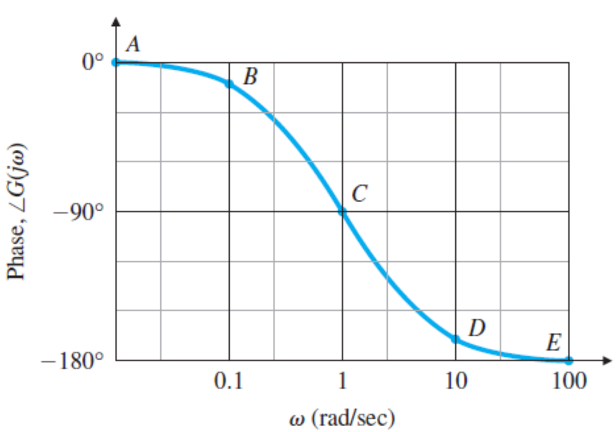
- Example 6.8: Nyquist Plot for a Second-Order System









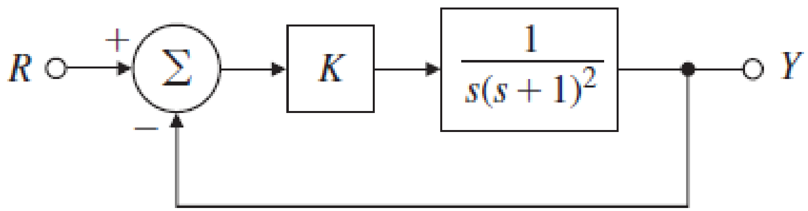


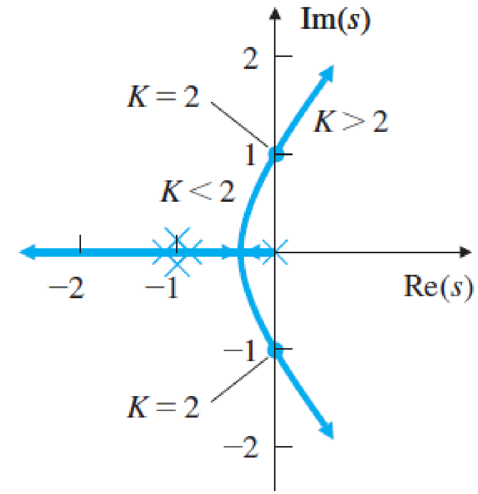
(b)

- N = 0: not encircle -1
- P = 0: no poles of G(s) in RHP
- Z = N + P → Z = 0,

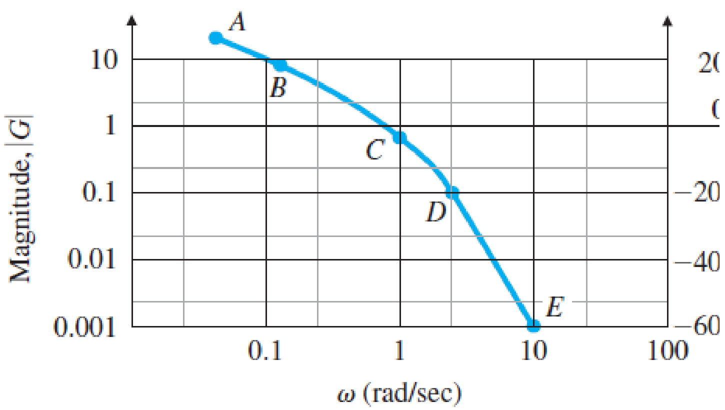
  no unstable roots for K = 1
- K > 0 also holds

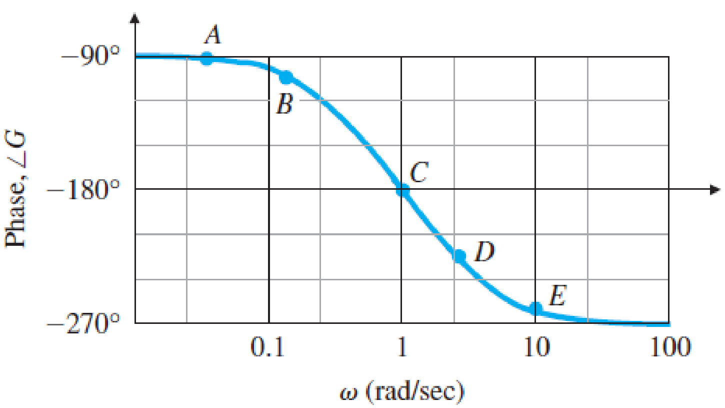
Example 6.9: Nyquist Plot for a Third-Order System



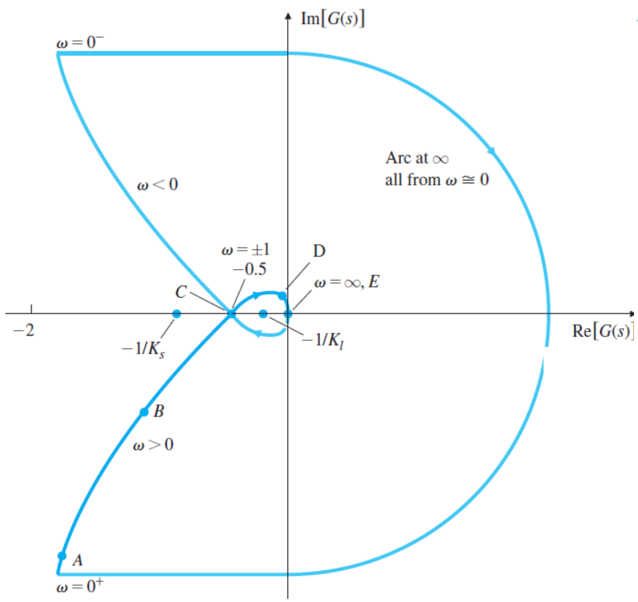


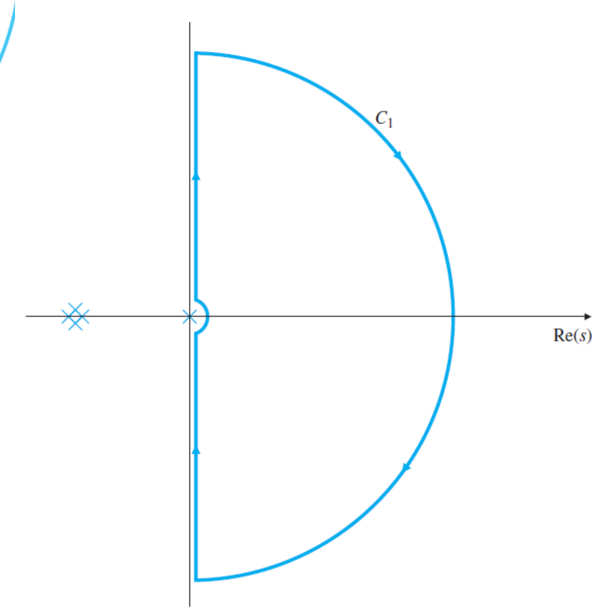
(b)



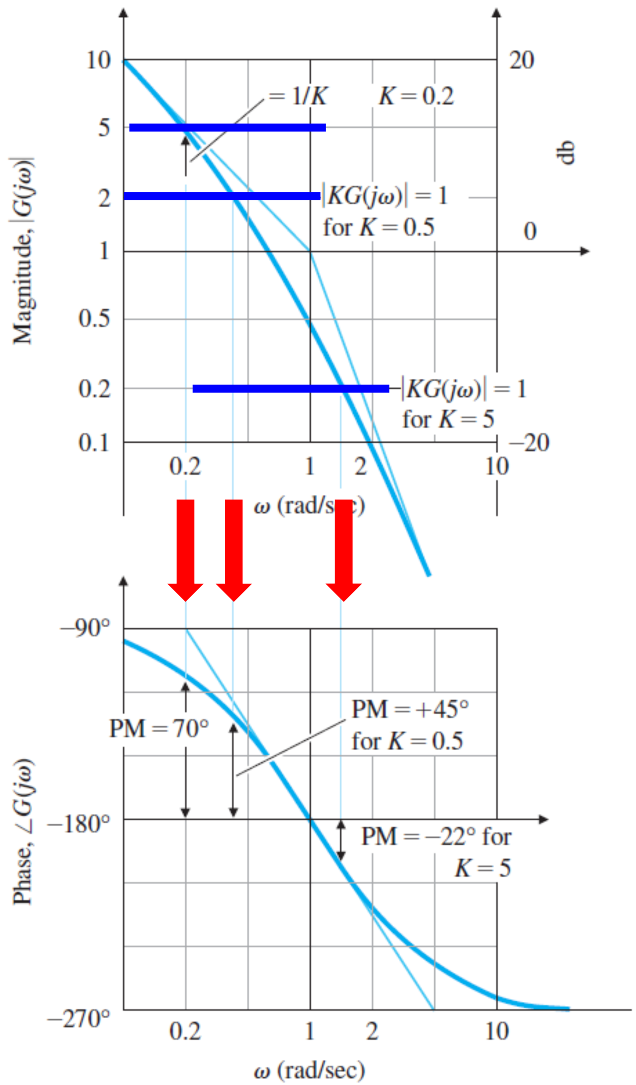


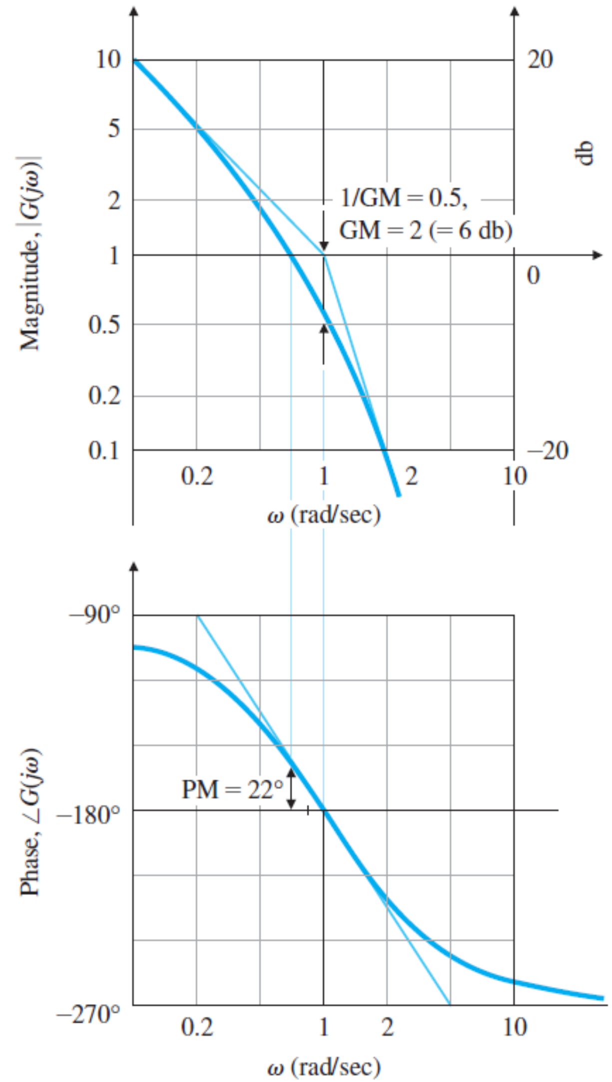
(b)





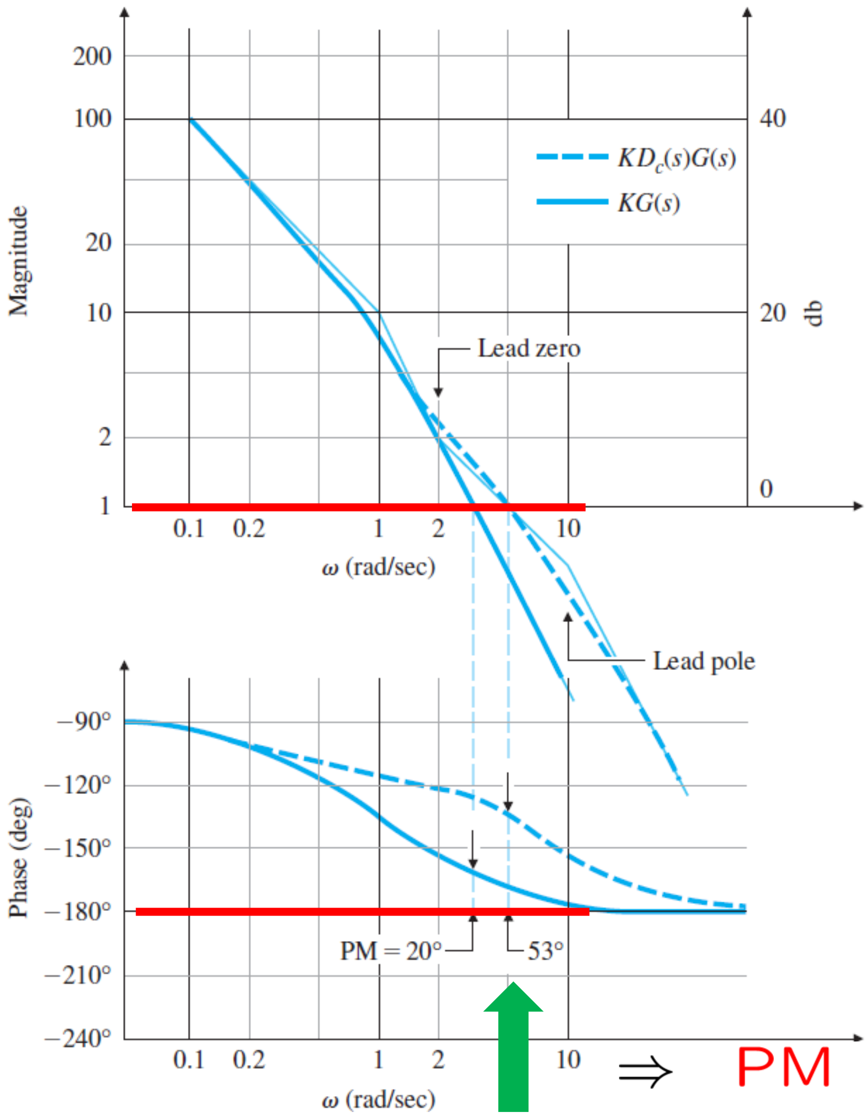
- PM vs K
- K = 5
- | KG(jw) | = 1
- PM = -22^o
- K = 0.5
- | KG(jw) | = 1
- PM = $+45^o$
- K = 0.2
- | KG(jw) | = 1
- PM = $+70^o$





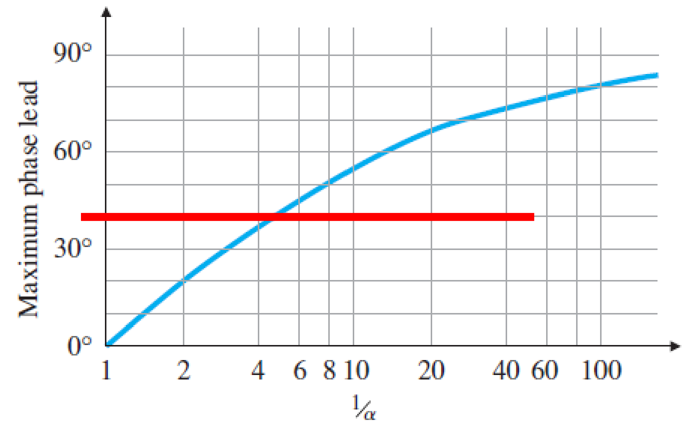
▪ Example 6.15: Lead Compensation for a DC Motor

$K\ G(s)$ $\qquad$ $K\ D_c(s)\ G(s)$



$\Rightarrow$ PM $\geq 25^o$, at $w_c = 3$

$\Rightarrow$ Phase lead $= 40^o$,



$$\Rightarrow \frac{1}{\alpha} = 5$$

a zero at $w = 2$ rad/sec

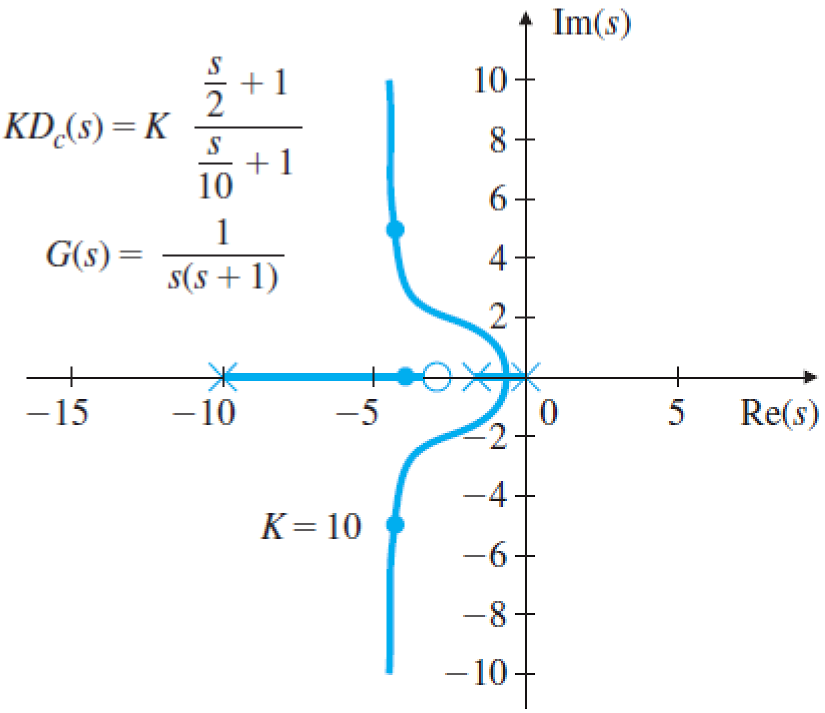
a pole at $w = 10$ rad/sec

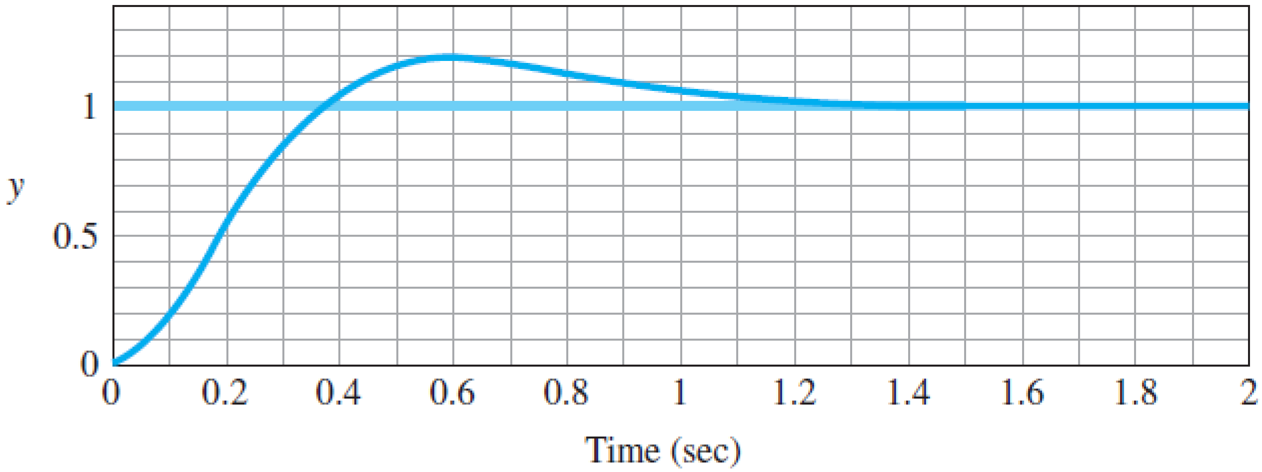
$$K\ D_c(s) = 10\ \frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$\Rightarrow$ PM $= 53^o$, at $w_c = 5$

- Example 6.15: Lead Compensation for a DC Motor

$$K\ D_c(s) \ = \ 10\,\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$





- Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

$$K\,G(s) = K \frac{10}{s\left(\frac{s}{2.5}+1\right)\left(\frac{s}{6}+1\right)}$$

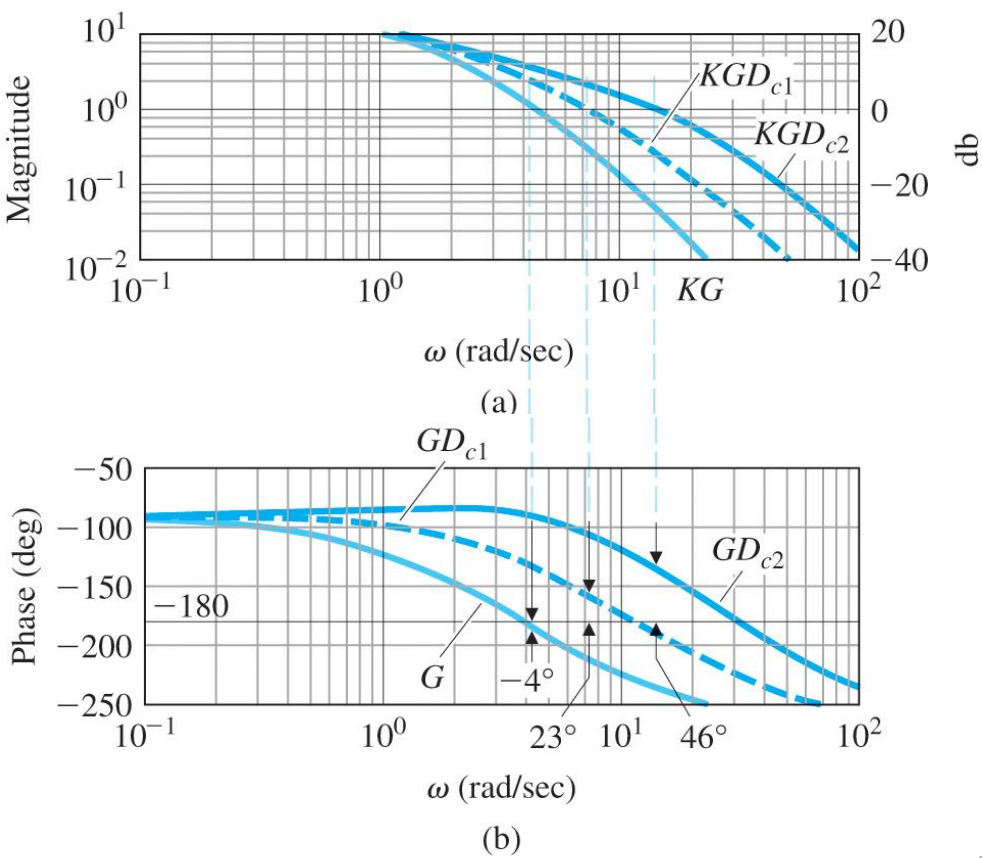
- $K_v = 10$
- $\text{PM} = 45^o$

1. Determine gain K:

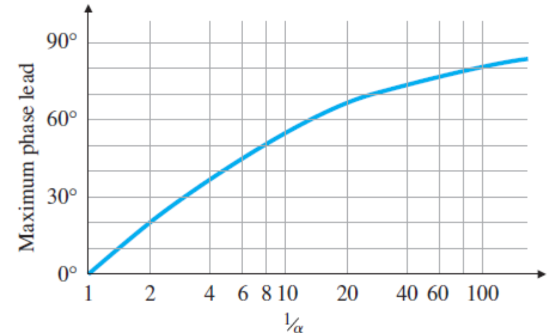
$$K_v = \lim_{s\to 0} s\,K\,G(s)$$
$$= K \times 10 = 10$$
$$\Rightarrow K = 1$$

2. Bode plot of KG(s), K = 1

→ PM ~= - 4, Wcp ~= 4



▪ Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System



3. Allow for 5^o of extra margin

$$\rightarrow \quad 45^o + 5^o - (-4^o) = 54^o$$

4. Pick α → $1/\alpha = 10$

5. Zero & Pole

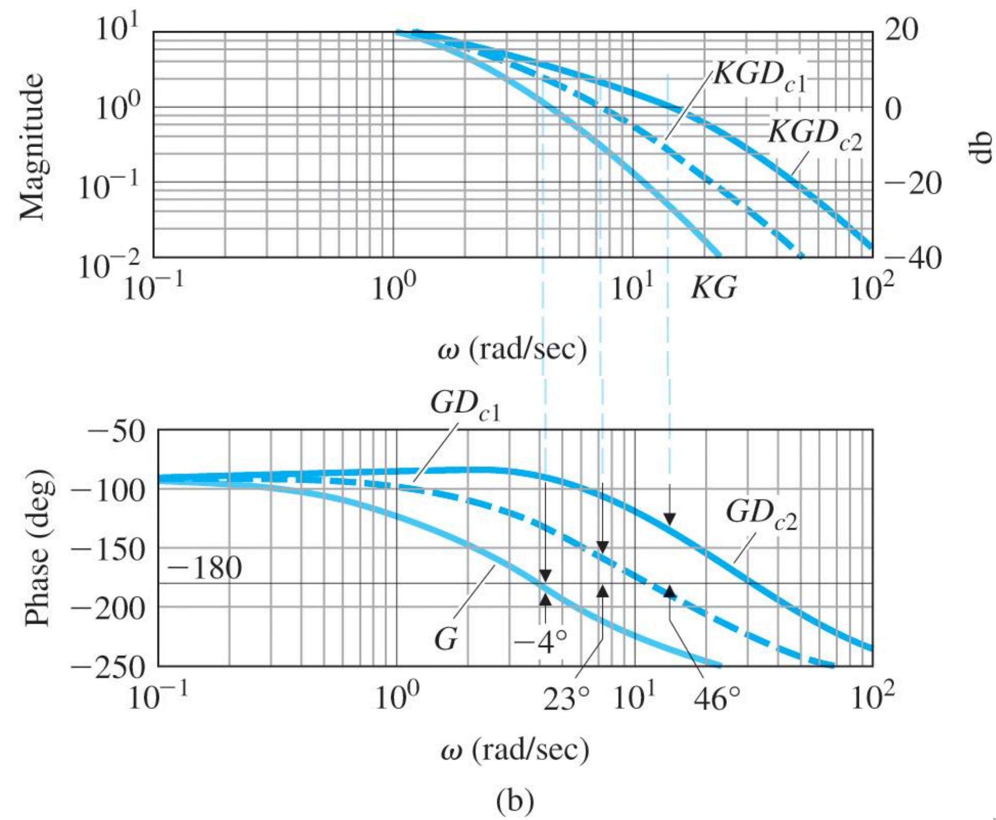
a zero at 2

a pole at 20

$$D_1(s) = \frac{(\frac{s}{2}+1)}{(\frac{s}{20}+1)}$$

$$= \frac{1}{0.1}\left(\frac{s+2}{s+20}\right)$$

→ PM ~= 23, Wcp ~= 7

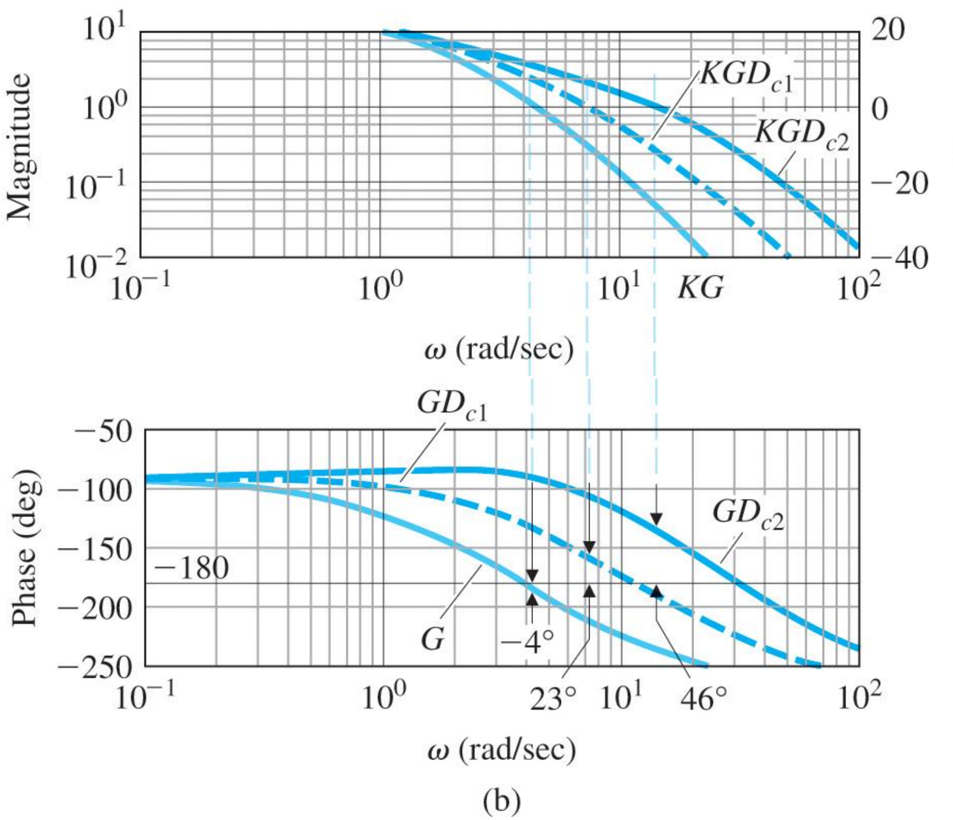


- Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

7. A double-lead compensator:

$$D_2(s) = \frac{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{\left(\frac{s}{20}+1\right)\left(\frac{s}{40}+1\right)} = \frac{1}{(0.1)^2}\frac{(s+2)(s+4)}{(s+20)(s+40)}$$

- PM $= 46^o$

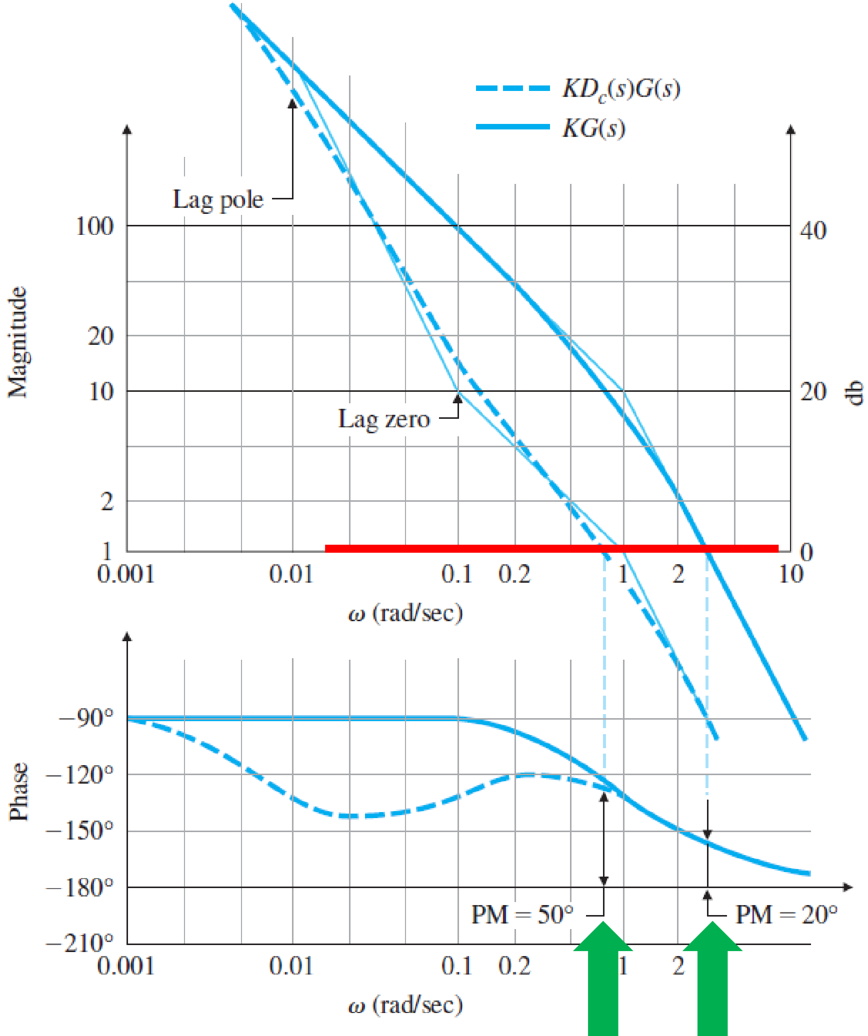


- Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- K = 10
- PM = 20^o at ω_c ~= 3
- Select break points
  - ✓ ω_c is lowered
  - ✓ more favorable PM results
- ➢ Lag zero = 0.10
- ➢ Lag pole = 0.01
  - ✓ PM = 50^o

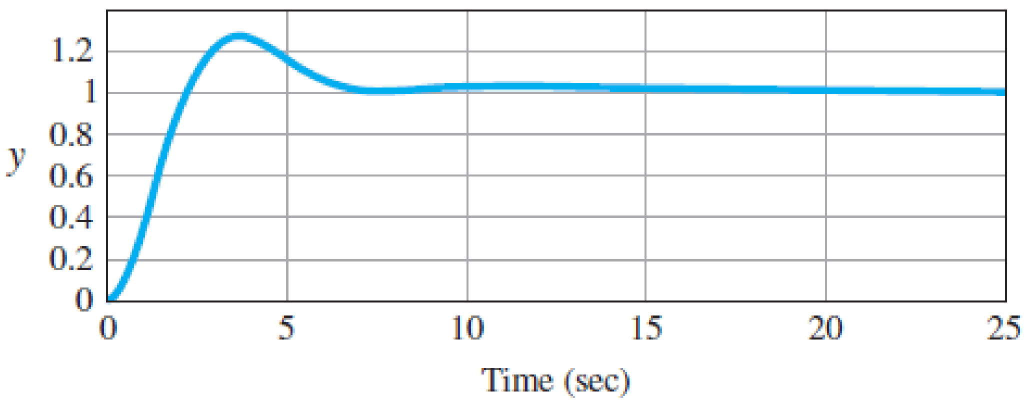
- Error constant: $K_v = 10$
- PM = 45^o



- Example 6.19: Lag Compensation for the DC Motor

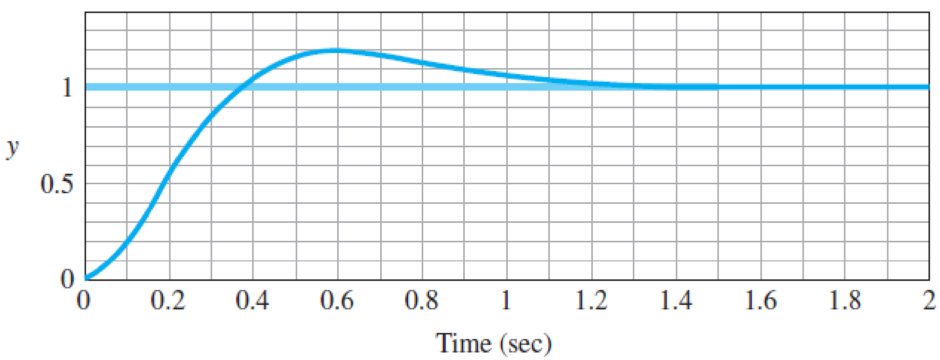
$$G(s) = \frac{1}{s(s+1)}$$

- Error constant: $K_v = 10$
- PM = 45^o



- No steady-state error
  - ✓ a Type 1 system
- Settling time ~= 25 sec
- Rise time ~= 2 sec

- Example 6.15: Lead Compensation



- Example 6.20: PID Compensation for Spacecraft Attitude Control

$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

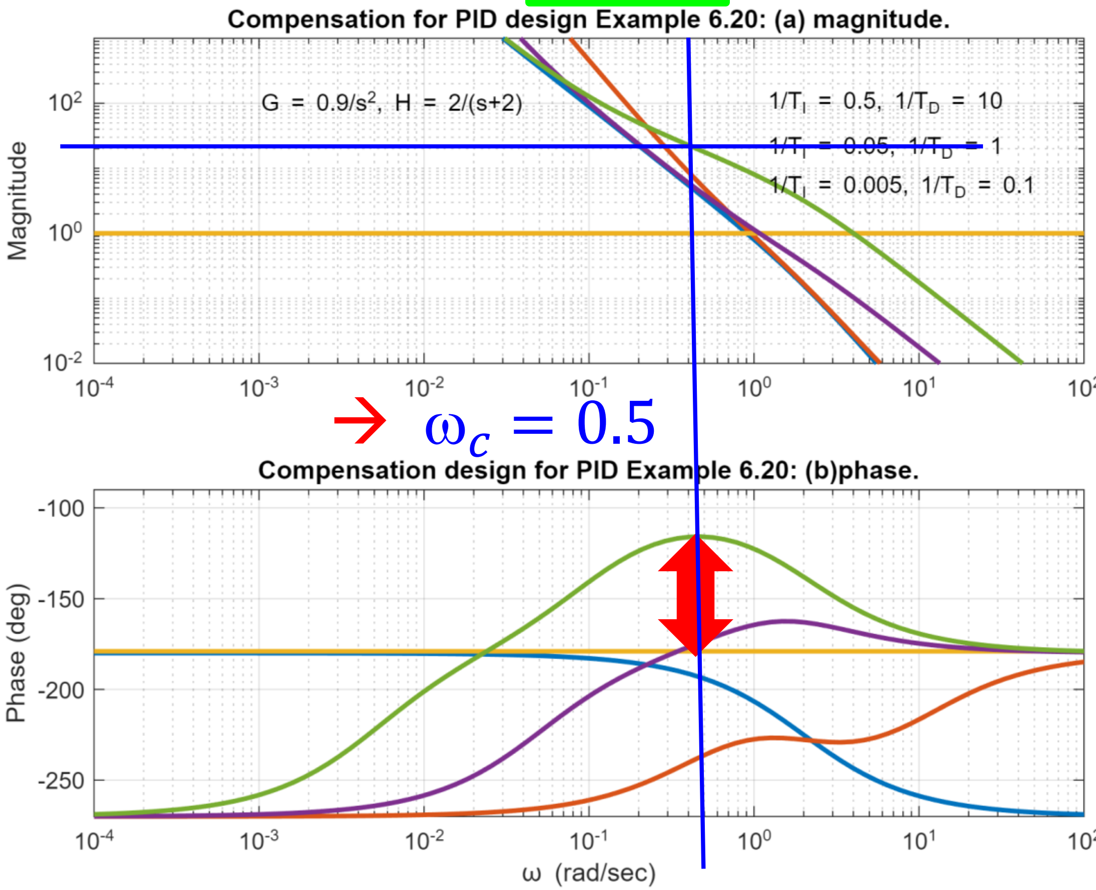
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s+2}$$

$\frac{1}{T_I} = 0.5$ $\frac{1}{T_D} = 10$

$\frac{1}{T_I} = 0.05$ $\frac{1}{T_D} = 1$

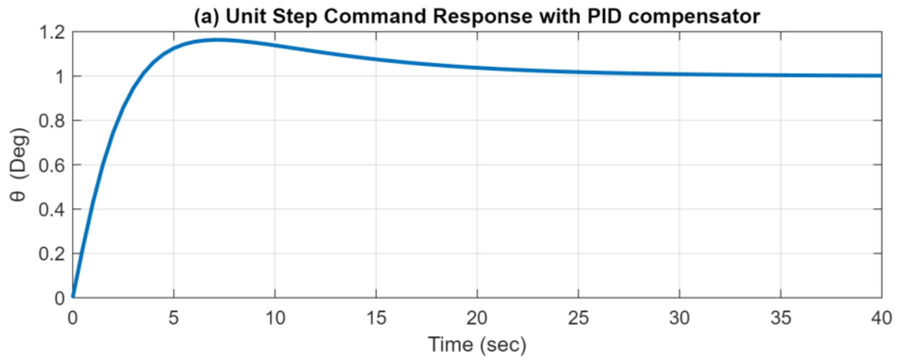
$\frac{1}{T_I} = 0.005$ $\frac{1}{T_D} = 0.1$



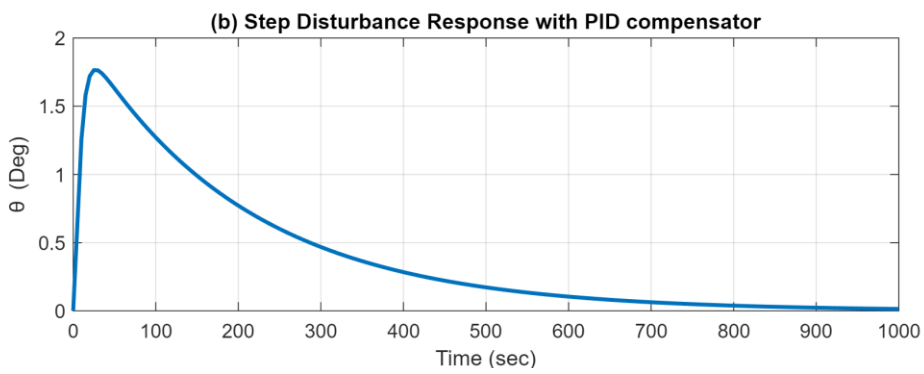
- PM = 65^o
- → Find K
- $|\, D_c(s)\, G(s)\, | = 20$
- 1/K = 20
- K = 0.05

- Example 6.20: PID Compensation for Spacecraft Attitude Control

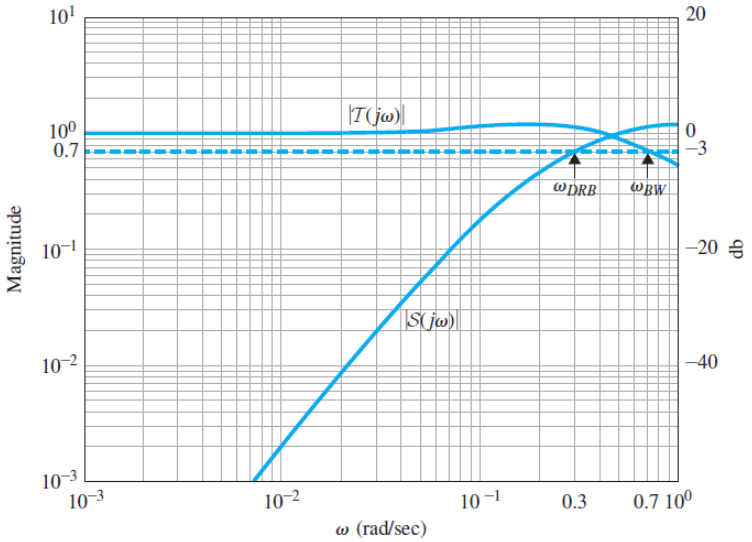
$$\mathcal{T}(s) = \frac{\Theta}{\Theta_{com}} = \frac{D_c\, G}{1 + D_c\, G\, H}$$



$$\frac{\Theta}{T_d} = \frac{G}{1 + D_c\, G\, H}$$

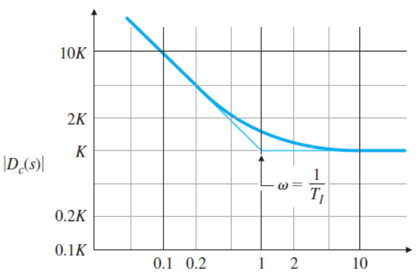


- Frequency Response of $T(s)$ and $S(s)$ are shown:

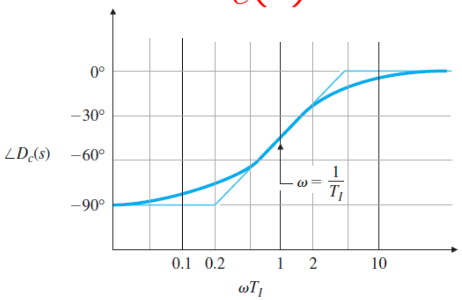


(PI, Lag) (PID, Lead-Lag) (PD, Lead)

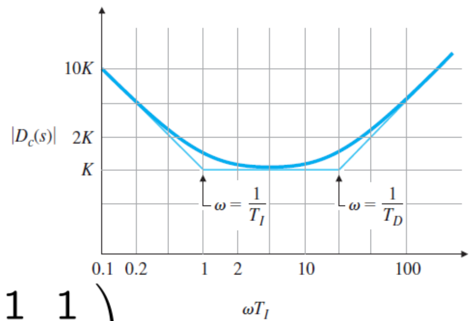
PI

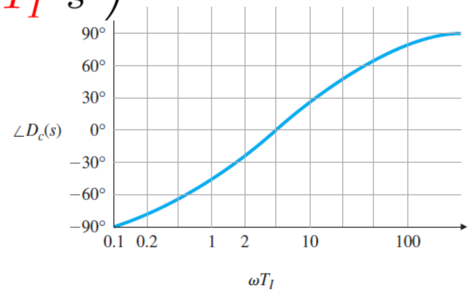


$$D_c(s) = K\left(1 + \frac{1}{T_I}\frac{1}{s}\right)$$



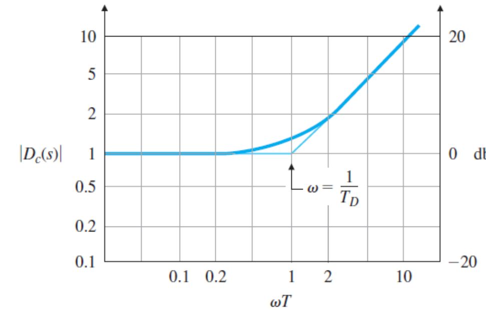
PID



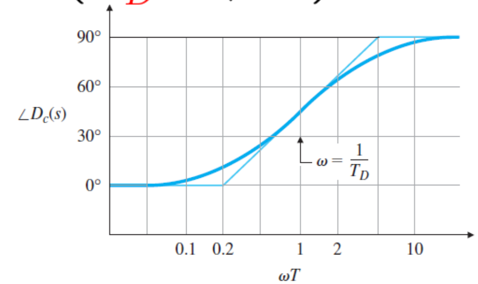


$$D_c(s) = \frac{K}{s}\left[(T_D\, s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

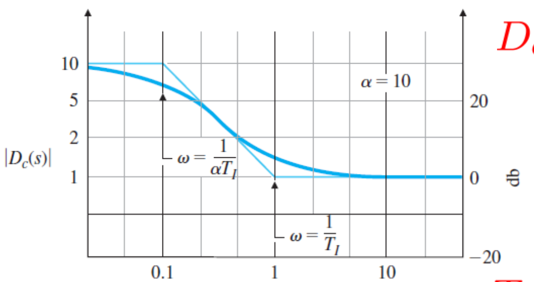
PD



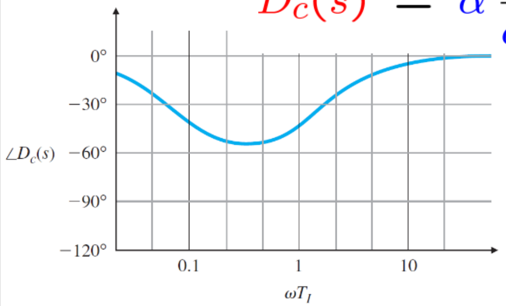
$$D_c(s) = (T_D\, s + 1)$$



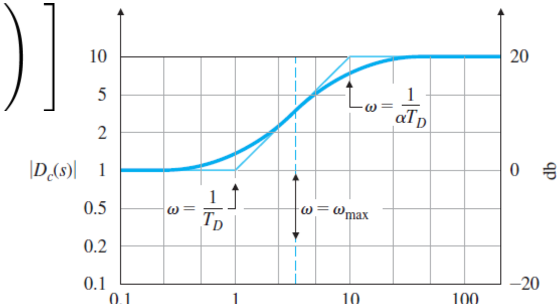
Lag



$$D_c(s) = \alpha\,\frac{T_I\, s + 1}{\alpha\, T_I\, s + 1}, \quad \alpha > 1$$



Lead



$$D_c(s) = \frac{T_D\, s + 1}{\alpha\, T_D\, s + 1}, \quad \alpha < 1$$