Fall 2022 (111-1)

控制系統
Control Systems

Unit 5F
Extensions of the Root-Locus Method

Feng-Li Lian
NTU-EE

Sep 2022 – Dec 2022

- Positive (180°) Root Locus VS Negative (0°) Root Locus

- One Parameter VS Two Parameters

Rules for Plotting a Negative (0∘) Root Locus

- For negative values of parameters
- Has a zero in the RHP (non-minimum phase)

$$\Rightarrow \; 1 + A\,(z_i - s)\,G'(s) \;=\; 0$$

$$\Rightarrow \; 1 + (-A)\,(s - z_i)\,G'(s) \;=\; 0$$

$$\Rightarrow \; 1 + K\,(s - z_i)\,G'(s) \;=\; 0$$

$$\Rightarrow \; K \;=\; -A \;<=\; 0$$

- For negative locus, the phase condition is:
  - The angle of L(s) is $0^\circ + 360^\circ(l-1)$ for s on the negative locus
  - Hence, a Negative Locus is referred as a 0∘ Root Locus

Rules for Plotting a Negative (0°) Root Locus

- Rule 1: (as before)
- The n branches of the locus leave the poles of L(s) and
- m of these branches approach the zeros of L(s) and
- n – m branches approach the asymptotes.

- Rule 2: (odd → even)
- The locus is on the real axis to the left of an even number of real poles and zeros.

- Rule 3: (180° → 0°)
- The asymptotes are described by:

$$\phi_l = \frac{0^o + 360^o(l - 1)}{n - m} \qquad l = 1, 2, \cdots, n - m$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n - m} = \frac{-a_1 + b_1}{n - m}$$

- Rule 4: (180∘ → 0∘)
- The angle of departure of a branch of the locus from repeated poles with multiplicity q is given by

$$q\ \phi_{l,dep} = \sum \psi_i - \sum_{i \neq l,dep} \phi_i - 0^o - 360^o(l-1)$$

$$l = 1, 2, \cdots, q$$

- The angle of arrival of a branch at a zero with multiplicity q is given by

$$q\ \psi_{l,arr} = \sum \phi_i - \sum_{i \neq l,arr} \psi_i + 0^o + 360^o(l-1)$$

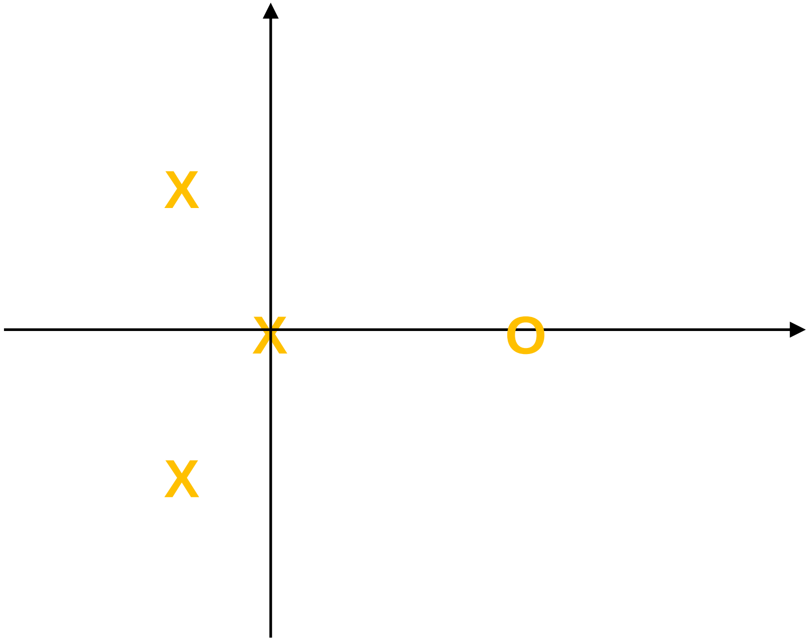
- Rule 5:
- The locus can have multiple roots at points on the locus and the branches will approach a point of q roots at angles separated by $\dfrac{180^o - 360^o(l-1)}{q}$
- And will depart at angles with same separation.

Example 5.13: Negative Root Locus for Airplane

$$G(s) = \frac{6-s}{s(s^2+4s+13)}$$

$$= -\frac{s-6}{s(s^2+4s+13)}$$

$$\Rightarrow 1 + K\frac{s-6}{s(s^2+4s+13)} = 0$$



- Rule 1:
- There are 3 branches and 2 asymptotes.
- Rule 2:
- One real-axis segment to the right of s = 6 and
- A segment is to the left of s = 0.

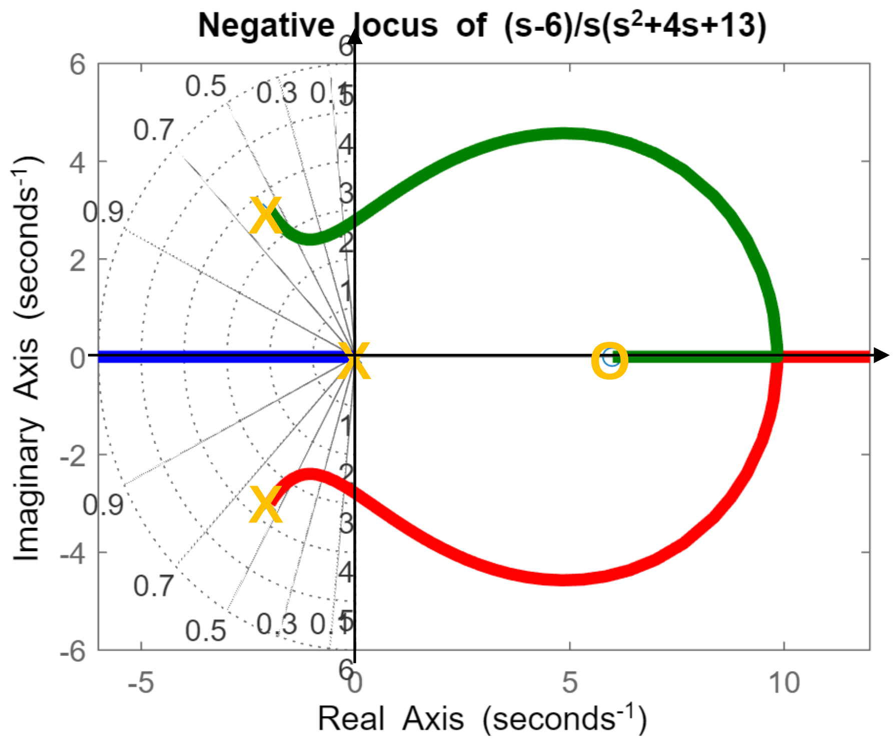
▪ Example 5.13: Negative Root Locus for Airplane

- Rule 3:

- The angles of asymptotes

$$\phi_l = \frac{360^o(l-1)}{3-1} = 0^o, 180^o$$

$$\alpha = \frac{-2-2-(6)}{3-1} = -5$$



- Rule 4:

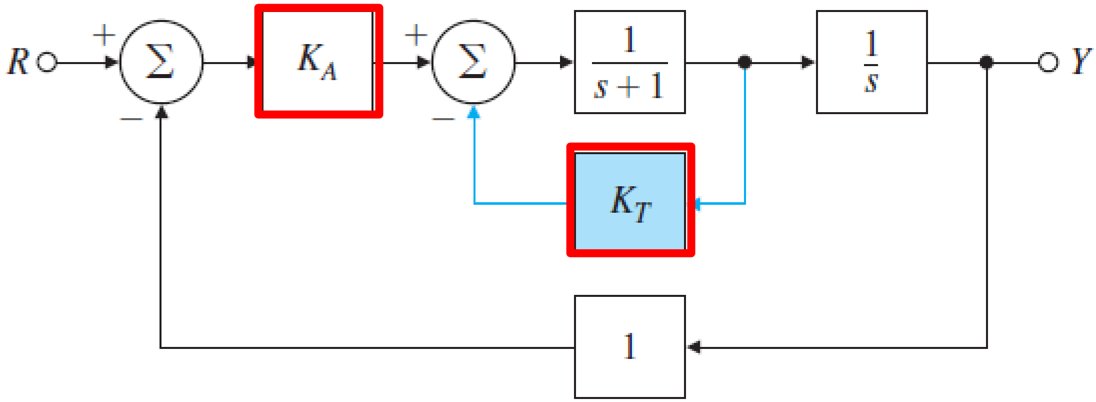
- Departs at $s = -2+j3$ at

$$\phi = \tan^{-1}\left(\frac{3}{-8}\right) - \tan^{-1}\left(\frac{3}{-2}\right) - 90^o + 360^o(l-1)$$

$$= 159.4^o - 123.7^o - 90^o + 360^o(l-1)$$

$$= -54.3^o$$

- Example 5.14: Root Locus Using 2 Parameters in Succession



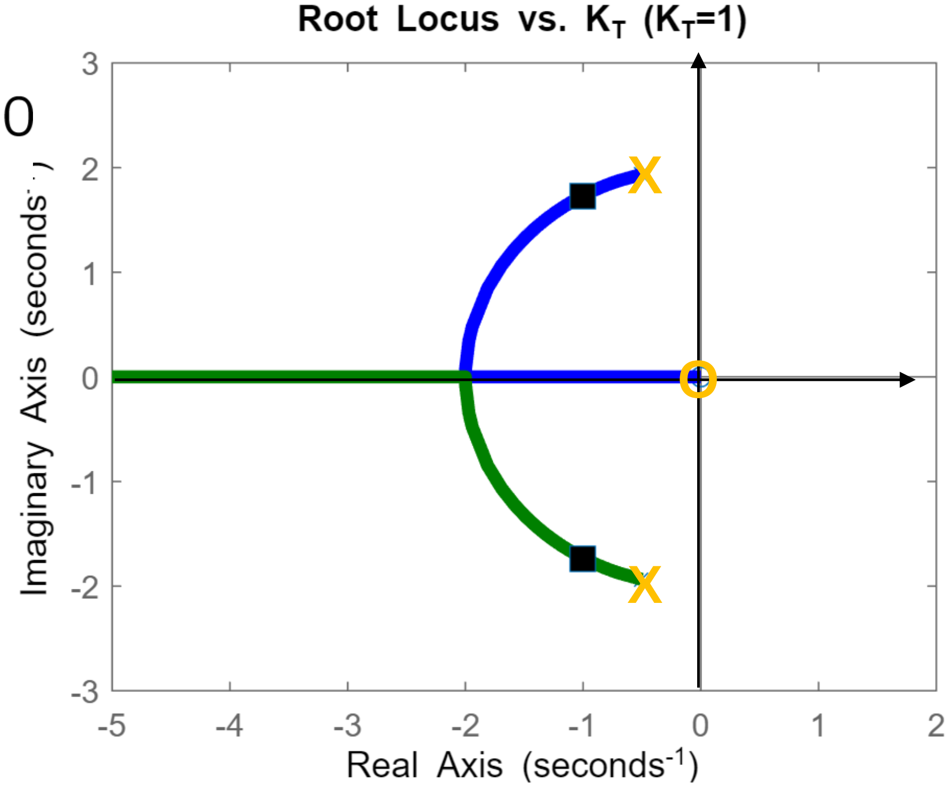
$$\Rightarrow 1 + \frac{K_A}{s(s+1)} + \frac{K_T}{(s+1)} = 0$$

$$\Rightarrow s^2 + s + K_A + K_T s = 0$$

$$\Rightarrow \text{with } K_A = 4$$

$$\Rightarrow 1 + K_T \frac{s}{s^2+s+4} = 0$$

$$\Rightarrow 1 + K\, L(s) = 0$$

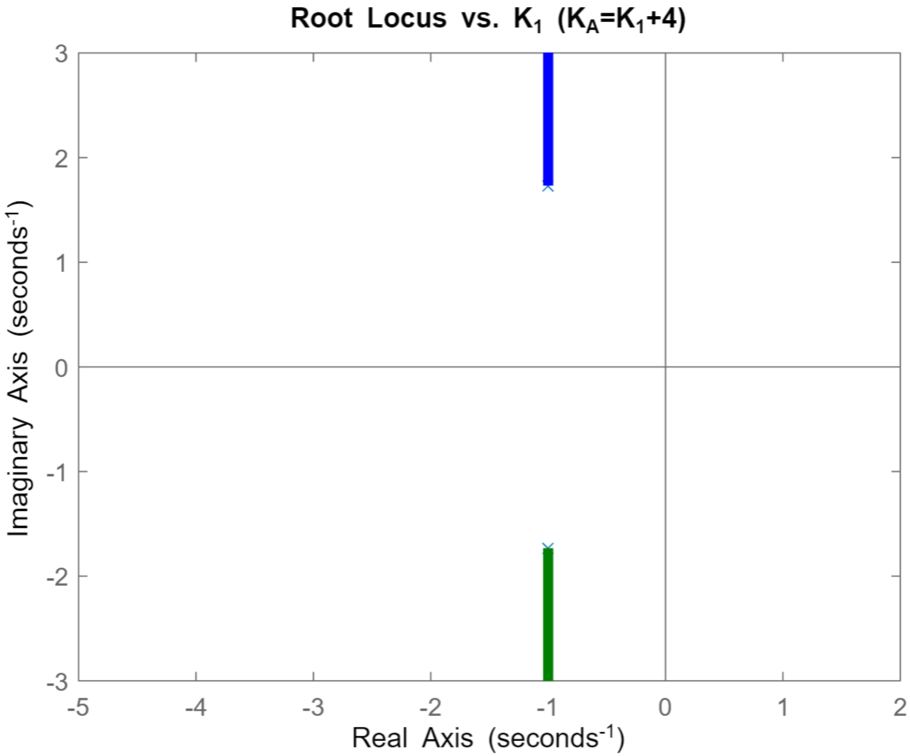


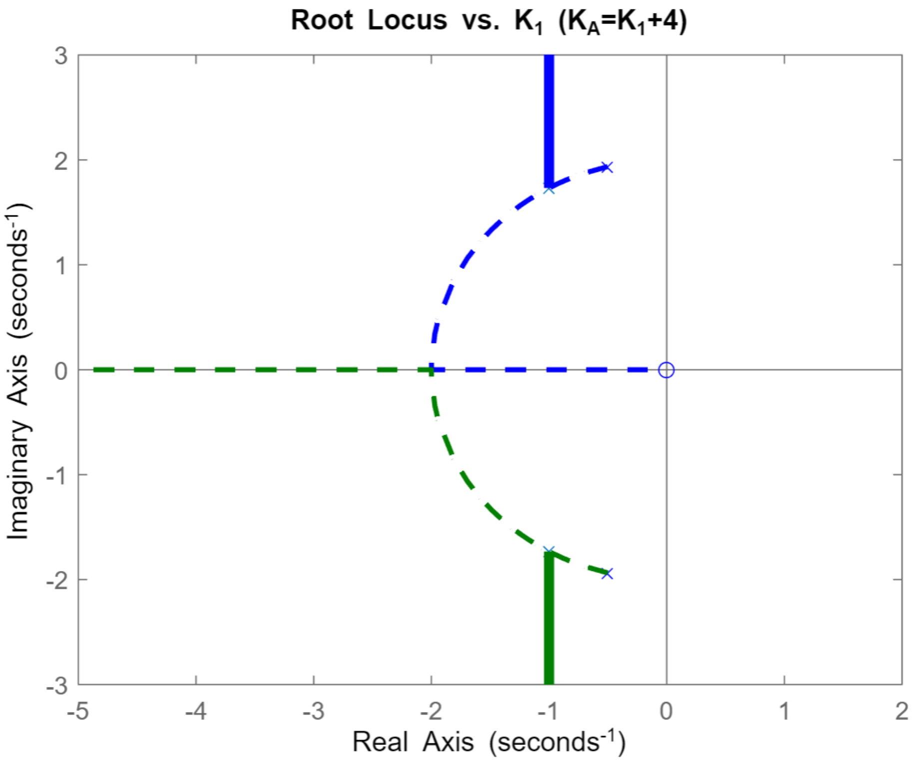
- Example 5.14: Root Locus Using 2 Parameters in Succession

$$\Rightarrow K_T = 1$$

$$\Rightarrow K_A = 4 + K_1$$

$$\Rightarrow 1 + K_1 \frac{1}{s^2 + 2s + 4} = 0 \qquad \Rightarrow 1 + K_T \frac{s}{s^2 + s + 4} = 0$$





Example 5.14: Root Locus Using 2 Parameters in Succession

$$\Rightarrow 1 + K_T \frac{s}{s^2 + s + 4} = 0$$

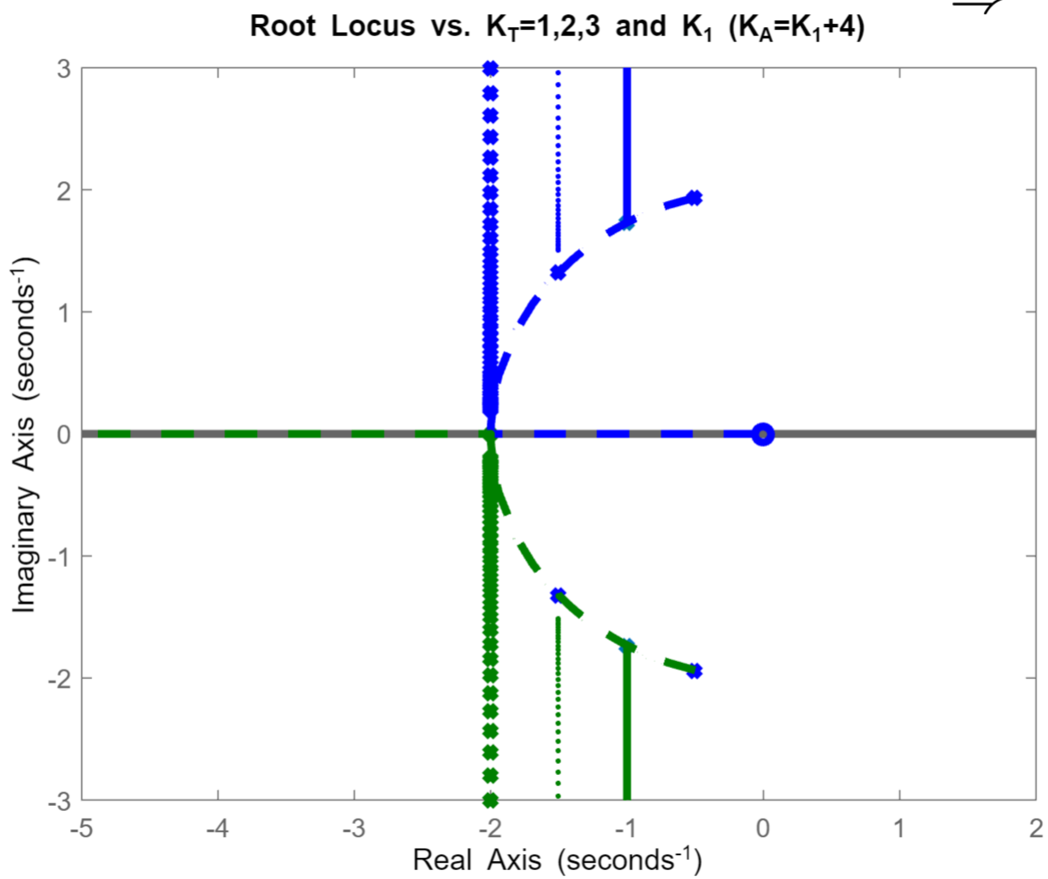
$$\Rightarrow K_T = 1, 2, 3$$

$$\Rightarrow K_A = 4 + K_1$$

$$\Rightarrow 1 + K_1 \frac{1}{s^2 + 2s + 4} = 0$$

$$\Rightarrow 1 + K_1 \frac{1}{s^2 + 3s + 4} = 0$$

$$\Rightarrow 1 + K_1 \frac{1}{s^2 + 4s + 4} = 0$$

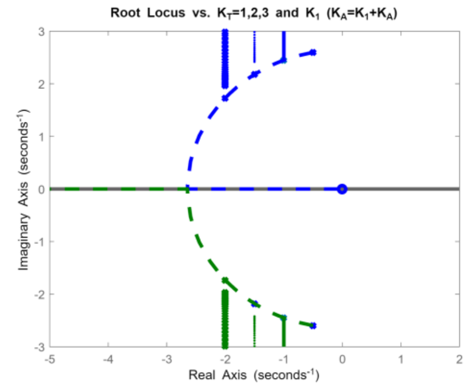


- Example 5.14: Root Locus Using 2 Parameters in Succession

$$\Rightarrow\ 1 + K_T \frac{s}{s^2 + s + \boxed{1}} = 0$$

$$\Rightarrow\ K_T = 1, 2, 3$$

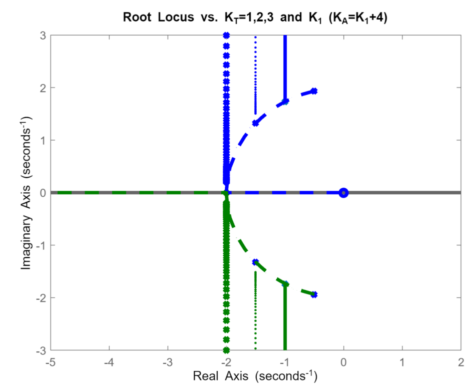
$$\Rightarrow\ K_A = 1 + K_1$$



$$\Rightarrow\ 1 + K_T \frac{s}{s^2 + s + \boxed{4}} = 0$$

$$\Rightarrow\ K_T = 1, 2, 3$$

$$\Rightarrow\ K_A = 4 + K_1$$



$$\Rightarrow\ 1 + K_T \frac{s}{s^2 + s + \boxed{7}} = 0$$

$$\Rightarrow\ K_T = 1, 2, 3$$

$$\Rightarrow\ K_A = 7 + K_1$$