

Fall 2022 (111-1)

控制系統  
Control Systems

Unit 5E  
Design Examples Using the Root Locus

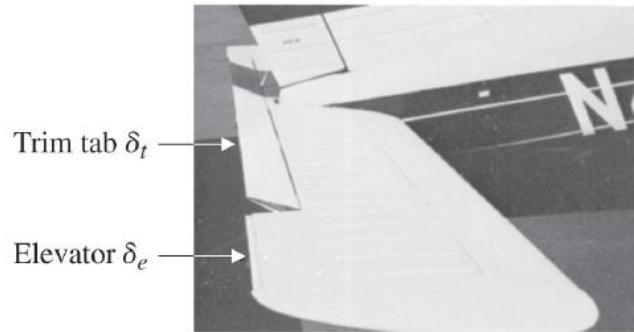
Feng-Li Lian

NTU-EE

Sep 2022 – Dec 2022

## ■ Example 5.12: Control of Small Airplane

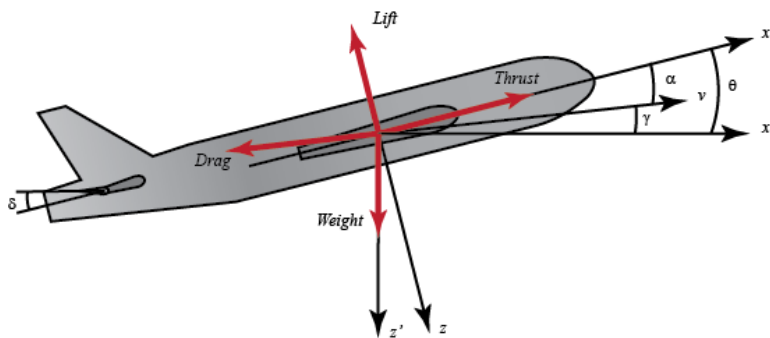
- Autopilot design in the **Piper Dakota**, showing **elevator and trim tab**



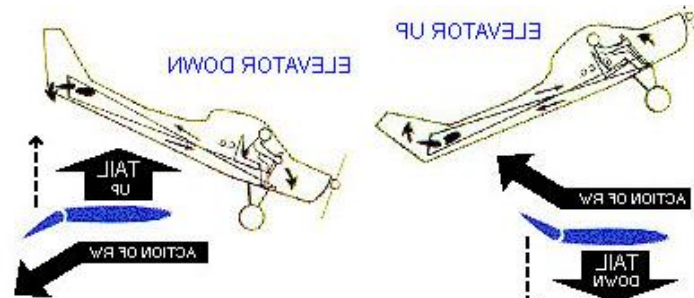
(b)



(a)

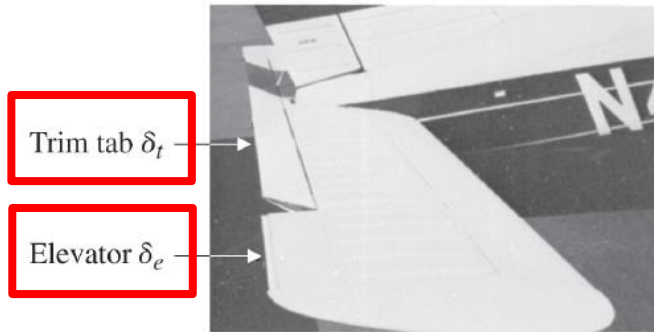


<http://avstop.com/ac/1-14.html>



## Example 5.12: Control of Small Airplane

- Autopilot design in the **Piper Dakota**, showing **elevator** and **trim tab**

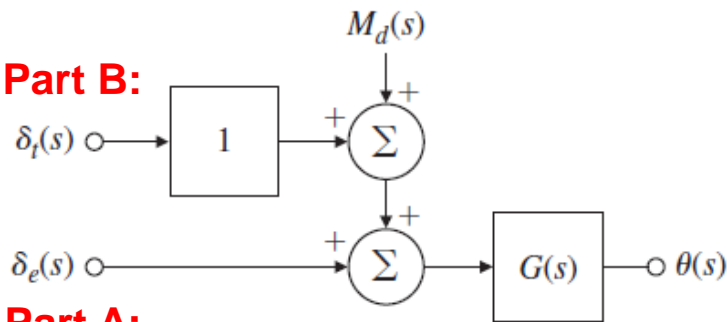


(b)

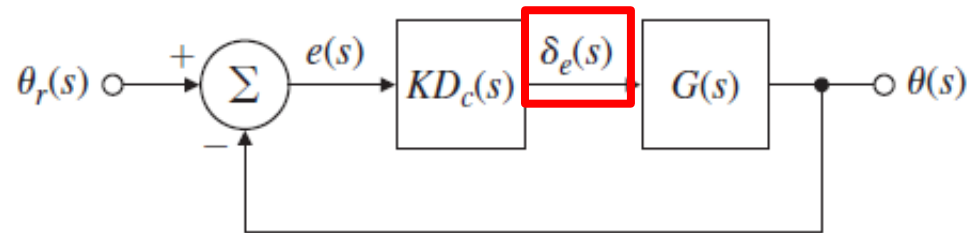


(a)

### Part B:



### Part A:



- Transfer Function** between **elevator input** and **pitch attitude**:

$$G(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{160 (s + 2.5) (s + 0.7)}{(s^2 + 5s + 40) (s^2 + 0.03s + 0.06)}$$

## Example 5.12: Control of Small Airplane

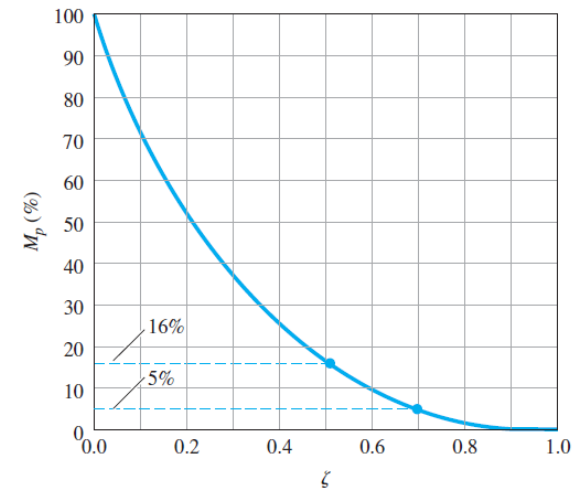
### Part A:

- Design an **autopilot** so that the response to a **step elevator input** has
  - a **rise time** of **1 sec or less** and
  - an **overshoot** less than **10%**.

### For ideal 2nd-order system:

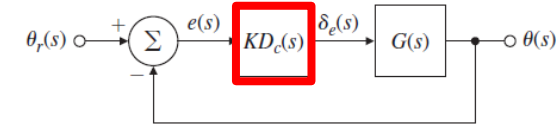
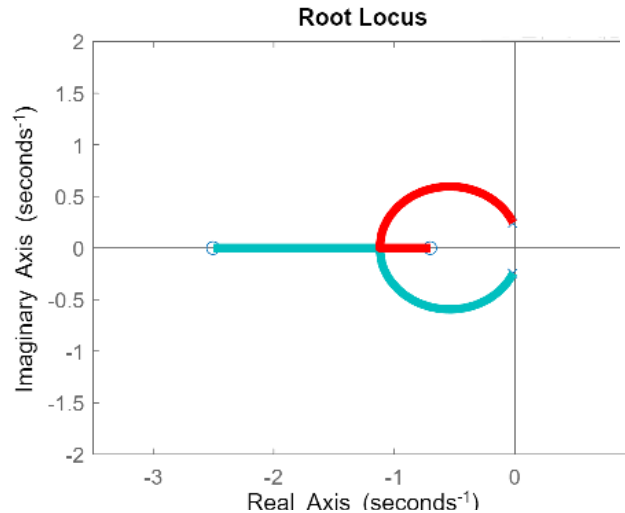
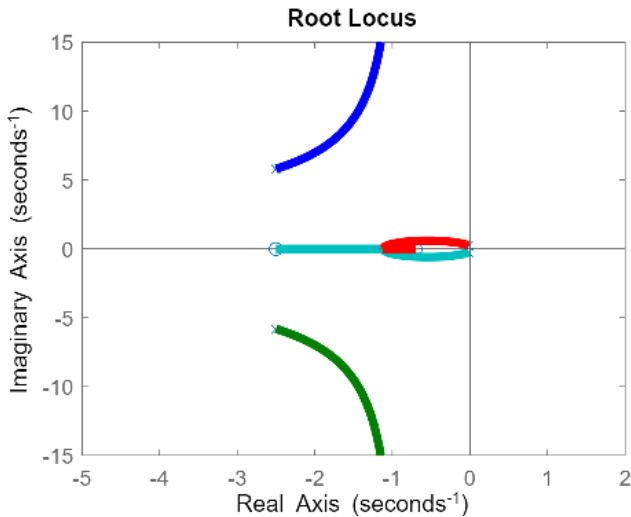
$$\Rightarrow t_r \leq 1 \quad \Rightarrow t_r \approx \frac{1.8}{\omega_n} \quad \Rightarrow \omega_n \geq \frac{1.8}{t_r} \quad \Rightarrow \omega_n \geq 1.8$$

$$\Rightarrow M_p \leq 10 \quad \Rightarrow \zeta \geq 0.6$$



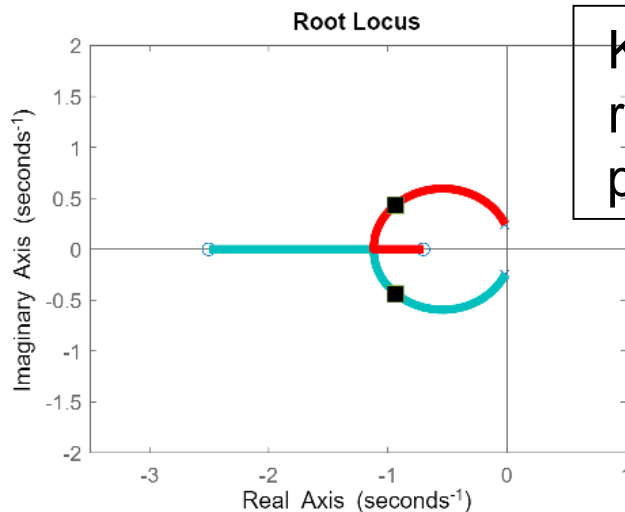
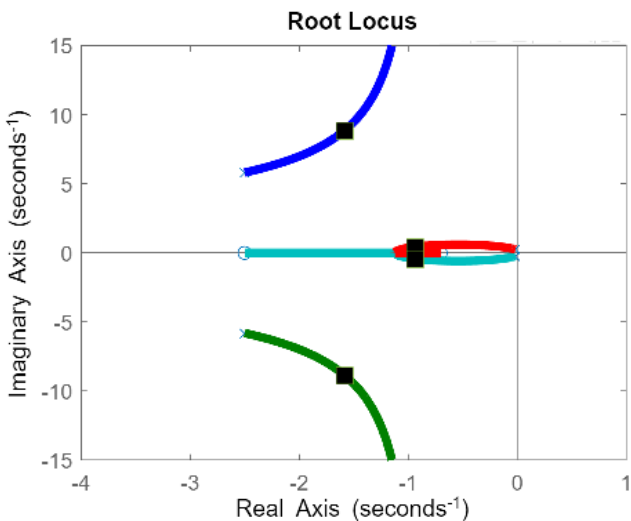
## Example 5.12: Control of Small Airplane

```
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
rlocus( sysG )
```



(b)

$$\Rightarrow KD_c(s) = K_p$$

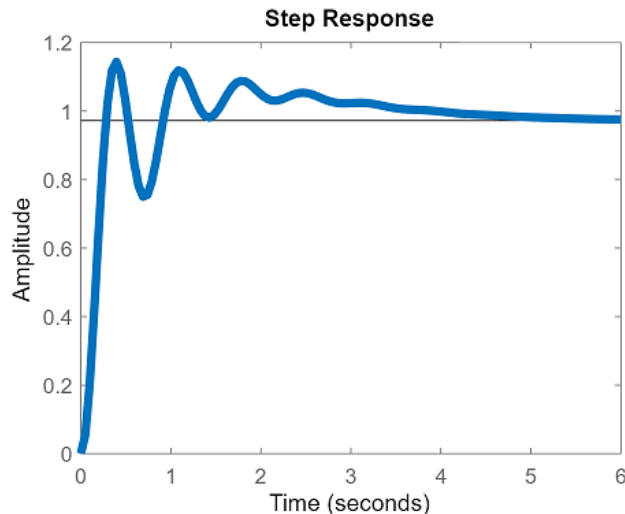


```
Kp = 0.3;
r1 = rlocus( sysG, Kp );
plot( r1, 's' )
```

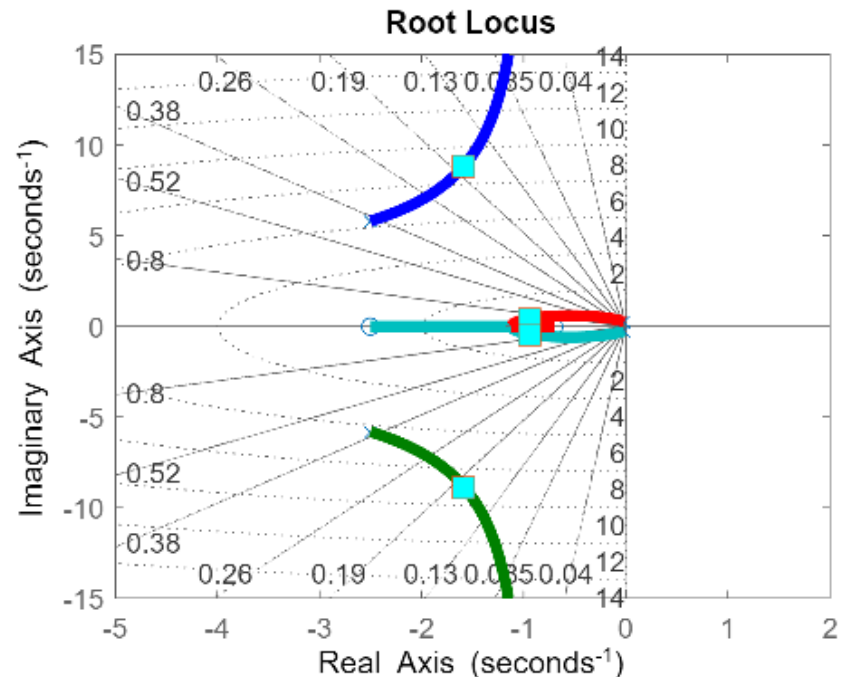
## Example 5.12: Control of Small Airplane

```

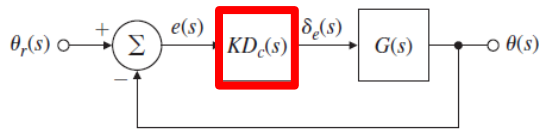
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
Kp = 0.3;
sysTp = feedback(Kp*sysG,1);
step( sysTp )
    
```



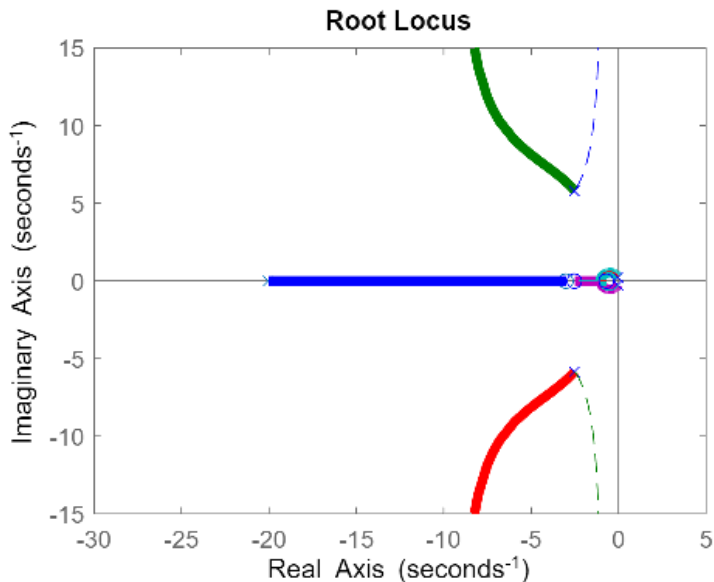
- Long-term settling
- $\zeta \leq 2.5/6.32 = 0.40$



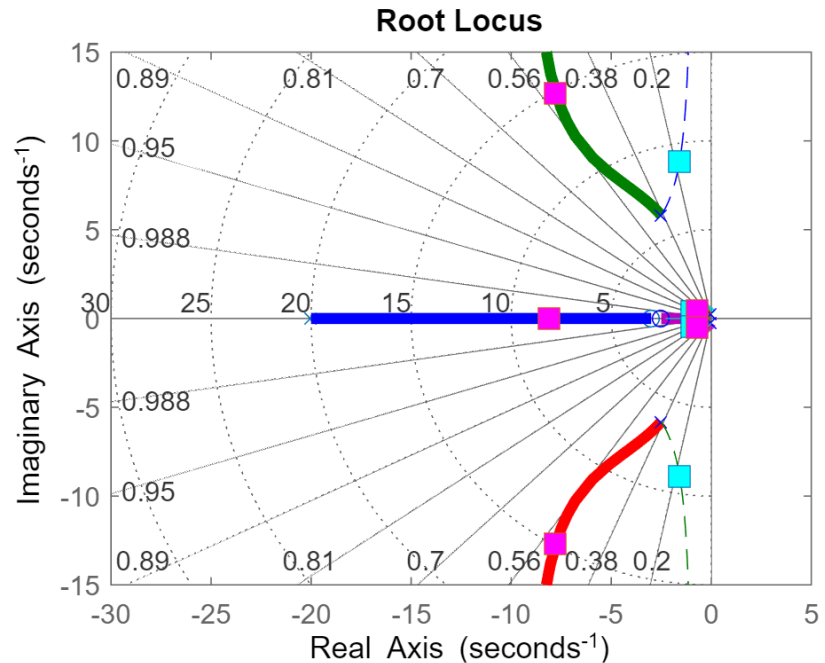
## Example 5.12: Control of Small Airplane



$$\Rightarrow KD_c(s) = K_c \frac{s + 3}{s + 20}$$



```
sysG = 160*(s+2.5)*(s+0.7)/
      ((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
rlocus( sysDG )
```

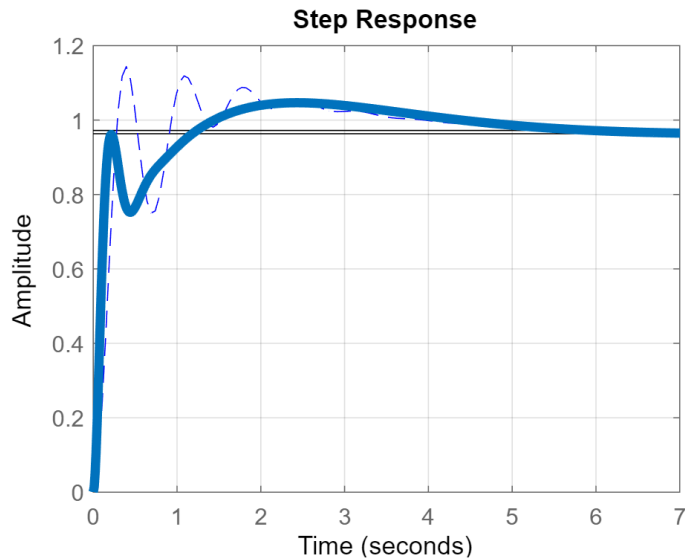


```
Kc = 1.5;
r2 = rlocus( sysDG, Kc );
plot( r2, 's' )
```

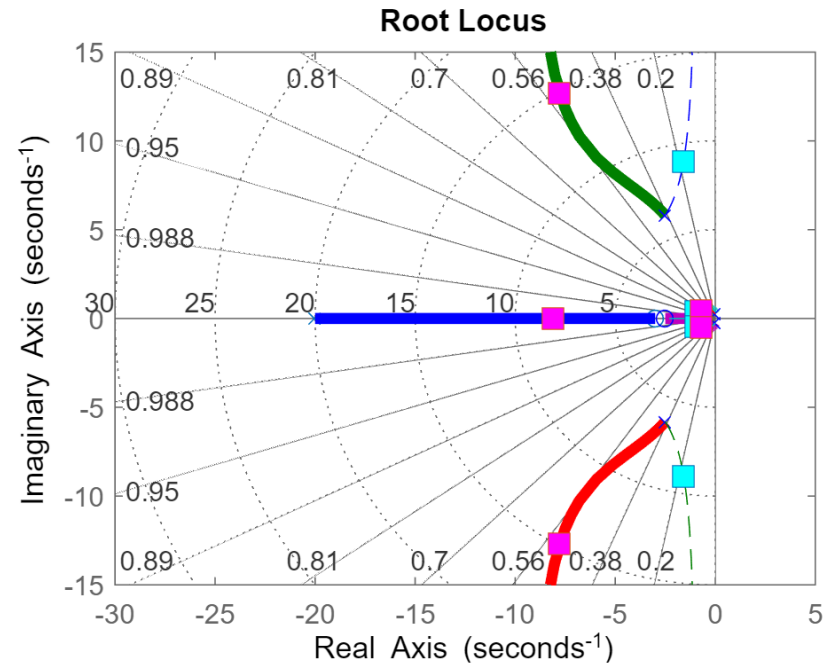
## Example 5.12: Control of Small Airplane

```

sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
Kp = 0.3;
sysTp = feedback(Kp*sysG,1);
step( sysTp )
    
```



- $t_r = 0.9$  sec
- $\zeta = 0.52$
- $M_p = 8\%$
- $\omega_n = 15$  rad/sec



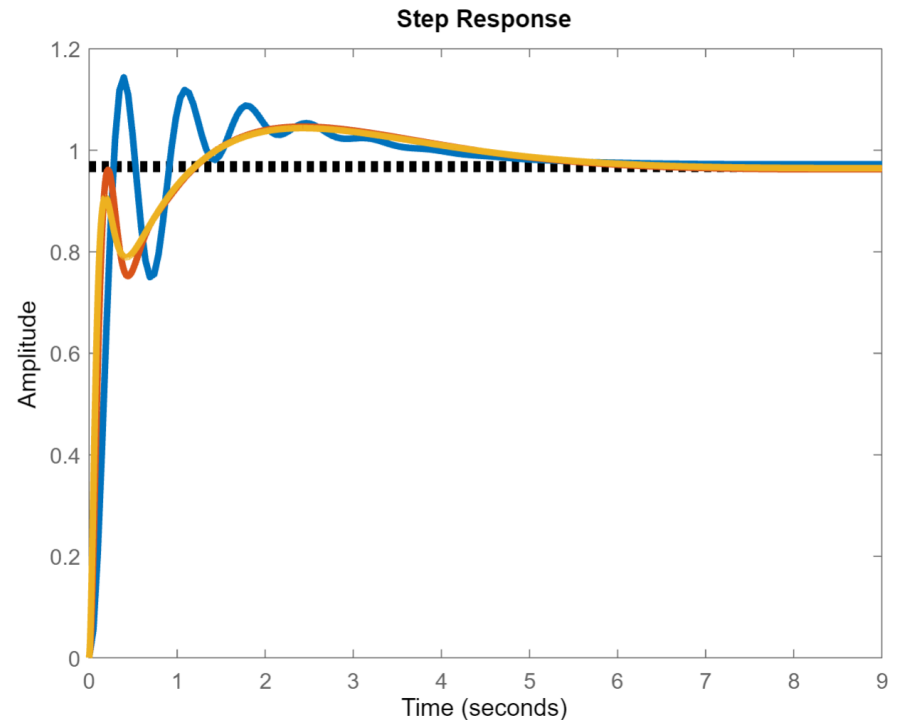
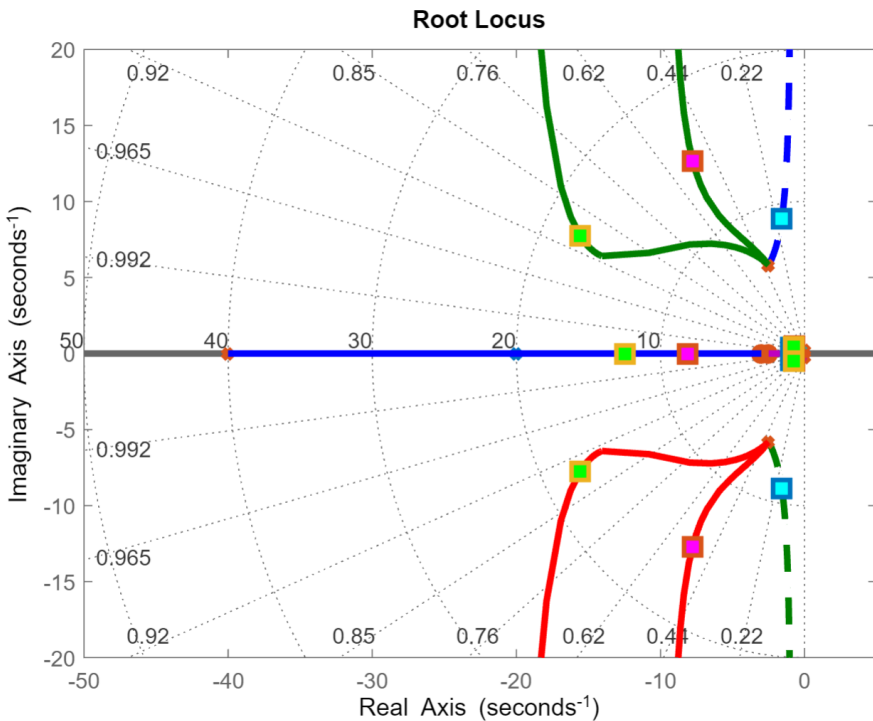


## Example 5.12: Control of Small Airplane

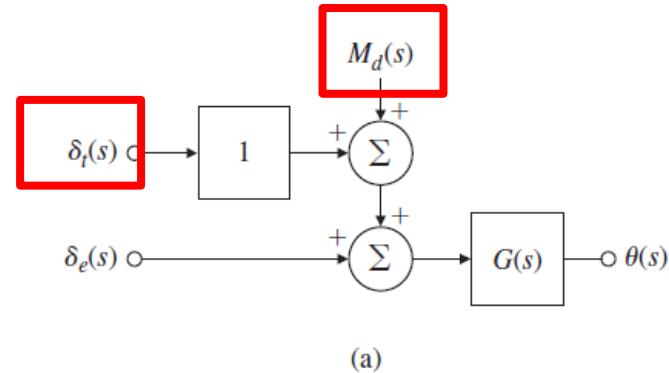
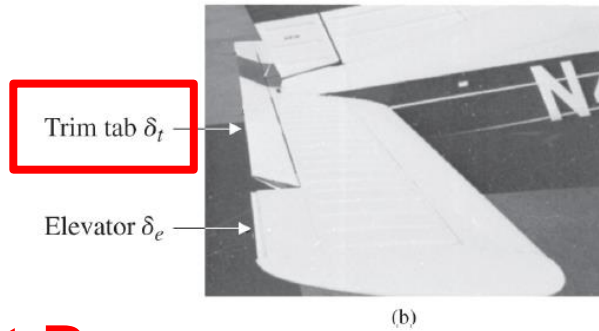
$$KD_c(s) = K_p = 0.3$$

$$KD_c(s) = 1.5 \frac{s + 3}{s + 20}$$

$$KD_c(s) = 3.2 \frac{s + 3}{s + 40}$$



## Example 5.12: Control of Small Airplane

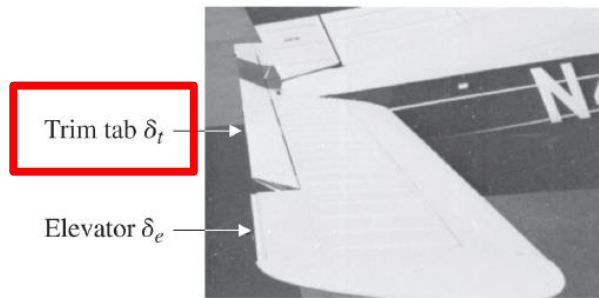


### Part B:

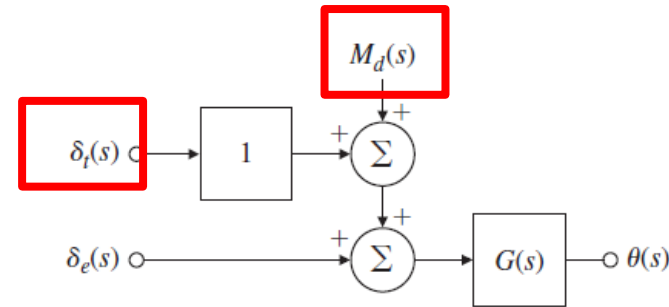
- When there is a **constant disturbing moment** acting on the aircraft so that the pilot must supply a **constant force** for steady-flight control
- It is said to be **out of trim**.



## Example 5.12: Control of Small Airplane



(b)



(a)

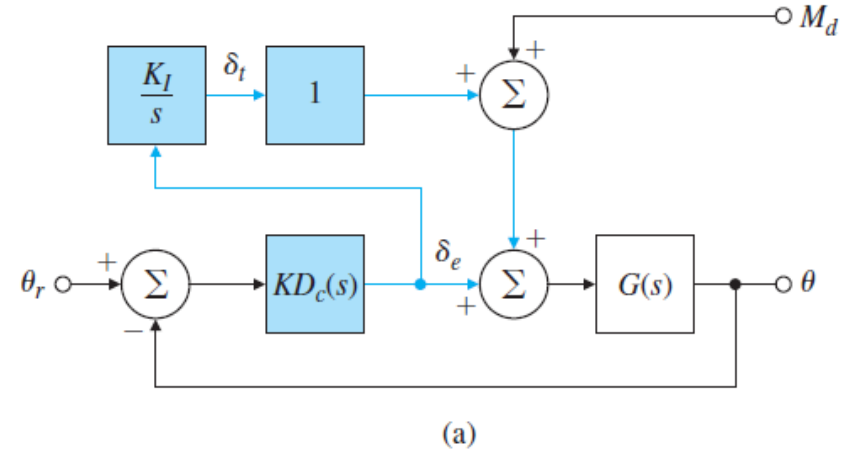
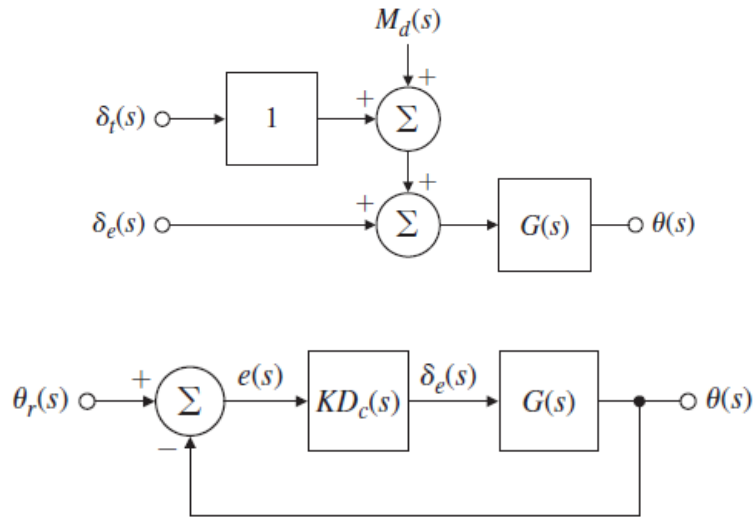
### Part B:

- When there is a **constant disturbing moment** acting on the aircraft so that the pilot must supply a **constant force** for steady-flight control
- It is said to be **out of trim**.
- **The transfer function** between the **disturbing moment** and the **attitude** is:

$$G(s) = \frac{\theta(s)}{M_d(s)} = \frac{160 (s + 2.5) (s + 0.7)}{(s^2 + 5s + 40) (s^2 + 0.03s + 0.06)}$$

- **No steady-state** control effort for **elevator**, that is,  $\delta_e = 0$
- Only command the **trim**  $\delta_t$  for arbitrary constant **moment**  $M_d$

## Example 5.12: Control of Small Airplane

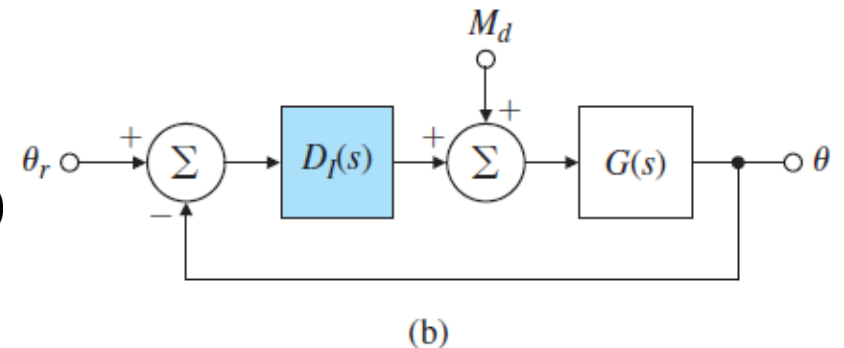


$$\Rightarrow D_I(s) = K D_c(s) \left( 1 + \frac{K_I}{s} \right)$$

$$\Rightarrow 1 + G(s) D_I(s) = 0$$

$$\Rightarrow 1 + K D_c G + \frac{K_I}{s} K D_c G = 0$$

$$\Rightarrow 1 + K_I \frac{\frac{1}{s} K D_c G}{1 + K D_c G} = 0 \quad \Rightarrow 1 + K_I L(s) = 0$$

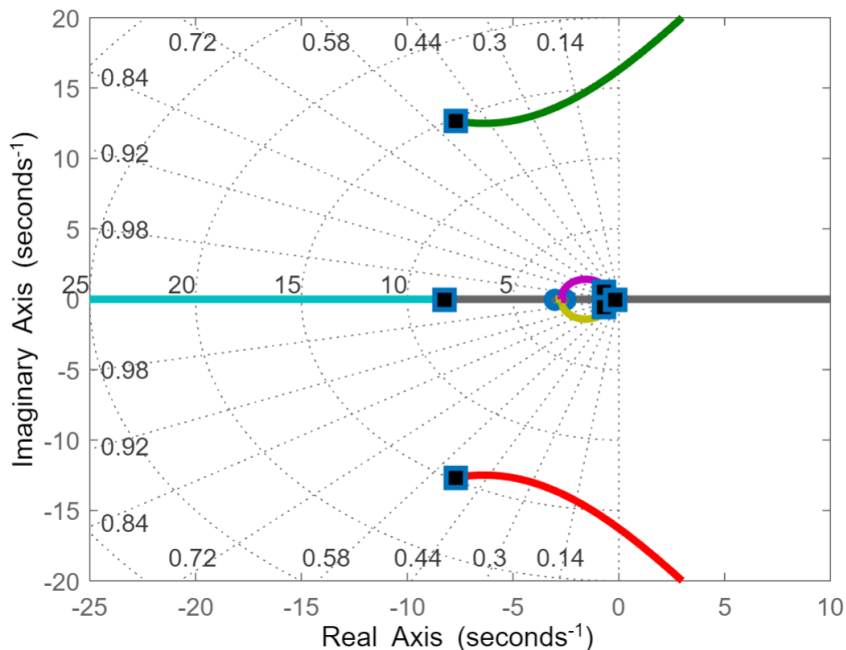


## Example 5.12: Control of Small Airplane

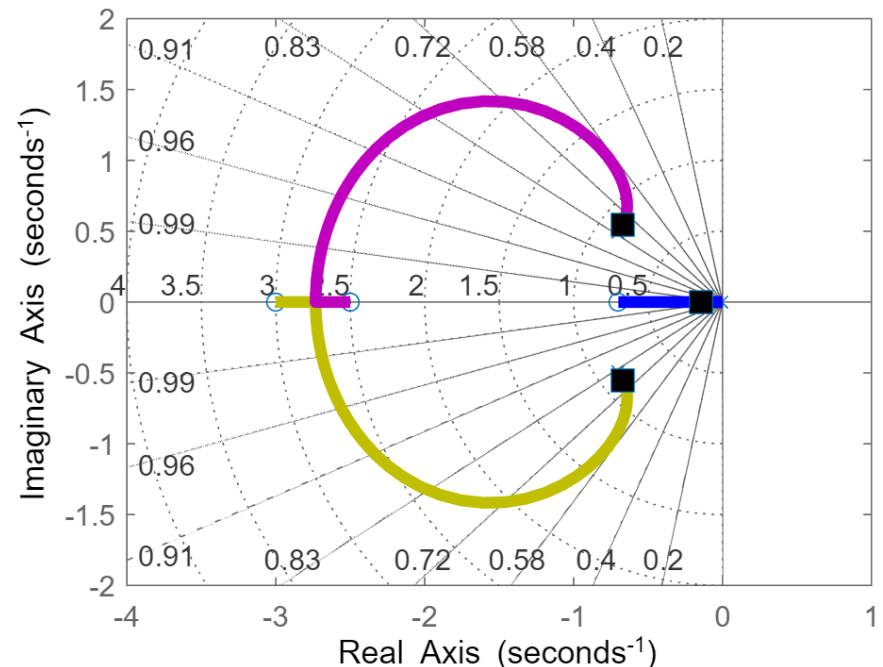
```

sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
Kc = 1.5;
sysT = feedback( Kc*sysDG, 1 );
sysL = sysT/s; % add integral control pole
rlocus(sysL)
KI = 0.15;
[R1] = rlocus( sysL, KI );      plot( R1,'s' )
    
```

Root Locus vs.  $K_i$  for autopilot



zoomed in



## Example 5.12: Control of Small Airplane

```

sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
Kc = 1.5;
KI = 0.15;
sysDI = Kc*sysD*(1+KI/s);
sysLICL=feedback(sysDI*sysG,1);
step(5*sysLICL,30) % X5 because a step command of 5 deg
sysFBDe=sysG*(1+KI/s);
sysDeCL=feedback(Kc*sysD,sysFBDe);
step( 5*sysDeCL,30 )

```

- Integral Term at  $s = -0.14$
- $t_s \geq 4.6/0.14 = 33$

