Fall 2022 (111-1)

控制系統
Control Systems

Unit 50
Root Locus (s-Domain)

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NTU-EE

Sep 2022 – Dec 2022

Unit 5

Root Locus

Summary of the Rules for Determining a Root Locus

- Rule 1:
- The n branches of the locus start at the poles of L(s) and
- m of these branches end on the zeros of L(s).

- Rule 2:
- The loci are on the real axis to the left of an odd number of poles and zeros.

- Rule 3:
- For large s and K, n-m branches of the loci are asymptotic to lines at angles ϕ radiating out from the point s = α on the real axis, where

$$\phi_l = \frac{180^o + 360^o(l - 1)}{n - m} \qquad l = 1, 2, \cdots, n - m$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n - m}$$

Summary of the Rules for Determining a Root Locus

- Rule 4: $\sum \psi_i - \sum \phi_i = 180^o + 360^o (l - 1)$

- The angle of departure of a branch of the locus from repeated poles with multiplicity q is given by

$$q \, \phi_{l,dep} = \sum \psi_i - \sum_{i \neq l,dep} \phi_i - 180^o - 360^o (l-1)$$

$$l = 1, 2, \cdots, q$$

- The angle of arrival of a branch at a zero with multiplicity q is given by

$$q \, \psi_{l,arr} = \sum \phi_i - \sum_{i \neq l,arr} \psi_i + 180^o + 360^o (l-1)$$

- Rule 5:

- The locus can have multiple roots at points on the locus and the branches will approach a point of q roots at angles separated by

$$\frac{180^o - 360^o (l-1)}{q}$$

- And will depart at angles with same separation.

Your Jobs are To

- By Hand:
  - **Hand Writing in Exam (40%)**
- Use the 5 rules of Root Locus Method

  to roughly sketch the root locus of any transfer function

  by identifying these critical root locations
- Properly choose some roots

  between these critical root locations

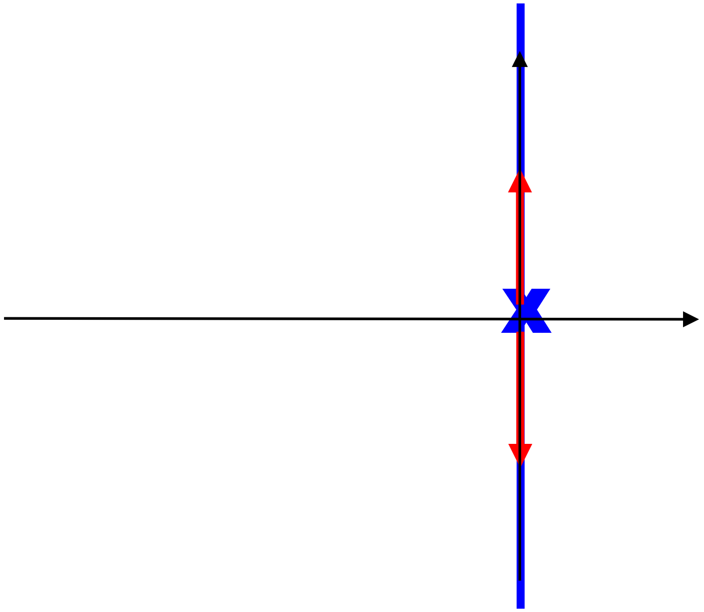
- By Computer:
  - **Multiple Choice in Exam (60%)**
- Use Matlab codes

  to draw the exact root locus of any transfer function
- Design proper transfer function and

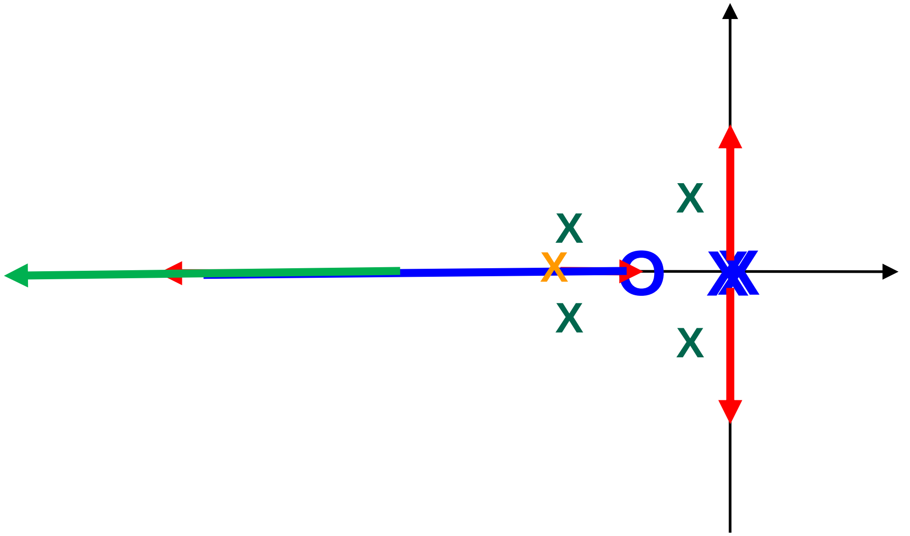
  select associated and reasonable gain value

Your Jobs By Hand

$$\Rightarrow \; 1 + K_P \frac{1}{s^2} = 0$$

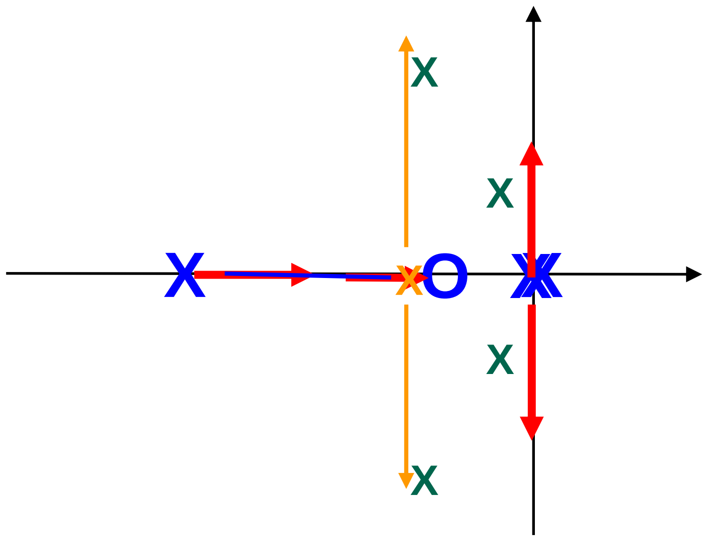


$$\Rightarrow \; 1 + K \frac{s+1}{s^2} = 0$$

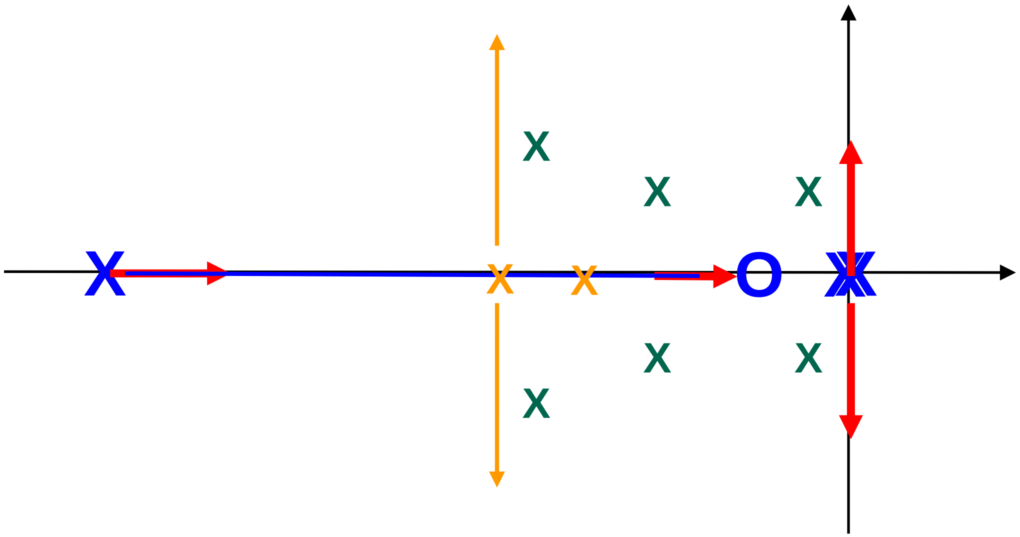


Your Jobs By Hand

$$\Rightarrow \; 1 + K \frac{(s+1)}{s^2\,(s+4)} = 0$$

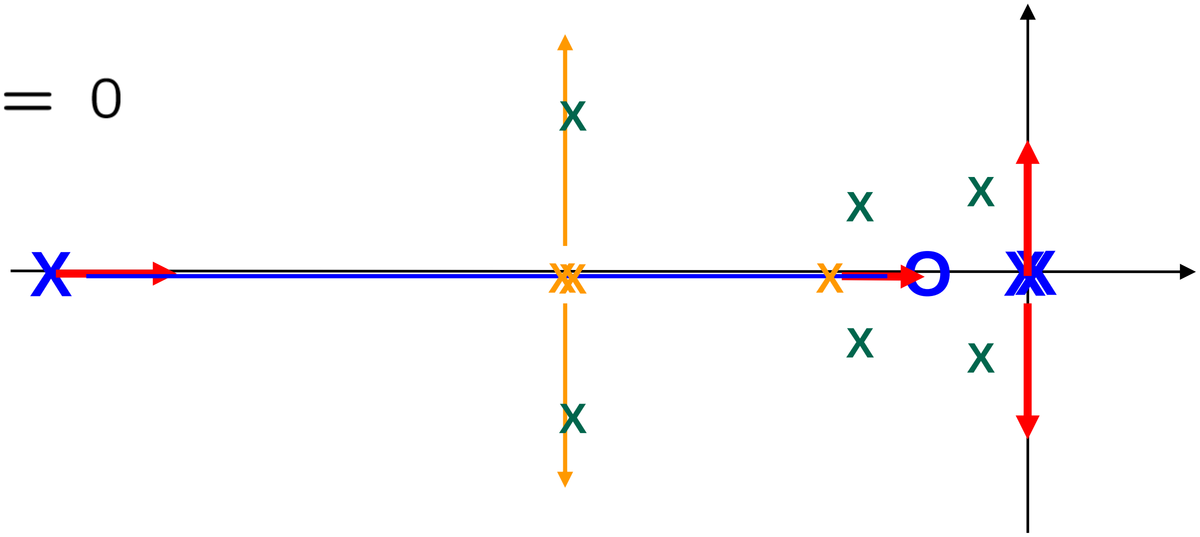


$$\Rightarrow \; 1 + K \frac{(s+1)}{s^2\,(s+9)} = 0$$



Your Jobs By Hand

$$\Rightarrow \; 1 + K \frac{(s+1)}{s^2\,(s+12)} = 0$$



Your Jobs By Hand

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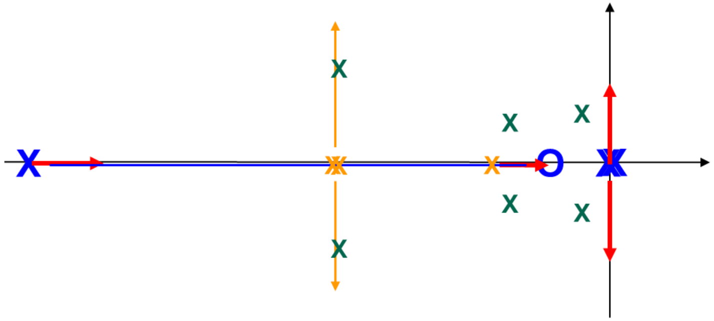

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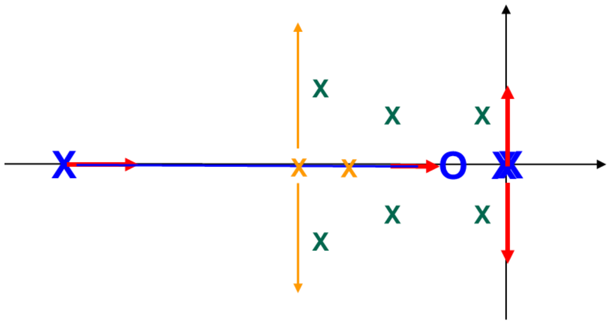
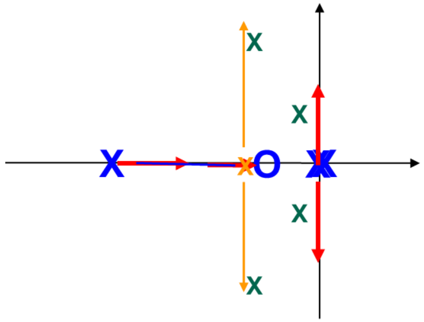
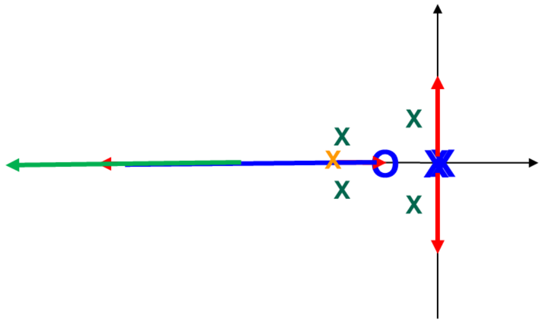


$$\Rightarrow \; 1 + K \frac{(s+1)}{s^2\,(s+4)} = 0$$



$$\Rightarrow \; 1 + K \frac{s+1}{s^2} = 0$$



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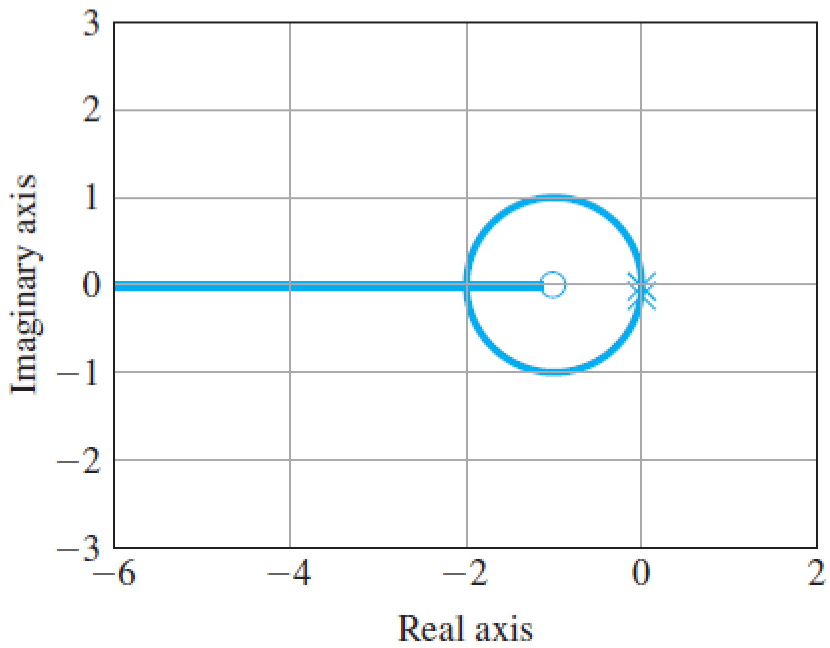

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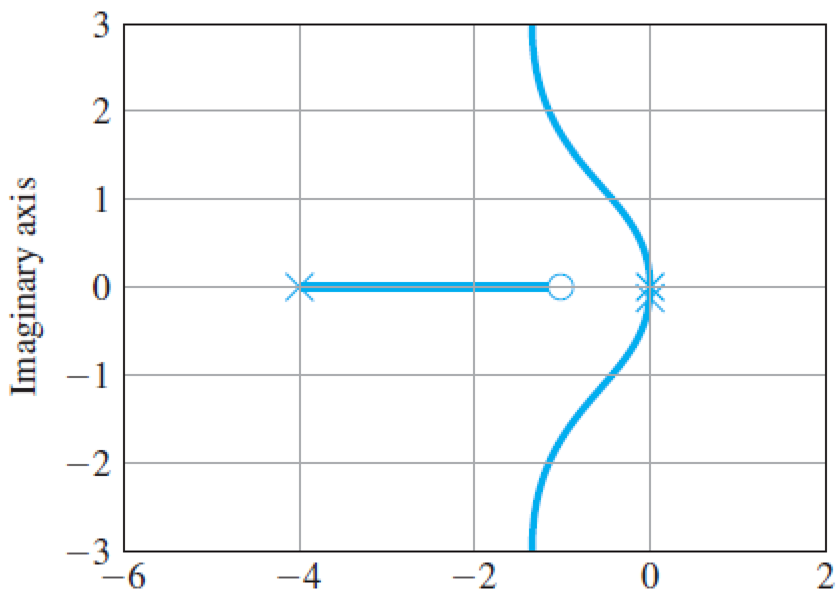

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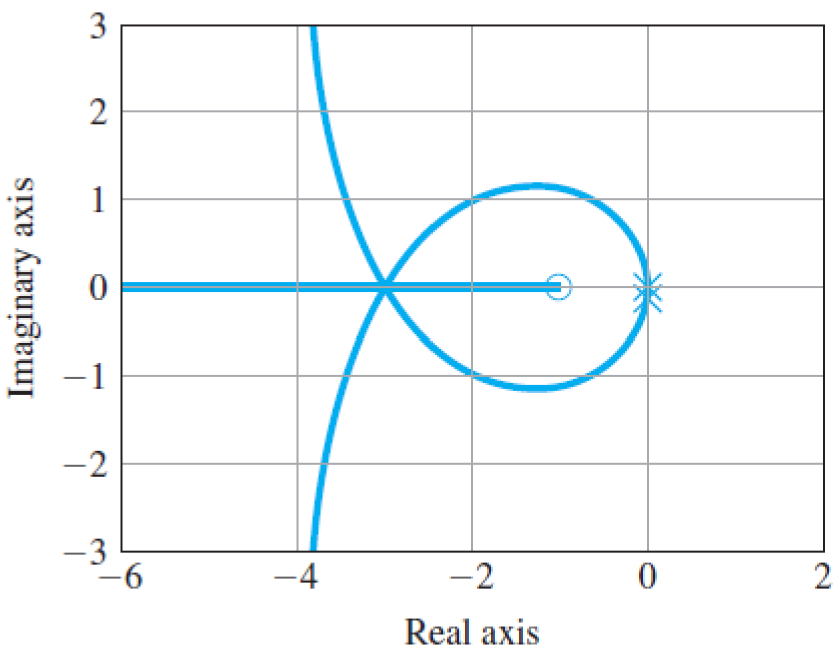

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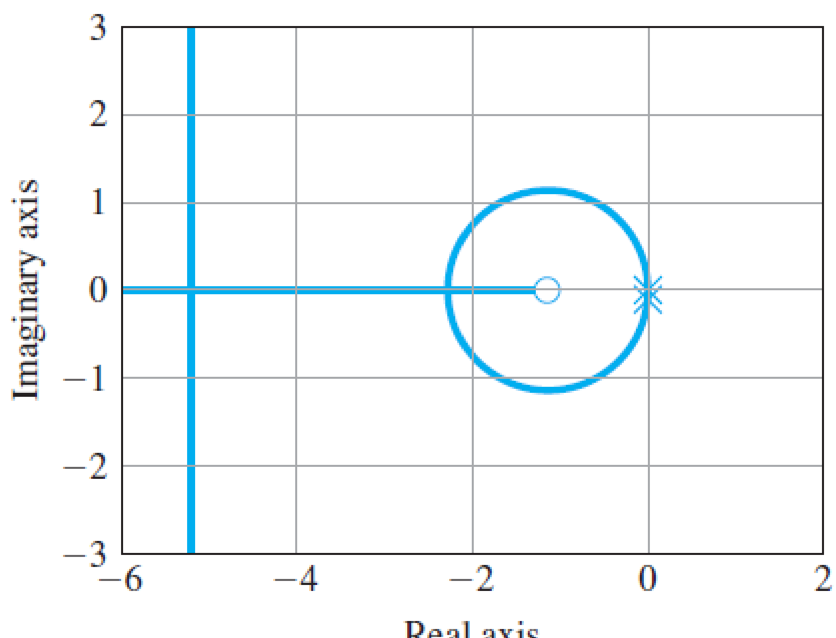
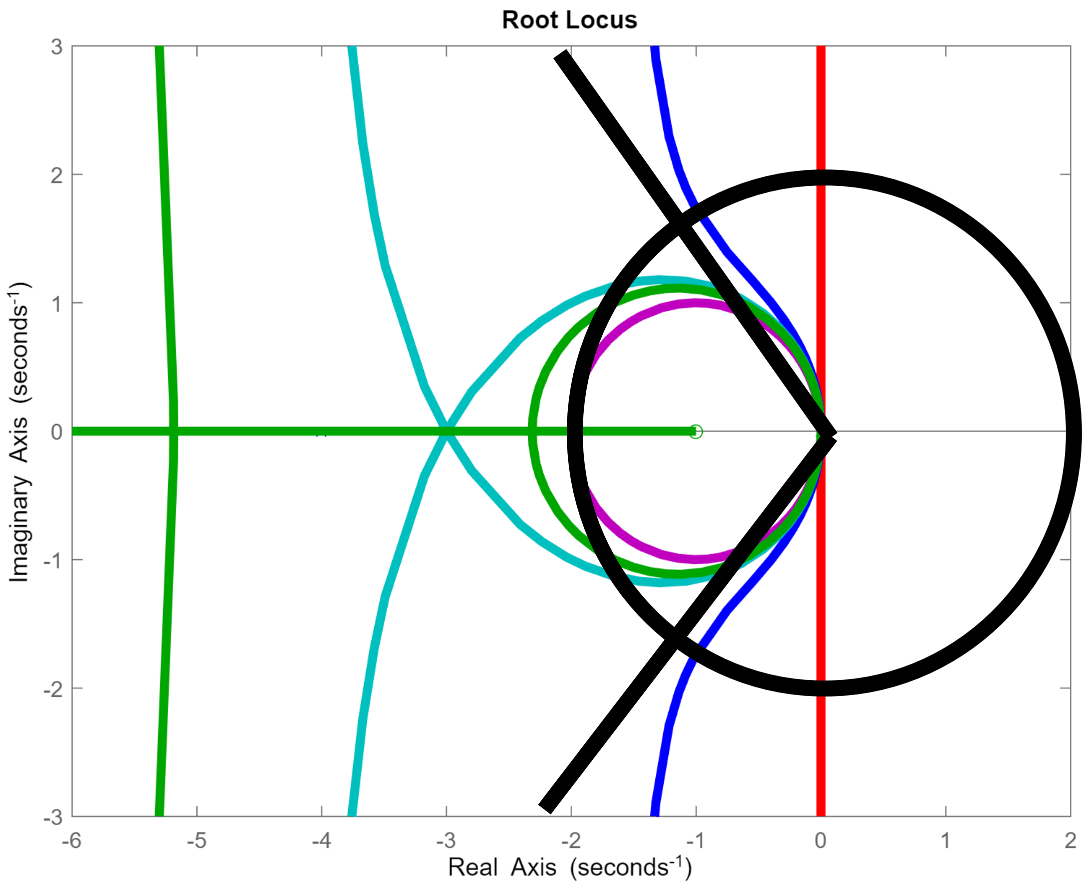
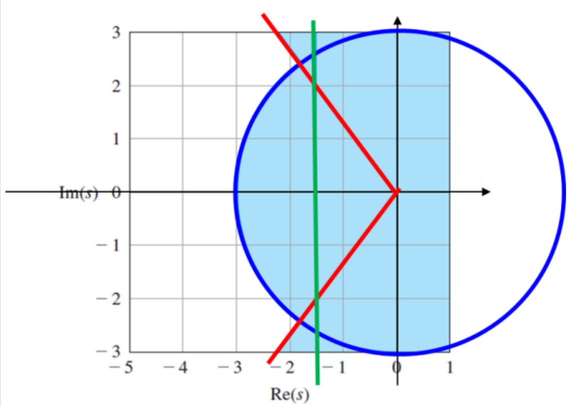


Your Jobs By Computer





- Proper transfer function
- Reasonable gain values