Fall 2022 (111-1)

控制系統
Control Systems

Unit 3D
Time-Domain Specifications

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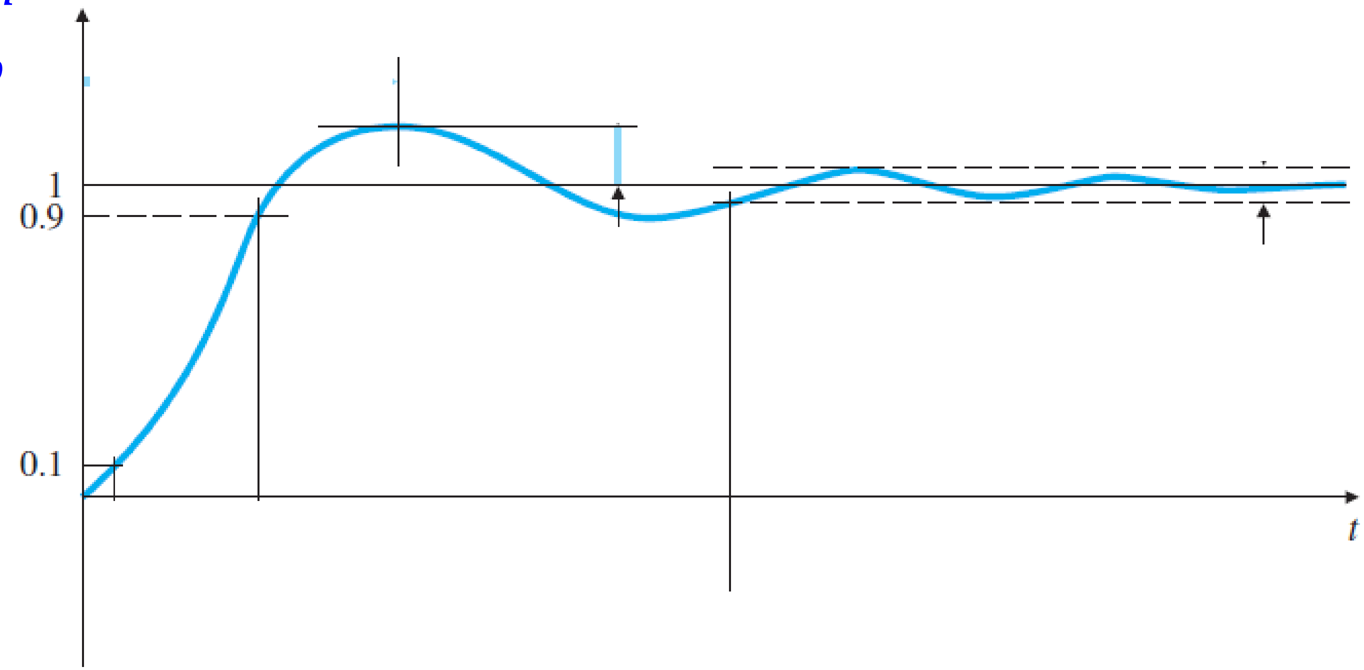
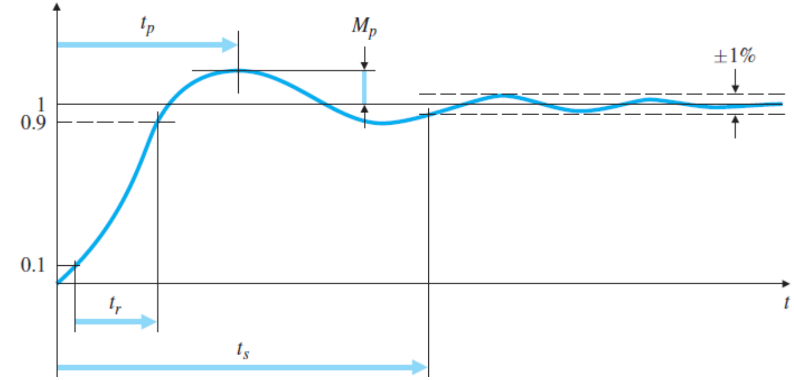
- Performance specification for a control system design often involve certain requirements associated with the time response of the system.
- The requirement for a step response are expressed in terms of the standard quantities illustrated in the figure.
- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p



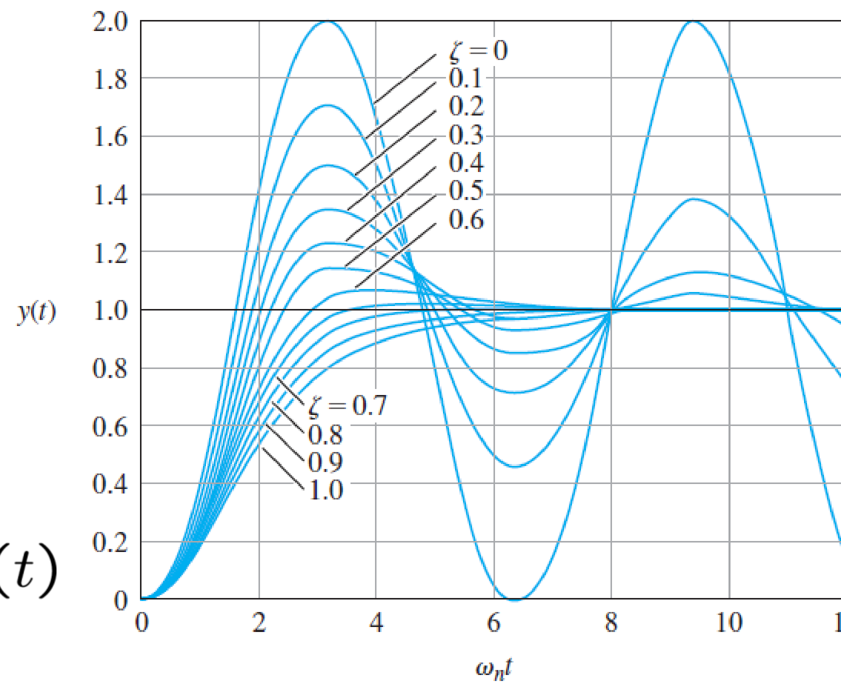
Time-Domain Specifications

- Rise time t_r
- The time from y=0.1 to y=0.9



- For $\zeta = 0.5$,
- Approximately, $w_n \, t_r = 1.8$

$$\Rightarrow t_r \cong \frac{1.8}{w_n}$$

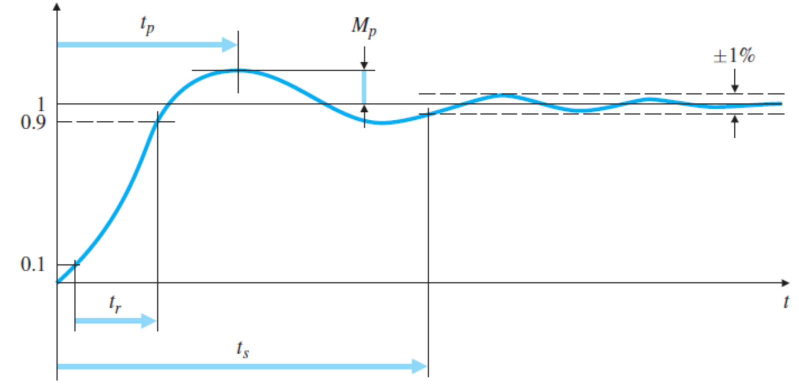


(b)

$$h(t) = \frac{w_n}{\sqrt{1-\zeta^2}} e^{-\sigma t} (\sin w_d t) \, 1(t)$$

$$H(s) = \frac{{w_n}^2}{s^2 + 2\,\zeta\, w_n\, s + {w_n}^2} = \frac{{w_n}^2}{(s+\zeta w_n)^2 + {w_n}^2(1-\zeta^2)}$$

- Overshoot M_p and Peak time t_p



$$Y(s) = H(s)\,\frac{1}{s}$$

$$\Rightarrow\ y(t) = 1 - e^{-\sigma t}\left(\cos w_d t + \frac{\sigma}{w_d}\sin w_d t\right)$$

$$\sigma = w_n\,\zeta$$

$$w_d = w_n\sqrt{1-\zeta^2}$$

$$A\sin(\alpha) + B\cos(\beta) = C\cos(\alpha-\beta)$$

$$C = \sqrt{A^2+B^2} = \frac{1}{\sqrt{1-\zeta^2}}$$

$$\beta = \tan^{-1}\left(\frac{A}{B}\right)$$

$$= \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = \sin^{-1}(\zeta)$$

$$\Rightarrow\ y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(w_d t - \beta)$$

$$\Rightarrow\ \dot{y}(t_p) = 0$$

- Overshoot M_p and Peak time t_p

$$y(t) = 1 - e^{-\sigma t}\left(\cos w_d t + \frac{\sigma}{w_d}\sin w_d t\right)$$

$$\Rightarrow \dot{y}(t) = \sigma e^{-\sigma t}\left(\cos w_d t + \frac{\sigma}{w_d}\sin w_d t\right) - e^{-\sigma t}\left(-w_d \sin w_d t + \sigma \cos w_d t\right)$$

$$= e^{-\sigma t}\left(\frac{\sigma^2}{w_d} + w_d\right)\sin w_d t$$

$$\Rightarrow \dot{y}(t_p) = 0$$

$$\Rightarrow \sin w_d t = 0$$

$$\Rightarrow t_p = \frac{\pi}{w_d}$$

$$\Rightarrow y(t_p) \triangleq 1 + M_p$$

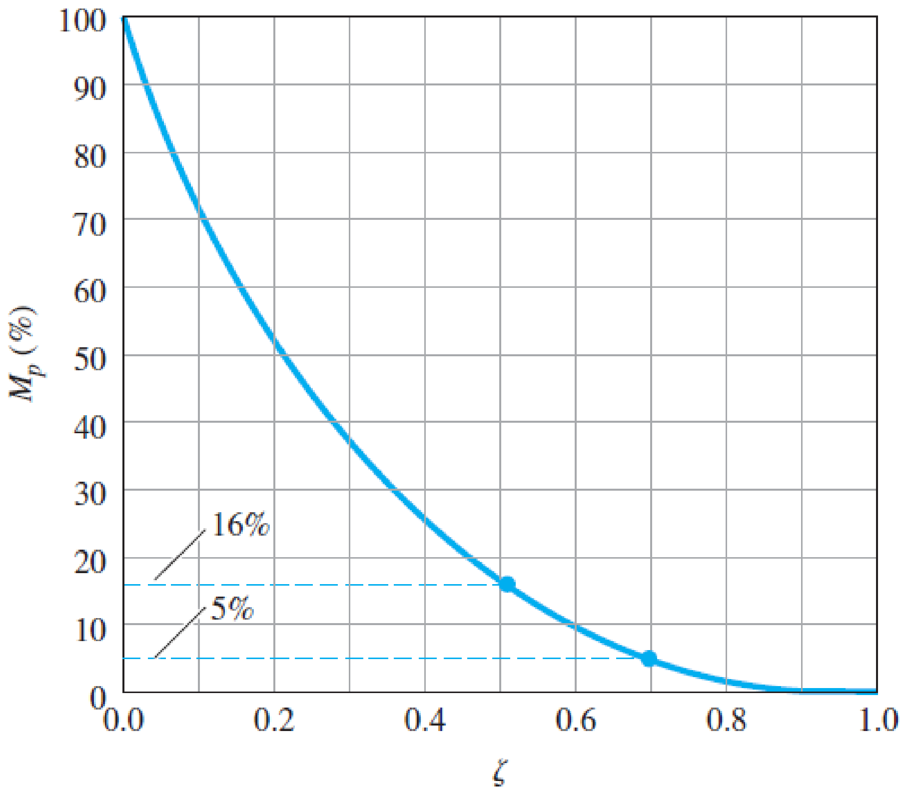
$$= 1 - e^{-\sigma \frac{\pi}{w_d}}\left(\cos \pi + \frac{\sigma}{w_d}\sin \pi\right)$$

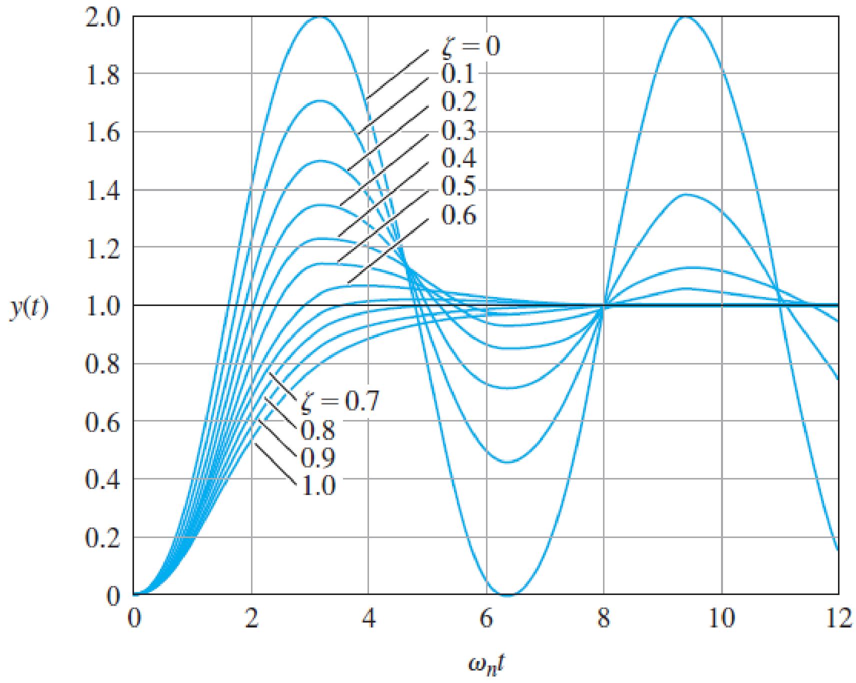
$$= 1 + e^{-\sigma \frac{\pi}{w_d}}$$

$$\Rightarrow M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad 0 \le \zeta < 1$$

- Overshoot M_p and Peak time t_p

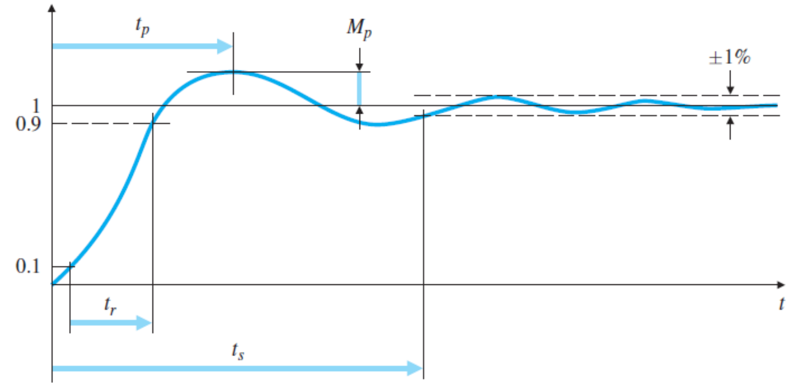
$$\Rightarrow \quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad 0 \le \zeta < 1$$





(b)

- Settling time t_s



$$\Rightarrow \quad e^{-\zeta w_n t_s} = 0.01$$

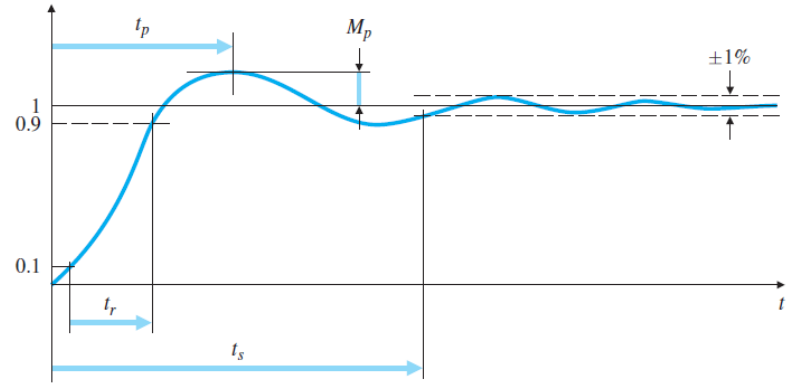
$$\Rightarrow \quad \zeta w_n t_s = 4.6$$

$$\Rightarrow \quad t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$

$$y(t) = 1 - e^{-\sigma t}\left(\cos w_d t + \frac{\sigma}{w_d} \sin w_d t\right)$$

Time-Domain Specifications

- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p



$$\Rightarrow t_r \cong \frac{1.8}{w_n}$$

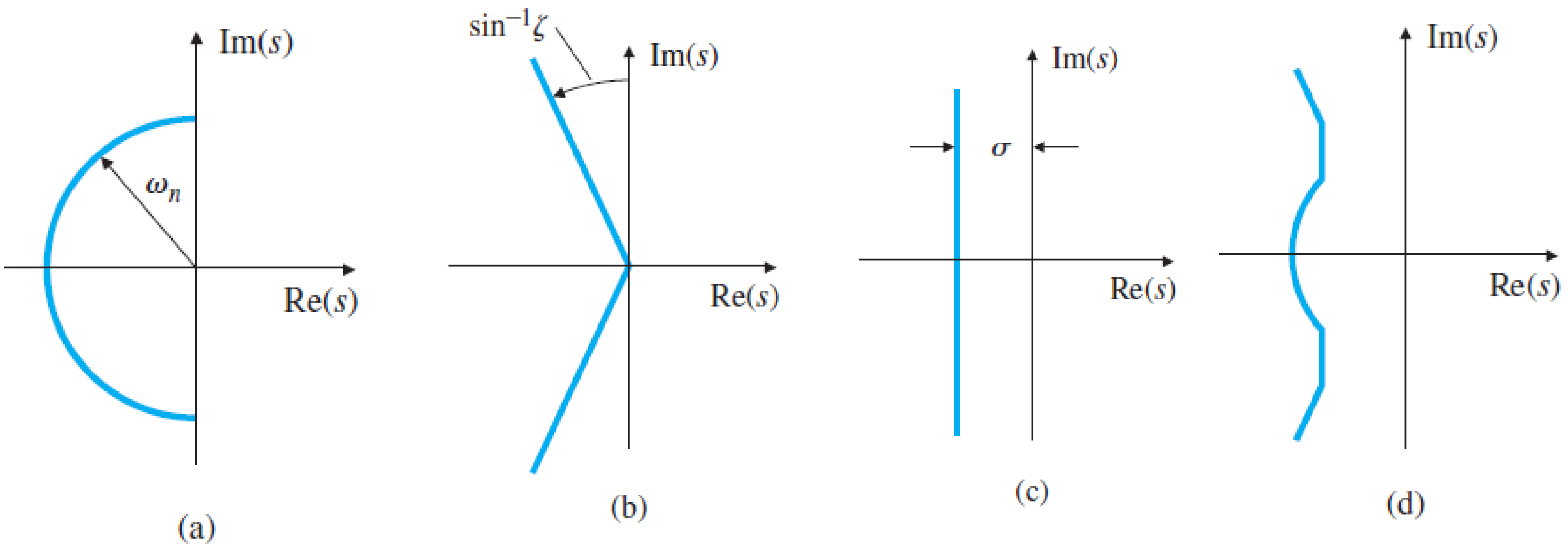
$$\Rightarrow w_n \geq \frac{1.8}{t_r}$$

$$\Rightarrow \zeta \geq fun(M_p)$$

$$\Rightarrow M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad 0 \leq \zeta < 1$$

$$\Rightarrow \sigma \geq \frac{4.6}{t_s}$$

- Graphs of regions in the s-plane delineated by certain transient requirements:
  (a) Rise time, (b) Overshoot, (c) Settling time,
  (d) Composite of all three requirements



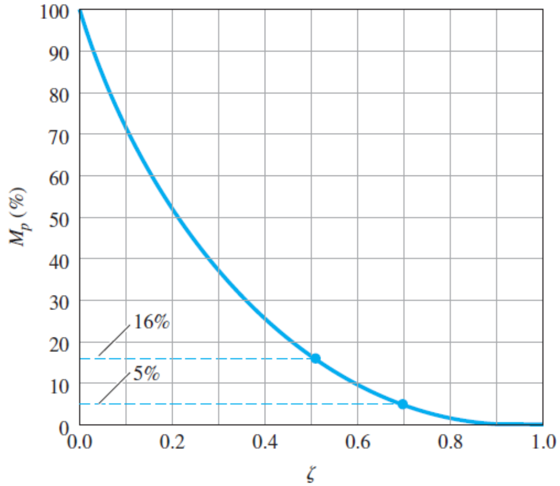
$$w_n \geq \frac{1.8}{t_r} \qquad \zeta \geq fun(M_p) \qquad \sigma \geq \frac{4.6}{t_s}$$

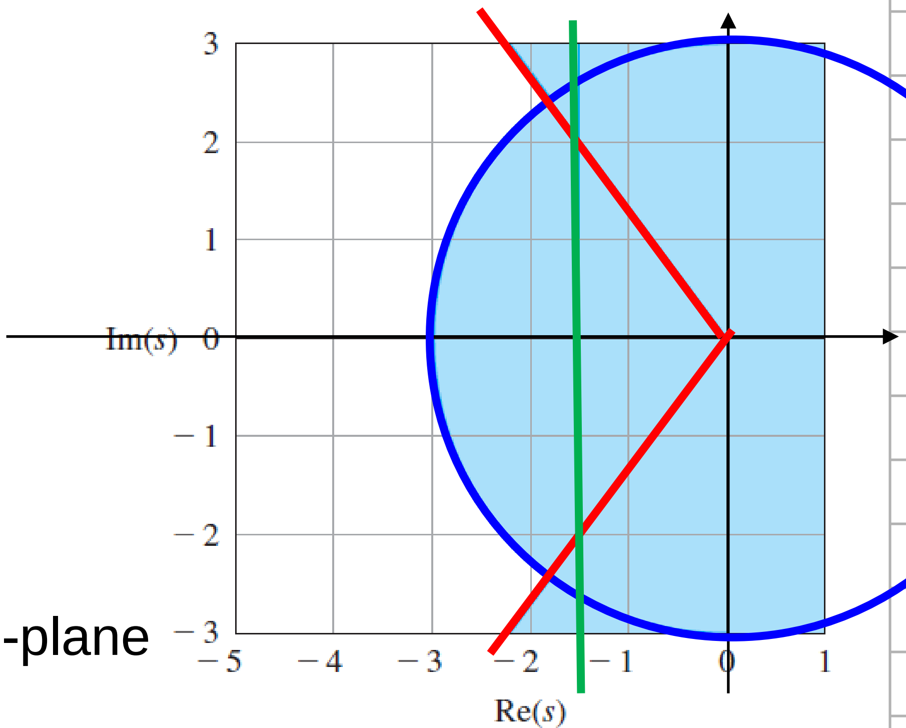
Example 3.27: Transformation of specifications to s-plane

$$\Rightarrow w_n \geq \frac{1.8}{t_r} = 3.0 \text{ rad/sec}$$

$$\Rightarrow \zeta \geq 0.6$$

$$\Rightarrow \sigma \geq \frac{4.6}{3} = 1.5 \text{ sec}$$





- Time domain specifications region in s-plane