

Fall 2022 (111-1)

控制系統
Control Systems

Unit 30
Dynamic Response

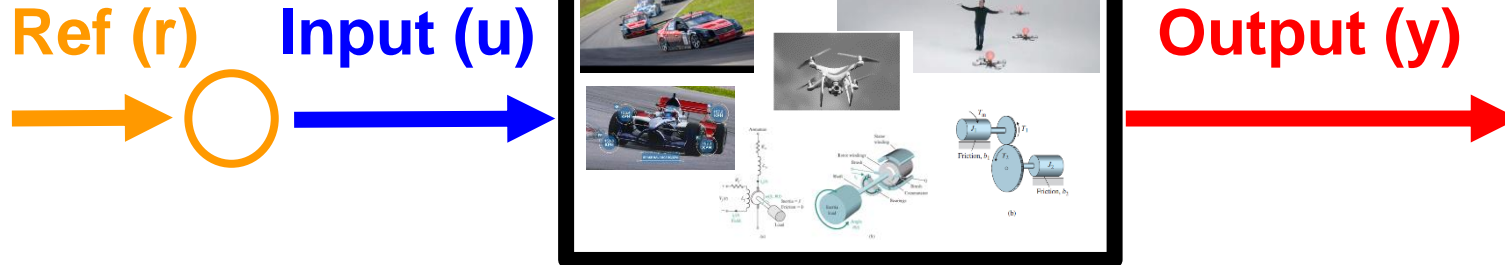
Feng-Li Lian

NTU-EE

Sep 2022 – Dec 2022

- U3A Review of Laplace Transforms
 - Laplace Transforms: From Differential Eqns to Algebraic Eqns
 - Transfer Functions, Frequency Response, Poles and Zeros
 - Step, Ramp, Impulse Functions
 - Laplace Transforms and Inverse LT, The Final Value Theorem
 - Using Laplace Transforms to Solve Differential Equations
- U3B System Modeling Diagrams: Model Visualization
- U3C Effect of Pole Locations
- U3D Time-Domain Specifications
 - Rise Time
 - Overshoot and Peak Time
 - Settling Time
- U3E Effects of Zeros and Additional Poles
- U3F Stability
 - Bounded Input–Bounded Output Stability
 - Stability of LTI Systems
 - Routh's Stability Criterion

Plant (P)



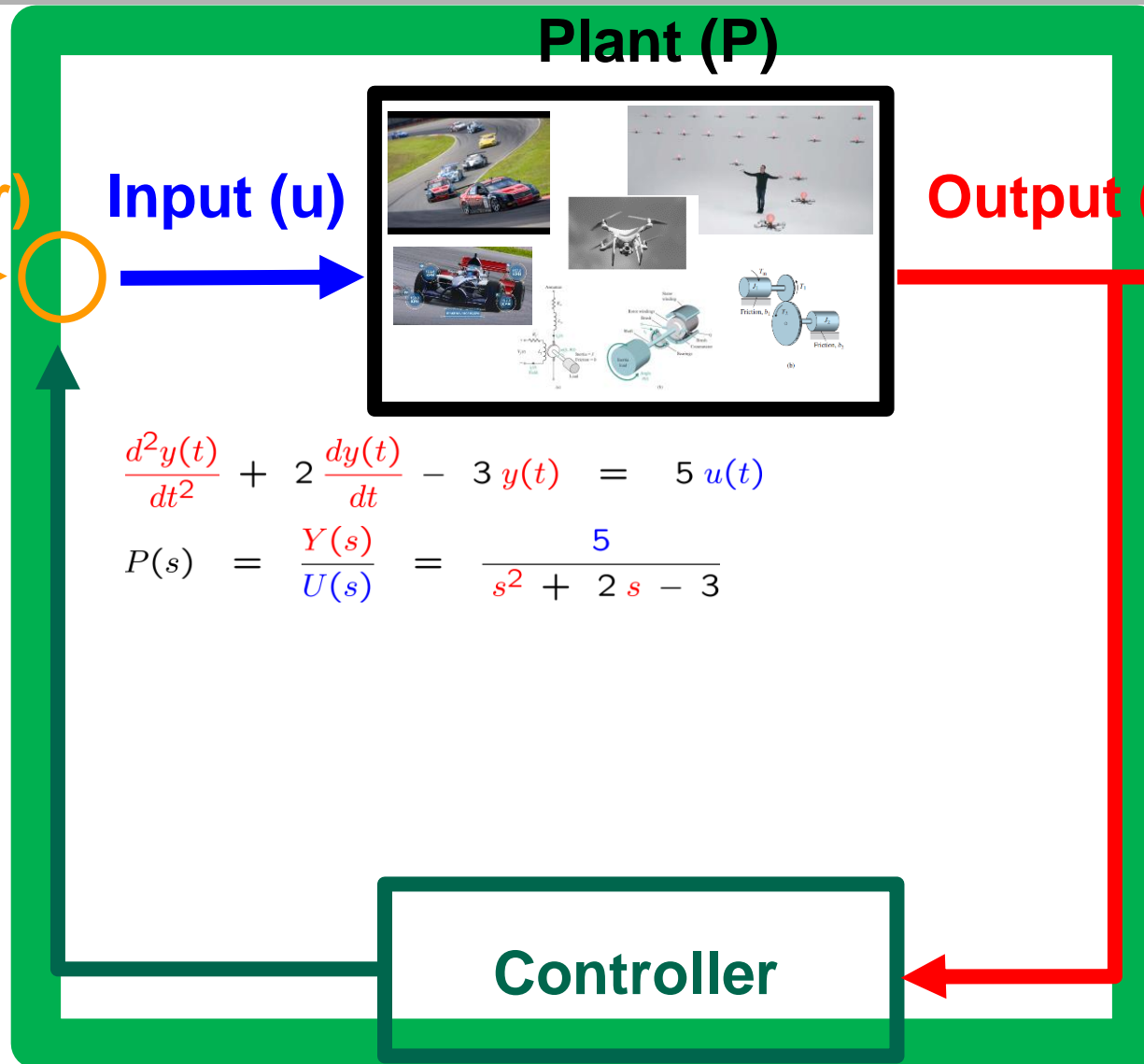
Signals & Systems

Control Systems

$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3y(t) = 5u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s - 3}$$

1. Model
2. Response
3. Analysis
4. Feedback
5. Control



Signals & Systems

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$$\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = 3r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4s + 3}$$

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$\frac{1}{s+a}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{s(s+a)}{a}$	$1 - e^{-at}$
12	$\frac{1}{s^2(s+a)}$	$\frac{1}{a}(at - 1 + e^{-at})$

Number	$F(s)$	$f(t), t \geq 0$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt} - ae^{-at}$
17	$\frac{s}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1 - e^{-at} \left(\cos bt + \frac{a}{b} \sin bt \right)$

Properties of Laplace Transforms

Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t-\lambda)$	Time delay ($\lambda \geq 0$)
3	$\frac{1}{ a } F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \dots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s)$	$\int_0^t f(\zeta) d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
8	$\lim_{s \rightarrow \infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s \rightarrow 0} sF(s)$	$\lim_{t \rightarrow \infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi} \int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega) d\omega$	$\int_0^\infty y(t)u(t) dt$	Parseval's Theorem
12	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

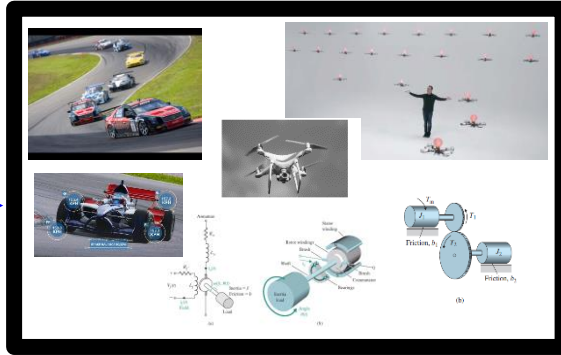
Plant (P)

Signals & Systems

Ref (r)

Input (u)

Output (y)



$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3y(t) = 5u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s - 3}$$

Control Systems

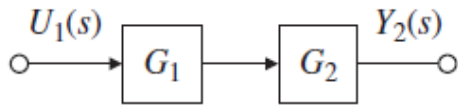
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Controller

$$\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = 3r(t)$$

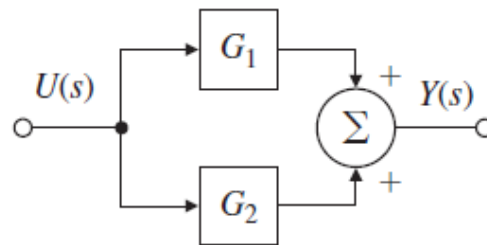
$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4s + 3}$$

- Elementary block diagrams: (a) series; (b) parallel; (c) feedback



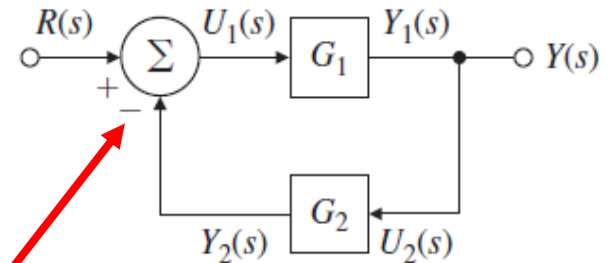
$$\frac{Y_2(s)}{U_1(s)} = G_2 G_1$$

(a)



$$\frac{Y(s)}{U(s)} = G_2 + G_1$$

(b)



$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_2 G_1}$$

(c)

- The **gain** of a **single-loop negative feedback** system

is given by

the **forward gain** divided by the sum of **1 plus the loop gain**.

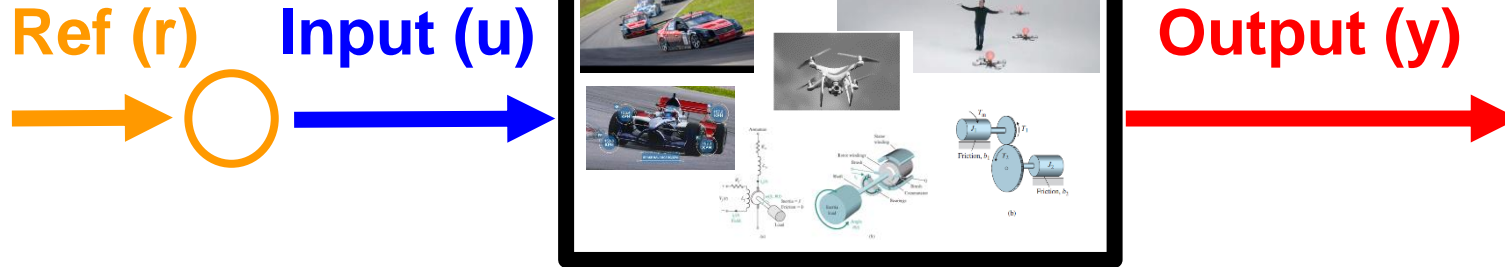
- Negative Feedback**
- Positive Feedback**
- Unity Feedback System**

$$\frac{Y}{R} = \frac{G_1}{1 + G_2 G_1}$$

$$\frac{Y}{R} = \frac{G_1}{1 - G_2 G_1}$$

$$G_2 = 1$$

Plant (P)



Signals & Systems

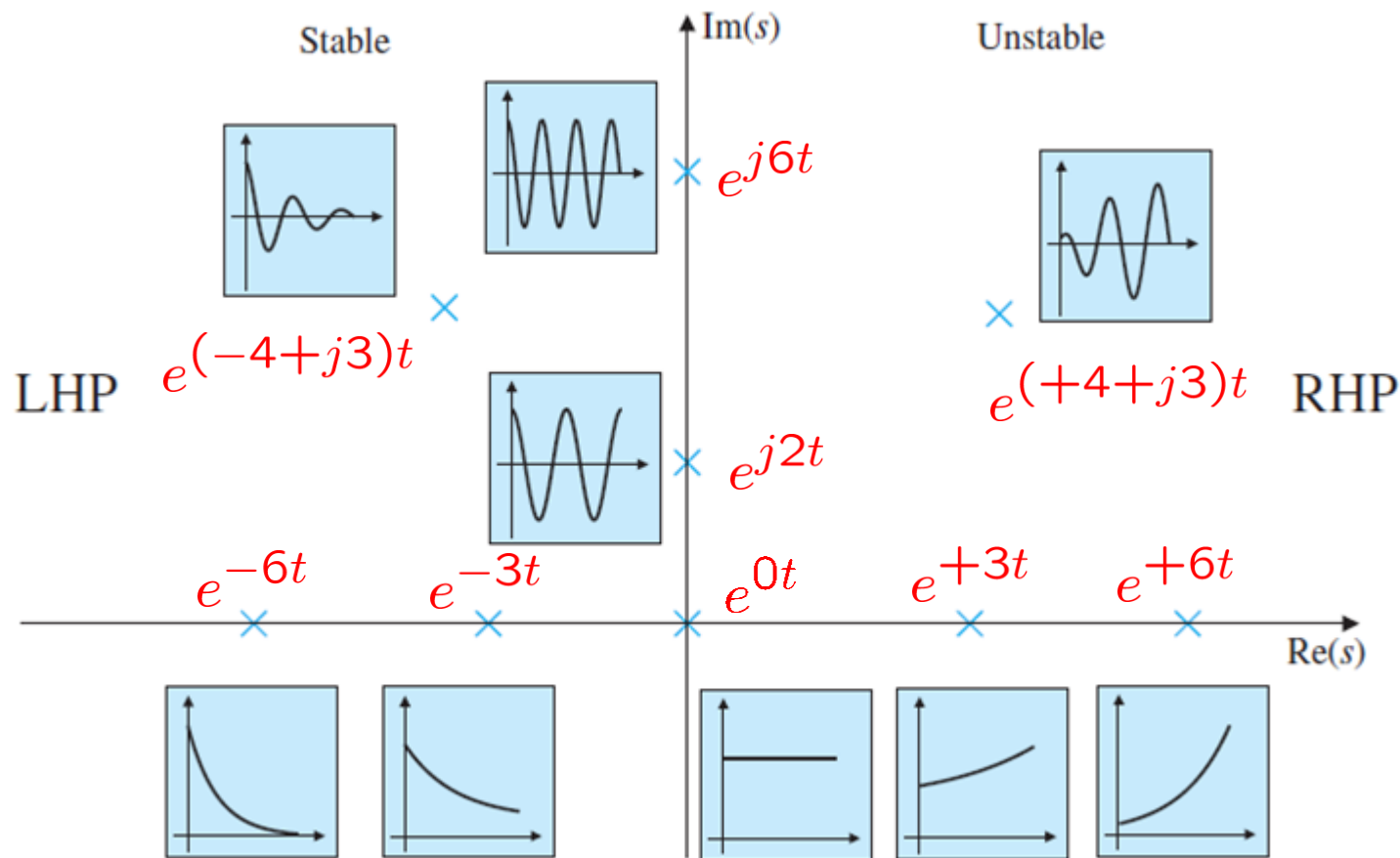
Control Systems

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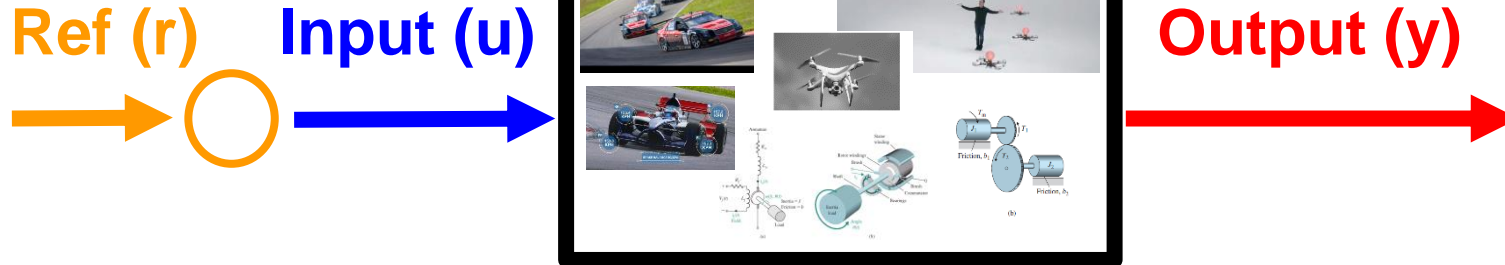
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- Time functions associated with points in the s-plane
(LHP, left half-plane; RHP, right half-plane)



Plant (P)

Signals & Systems

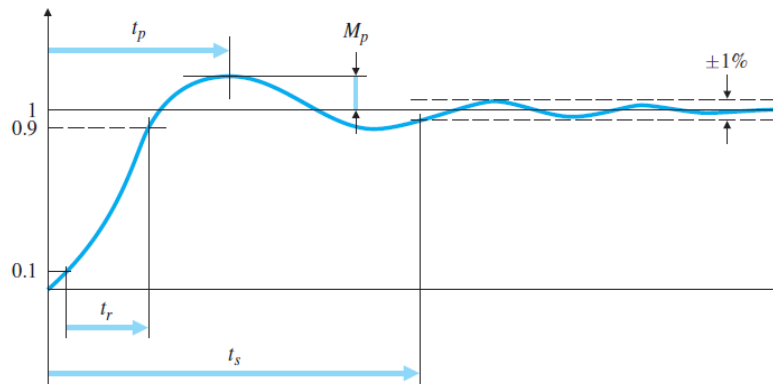
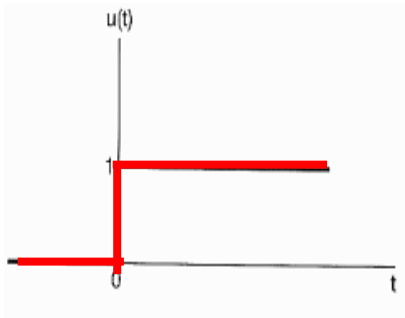


Control Systems

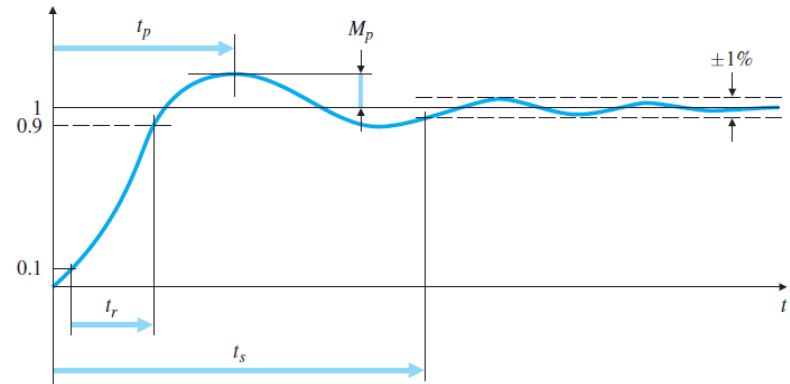
$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3y(t) = 5u(t)$$

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- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p



$$\Rightarrow t_r \approx \frac{1.8}{\omega_n}$$

$$\Rightarrow \omega_n \geq \frac{1.8}{t_r}$$

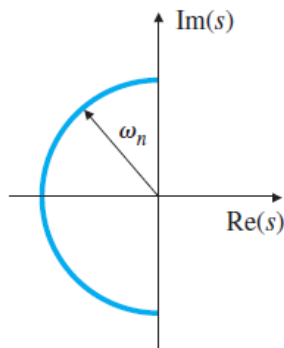
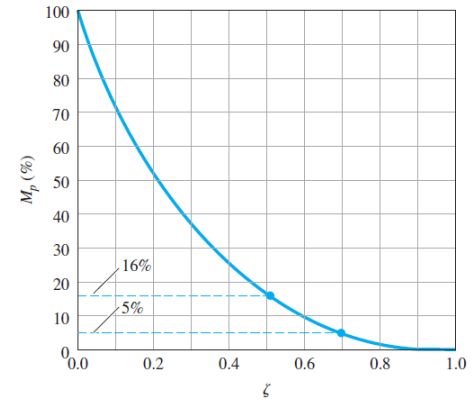
$$\Rightarrow t_s = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\sigma}$$

$$\Rightarrow \zeta \geq \zeta(M_p)$$

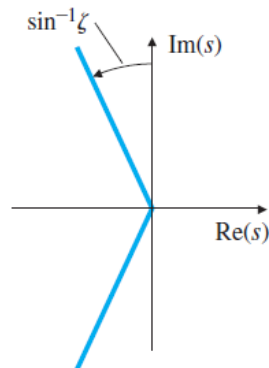
$$\Rightarrow M_p = e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}}$$

$$0 \leq \zeta < 1$$

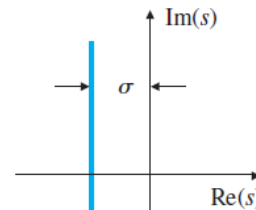
$$\Rightarrow \sigma \geq \frac{4.6}{t_s}$$



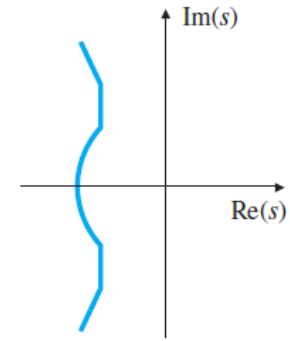
(a)



(b)

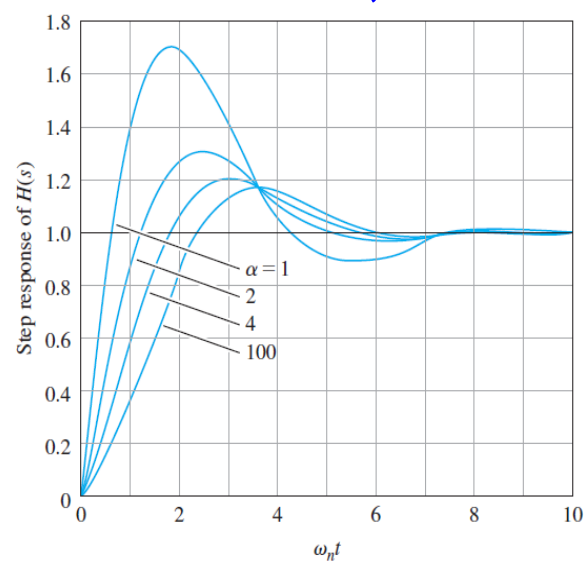


(c)

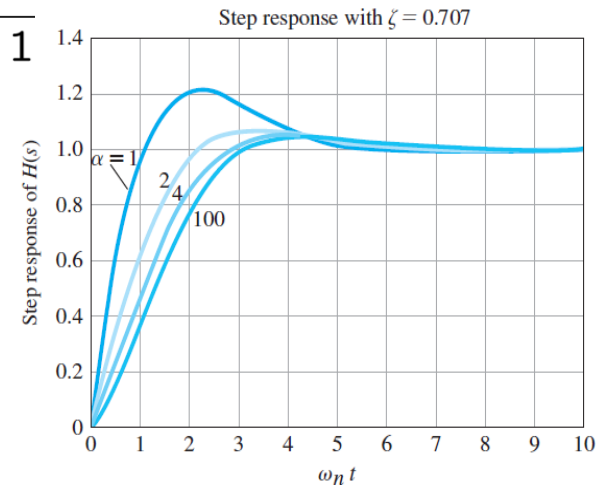


(d)

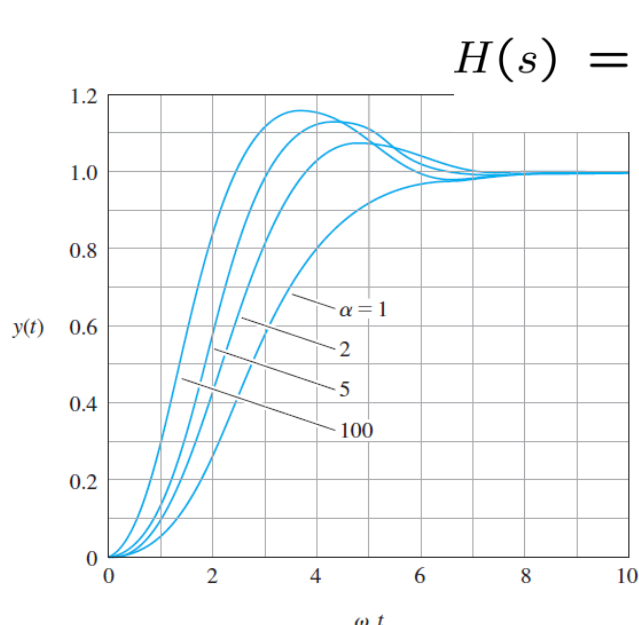
Same Poles, Different Zeros



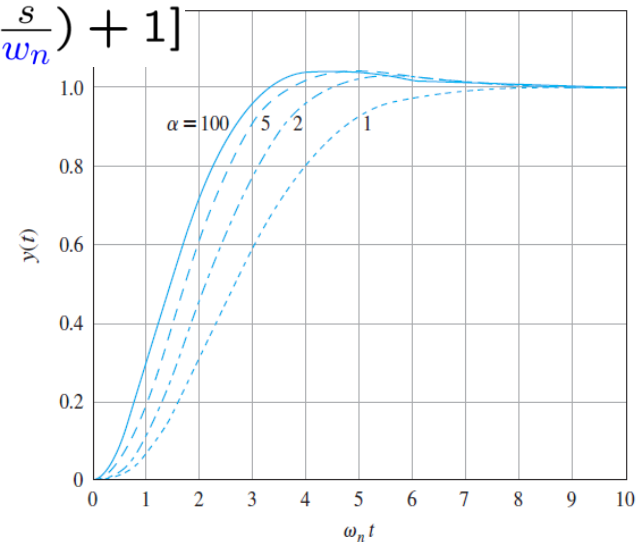
$$H(s) = \frac{\frac{s}{\alpha\zeta\omega_n} + 1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$$



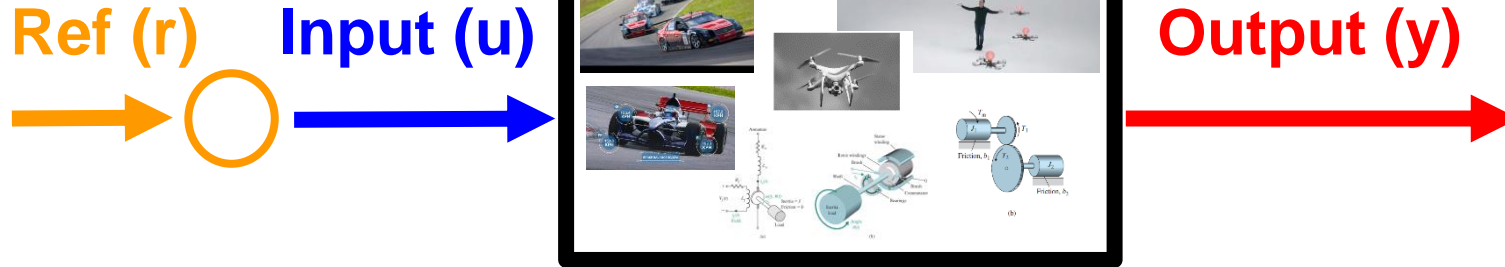
Effects of Pole-Zero Patterns on Dynamic Response



$$H(s) = \frac{1}{\left(\frac{s}{\alpha\zeta\omega_n} + 1\right)\left[\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1\right]}$$



Plant (P)



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