

Fall 2022 (111-1)

控制系統
Control Systems

Unit 2E
Electric Circuits

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NTU-EE

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- The basic equations of electric circuits are the **Kirchhoff's laws**

- **Kirchhoff's Current Law (KCL):**

The algebraic sum of the currents leaving a node

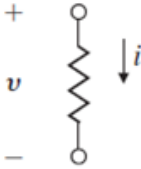
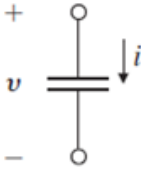
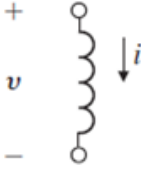
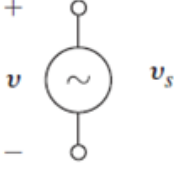



The algebraic sum of the currents entering that node

- **Kirchhoff's Voltage Law (KVL):**

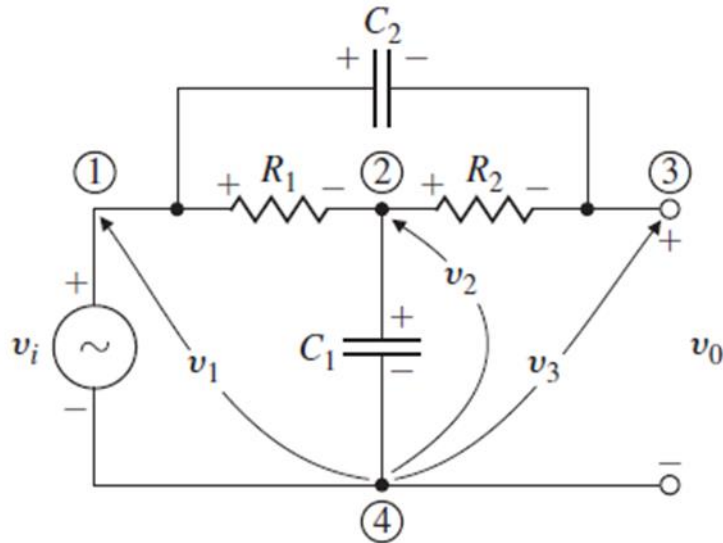
The algebraic sum of all voltages

taken around a closed path in a circuit is zero

	Symbol	Equation
Resistor		$v = Ri$
Capacitor		$i = C \frac{dv}{dt}$
Inductor		$v = L \frac{di}{dt}$
Voltage source		$v = v_s$
Current source		$i = i_s$

Example 2.8 the Bridged Tee Circuit

● Bridged Tee Circuit



■ Model (Equations of Motion)

- Select node 4 as the reference
- v_1, v_2, v_3 as the unknowns
- By KVL, $v_1 = v_i$

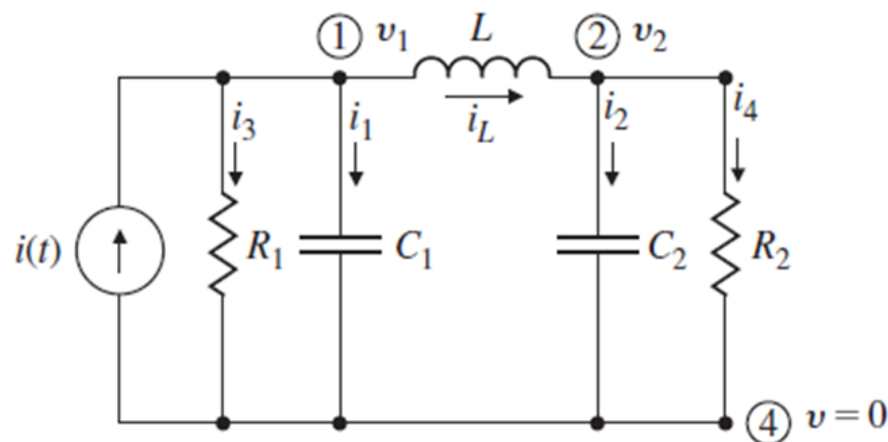
- At node 2, the KCL is
$$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1 \frac{dv_2}{dt} = 0$$

- At node 3, the KCL is
$$\frac{v_3 - v_2}{R_2} + C_2 \frac{d(v_3 - v_1)}{dt} = 0$$

- Transfer function from input v_i to output v_o can be derived

Example 2.9 a Circuit with a Current Source

- Circuit with a current source



- Model (Equations of Motion)

- Select node 4 as the reference
- v_1, v_2, i_L as the unknowns

- At node 1, the KCL is

$$i(t) = i_3 + i_1 + i_L$$

- At node 2, the KCL is

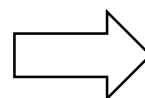
$$i_L = i_2 + i_4$$

- We also have the relations

$$i_3 = \frac{v_1}{R_1}, \quad i_1 = C_1 \frac{dv_1}{dt},$$

$$i_2 = C_2 \frac{dv_2}{dt}, \quad i_4 = \frac{v_2}{R_2},$$

$$v_1 - v_2 = L \frac{di_L}{dt},$$

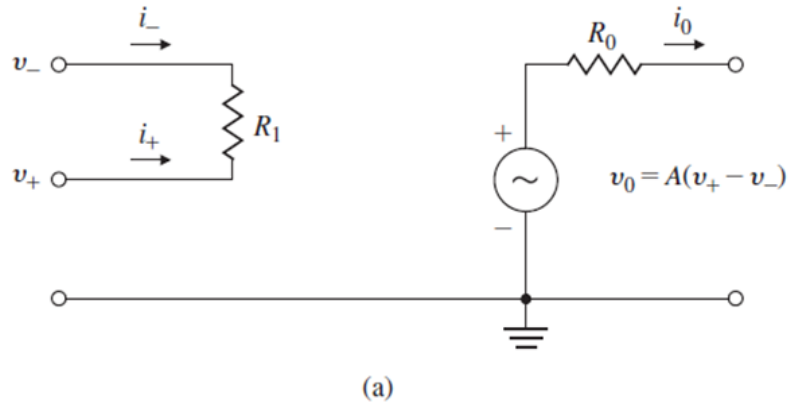


$$i(t) = \frac{v_1}{R_1} + C_1 \frac{dv_1}{dt} + i_L,$$

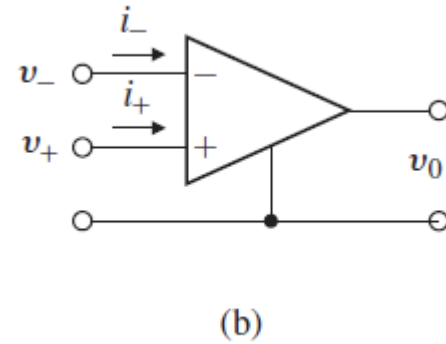
$$i_L = C_2 \frac{dv_2}{dt} + \frac{v_2}{R_2}$$

$$v_1 = L \frac{di_L}{dt} + v_2$$

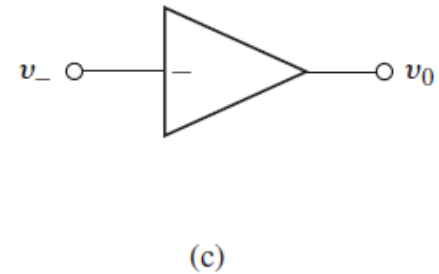
- Simplified circuit of op-amp



- op-amp schematic symbol



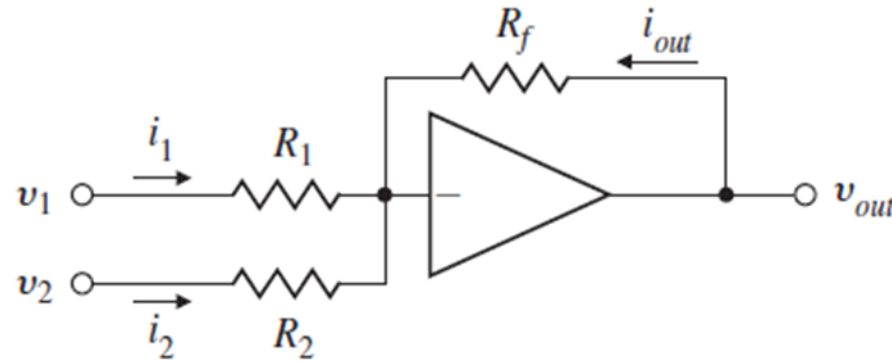
- Assume connected to ground, $v_+ = 0$



- Assume ideal op-amp, $R_1 = \infty$, $R_0 = 0$, $A = \infty$

$$i_+ = i_- = 0, \quad v_+ - v_- = 0$$

- The op-amp summer



- From $v_+ - v_- = 0$, we have $v_- = 0$

- Thus, $i_1 = \frac{v_1}{R_1}$, $i_2 = \frac{v_2}{R_2}$, $i_{out} = \frac{v_{out}}{R_f}$

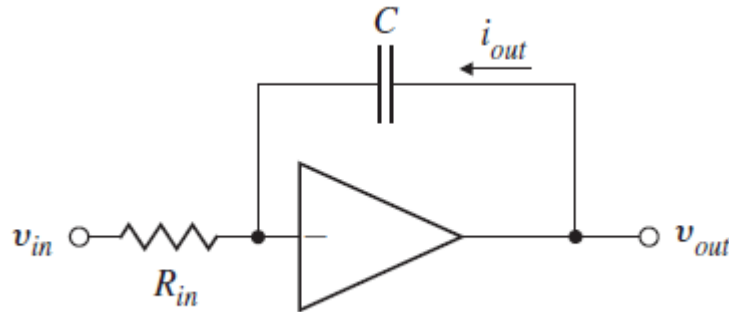
- From $i_+ = i_- = 0$, we have $i_1 + i_2 + i_{out} = 0$, $\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_{out}}{R_f} = 0$

- Model (Equations of Motion)

$$v_{out} = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right] \quad (\text{Output is the weighted sum of input voltages})$$

Example 2.11 Integrator

- The op-amp integrator



$$i_{in} + i_{out} = 0$$

$$\frac{v_{in}}{R_{in}} + C \frac{dv_{out}}{dt} = 0$$

- Model (Equations of Motion)

$$v_{out} = -\frac{1}{R_{in}C} \int_0^t v_{in}(\tau) d\tau + v_{out}(0)$$

- Transfer Function

$$V_{out}(s) = -\frac{1}{s R_{in}C} V_{in}(s) \quad (\text{Assume zero initial condition})$$

■ Table 2.1 [Dorf & Bishop 2017]

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, \mathcal{T}_{21}	

Table 2.2-1 [Dorf & Bishop 2017]



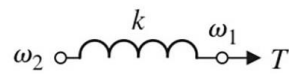

Type of Element	Physical Element	Governing Equation	Energy E or Power \mathcal{P}	Symbol
Inductive storage	Electrical inductance	$v_{21} = L \frac{di}{dt}$	$E = \frac{1}{2} Li^2$	
	Translational spring	$v_{21} = \frac{1}{k} \frac{dF}{dt}$	$E = \frac{1}{2} \frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k} \frac{dT}{dt}$	$E = \frac{1}{2} \frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I \frac{dQ}{dt}$	$E = \frac{1}{2} IQ^2$	

Table 2.2-2


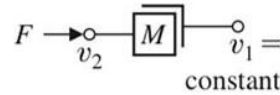
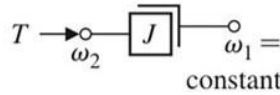
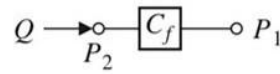
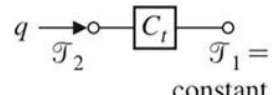

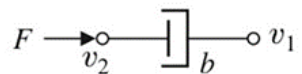
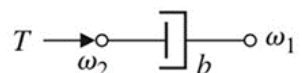

Capacitive storage	Electrical capacitance	$i = C \frac{dv_{21}}{dt}$	$E = \frac{1}{2} Cv_{21}^2$	
	Translational mass	$F = M \frac{dv_2}{dt}$	$E = \frac{1}{2} Mv_2^2$	
	Rotational mass	$T = J \frac{d\omega_2}{dt}$	$E = \frac{1}{2} J\omega_2^2$	
	Fluid capacitance	$Q = C_f \frac{dP_{21}}{dt}$	$E = \frac{1}{2} C_f P_{21}^2$	
	Thermal capacitance	$q = C_t \frac{d\mathcal{T}_2}{dt}$	$E = C_t \mathcal{T}_2$	

Table 2.2.3 [Dorf & Bishop 2017]

Type of Element	Physical Element	Governing Equation	Energy E or Power \mathcal{P}	Symbol
Energy dissipators	Electrical resistance	$i = \frac{1}{R} v_{21}$	$\mathcal{P} = \frac{1}{R} v_{21}^2$	
	Translational damper	$F = b v_{21}$	$\mathcal{P} = b v_{21}^2$	
	Rotational damper	$T = b \omega_{21}$	$\mathcal{P} = b \omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f} P_{21}$	$\mathcal{P} = \frac{1}{R_f} P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t} \mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t} \mathcal{T}_{21}^2$	