Fall 2022 (111-1)

控制系統
Control Systems

Unit 2C
Mechanical Systems –
Combined Rotation and Translation

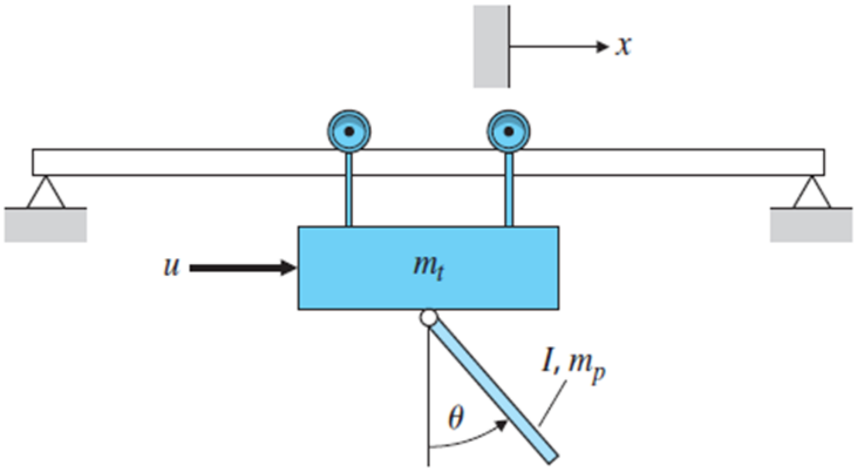
Feng-Li Lian

NTU-EE

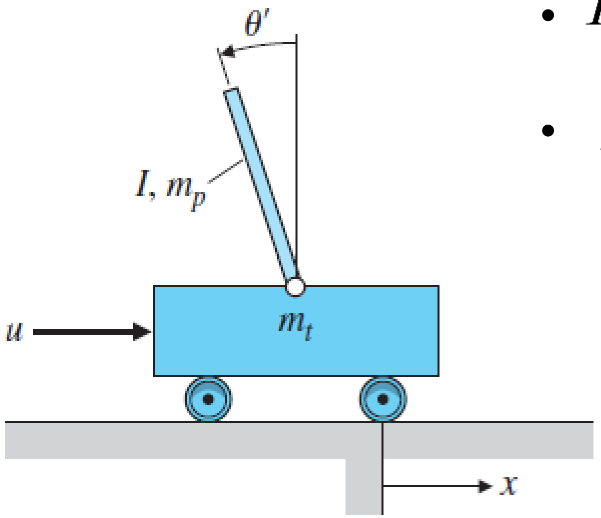
Sep 2022 – Dec 2022

Example 2.7 Hanging Crane (Rotational+ translational motion)

- Crane with hanging load



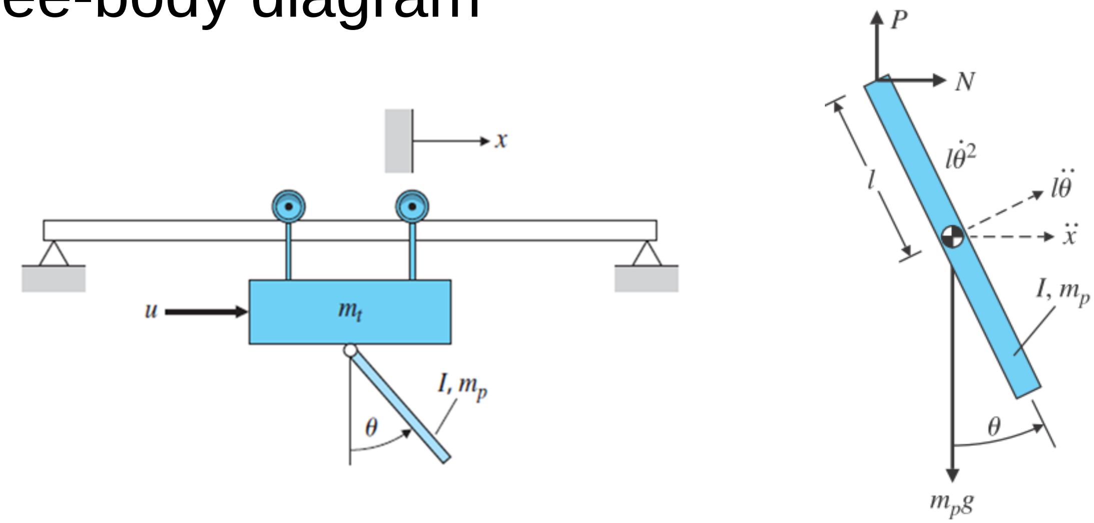
- Linearize the system at $\theta = \pi$ will result to the inverted pendulum



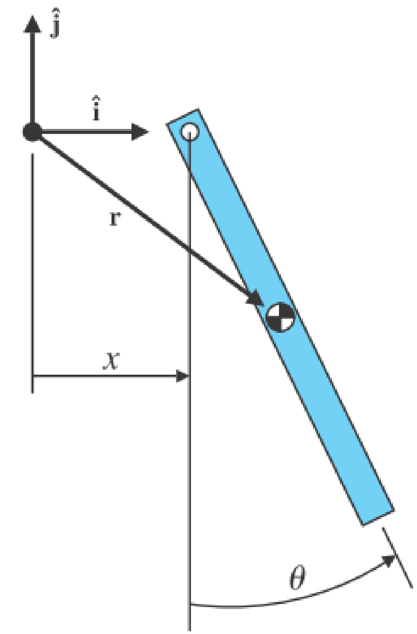
- I : the inertia about mass center of the pendulum
- $I + m_p \cdot l^2$: Moments of inertia of the pendulum about the pivot point

Example 2.7 Hanging Crane (Rotational+ translational motion)

- Free-body diagram



The free-body diagram of the trolley and pendulum



Position vector of the pendulum

- The Equations of Motion (see APPENDIX W2.1.4)

$$(I + m_p l^2)\ddot{\theta} + m_p g l \sin\theta = -m_p l \ddot{x} \cos\theta$$

$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} \cos\theta - m_p l \dot{\theta}^2 \sin\theta = u$$

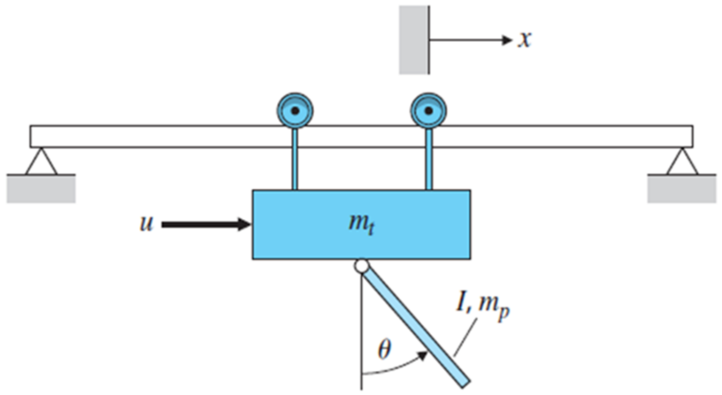
Example 2.7 Hanging Crane (Rotational+ translational motion)

Model (Equations of Motion)

- Assume small angles. For the small motion $\theta \approx 0$ we consider $\cos\theta \approx 1, \quad \sin\theta \approx \theta, \quad \dot{\theta}^2 \approx 0$

$$(I + m_p l^2)\ddot{\theta} + m_p g l \theta = -m_p l \ddot{x}$$

$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} = u$$



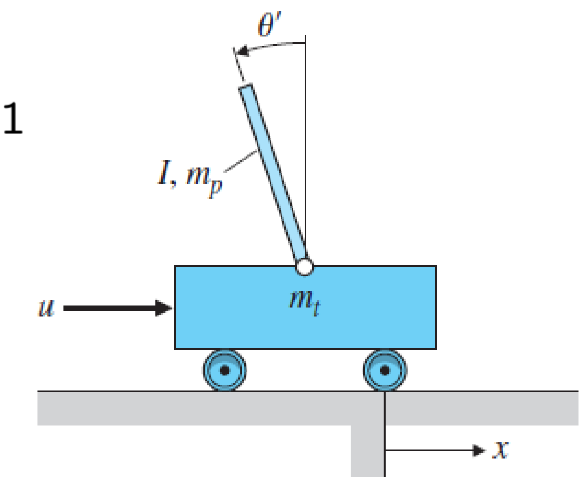
Transfer Function

- Neglecting the friction term, let $b = 0$

$$\frac{\Theta(s)}{U(s)} = \frac{-m_p l}{s^2\left((I + m_p l^2)(m_t + m_p) - m_p^2 l^2\right) + m_p g l (m_t + m_p)}$$

Model (Equations of Motion)

- For inverted pendulum, we consider $\theta \approx \pi$
- Assume $\theta = \pi + \theta' \quad \Rightarrow \sin\theta' \approx -\theta', \cos\theta' \approx -1$



$$(I + m_p l^2)\ddot{\theta}' + m_p g l \theta' = -m_p l \ddot{x}$$

$$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta}' = u$$

Transfer Function

- Neglecting the friction term, let $b = 0$

$$\frac{\Theta'(s)}{U(s)} = \frac{-m_p l}{s^2\left((I + m_p l^2)(m_t + m_p) - m_p^2 l^2\right) + m_p g l(m_t + m_p)}$$