

# Causality in Quantiles and Dynamic Stock Return-Volume Relations

**Chia-Chang Chuang**

Department of International Business  
National Taipei College of Business

**Chung-Ming Kuan**

Department of Finance  
National Taiwan University

**Hsin-yi Lin**

Department of Economics  
National Chengchi University

This version: January 22, 2009

† Author for correspondence: Chung-Ming Kuan, Department of Finance, National Taiwan University, Taipei 106, Taiwan; E-mail address: ckuan@ntu.edu.tw., phone: +886.2.3366.1072.

†† We would like to thank a referee and the managing editor for very useful comments and suggestions. We also benefit from the comments by Zongwu Cai, Yongmiao Hong, Po-Hsuan Hsu, Mike McAleer, Essie Maasoumi, Shouyang Wang, and Arnold Zellner. This paper is part of the project “Advancement of Research on Econometric Methods and Applications” (AREMA) and was completed while C.-M. Kuan was visiting USC. Kuan wishes to express his sincere gratitude to Cheng Hsiao and USC for arranging his visit. All remaining errors are ours.

## Abstract

This paper investigates the causal relations between stock return and volume based on quantile regressions. We first define Granger non-causality in all quantiles and propose testing non-causality by a sup-Wald test. Such a test is consistent against any deviation from non-causality in distribution, as opposed to the existing tests that check only non-causality in certain moment. This test is readily extended to test non-causality in different quantile ranges. In the empirical studies of 3 major stock market indices, we find that the causal effects of volume on return are usually heterogeneous across quantiles and those of return on volume are more stable. In particular, the quantile causal effects of volume on return exhibit a spectrum of (symmetric) *V*-shape relations so that the dispersion of return distribution increases with lagged volume. This is an alternative evidence that volume has a positive effect on return volatility. Moreover, the inclusion of the squares of lagged returns in the model may weaken the quantile causal effects of volume on return but does not affect the causality per se.

**JEL Classification No:** C12, G14

**Keywords:** Granger non-causality, quantile causal effect, quantile regression, return-volume relation, sup-Wald test

# 1 Introduction

The relationship between financial asset return and trading volume, henceforth the return-volume relation, is important for understanding operational efficiency and information dynamics in asset markets. Models related to this topic include, e.g., the sequential information arrival model (Copeland, 1976; Jennings, Starks, and Fellingham, 1981; Jennings and Barry, 1983) and mixture of distributions model (Clark, 1973; Epps and Epps, 1976; Tauchen and Pitts, 1983). There are also equilibrium models that emphasize the information content of volume, e.g., Harris and Raviv (1993), Blume, Easley, and O'Hara (1994), Wang (1994), and Suominen (2001). For instance, Blume, Easley, and O'Hara (1994) stress that volume carries information that is not contained in price statistics and hence is useful for interpreting the price (return) behavior. On the empirical side, there have been numerous studies on contemporaneous return-volume relation since Granger and Morgenstern (1963) and Ying (1966); see Gallant, Rossi, and Tauchen (1992) and also Karpoff (1987) for a review. Yet, as far as prediction and risk management are concerned, the dynamic (causal) relation between return and volume is more informative.

Causal relations between variables are typically examined by testing Granger non-causality. While Granger non-causality is defined in terms of conditional distribution, it is more common to test non-causality in conditional mean based on a linear model (Granger, 1969, 1980). Granger, Robins, and Engle (1986) and Cheung and Ng (1996) consider testing non-causality in conditional variance, whereas Hiemstra and Jones (1994) derive a test for nonlinear causal relations. These tests have been widely used in the literature (e.g., Fujihara and Mougoué, 1997; Silvapulle and Choi, 1999; Chen, Firth, and Rui, 2001; Ciner, 2002; Lee and Rui, 2002). A serious limitation of this approach is that non-causality in mean (or in variance) need not carry over to other distribution characteristics or different parts of the distribution. Diks and Panchenko (2005) also give examples that the test of Hiemstra and Jones (1994) may not test Granger non-causality. These motivate us to consider characterizing and testing causality differently.

This paper investigates causal relations from the perspective of conditional quantiles. We first define Granger non-causality in a given quantile range and non-causality in *all* quantiles. The quantile causal effects are then estimated by means of quantile regressions (Koenker and Basett 1978; Koenker, 2005). The hypothesis of non-causality in all quantiles is tested by the sup-Wald test of Koenker and Machado (1999). This test checks significance of the entire parameter process in quantile regression models and hence is consistent against any deviation from non-causality in distribution, as opposed to the

conventional tests of non-causality in a moment and the tests of Lee and Yang (2006) and Hong, Liu, and Wang (2008). The test of Koenker and Machado (1999) is easily extended to evaluate non-causality in different quantile ranges and enables us to identify the quantile range for which causality is relevant. Our approach thus provides a detailed description of the causal relations between return and volume.

In the empirical study we examine the causal relations between return and (log) volume in three stock market indices: New York Stock Exchange (NYSE), Standard & Poor 500 (S&P 500), and Financial Times-Stock Exchange 100 (FTSE 100). Despite that the conventional test may suggest no causality in mean, there are strong evidences of causality in quantiles in these indices. For NYSE and S&P 500, we find two-way Granger causality in quantiles between return and volumes; for FTSE 100, only volume Granger causes return in quantiles. In particular, the causal effects of volume on return are heterogeneous across quantiles, in the sense that they possess opposite signs at lower and upper quantiles and are stronger at more extreme quantiles. On the other hand, the causal effects of return on volume, if exist, are mainly negative and remain stable across quantiles.

With log volume on the vertical axis and return on the horizontal axis, the quantile causal effects of volume on return exhibit a spectrum of symmetric  $V$ -shape relations for NYSE and S&P 500. While many existing results (e.g., Karpoff, 1987) find a simple  $V$ -shape relation based on a least-squares regression of absolute return on volume, our  $V$ -shape results are very different. First, what we find are dynamic rather than contemporaneous relations. Second, these relations hold across quantiles rather than at the mean only. Moreover, the identified  $V$  spectrum suggests that distribution dispersion increases with lagged volume. This constitutes an alternative evidence that volume has a positive effect on return volatility and is compatible with the empirical finding based on conditional variance models (e.g., Lamoureux and Lastrapes, 1990; Gallant, Rossi, and Tauchen, 1992).

It is interesting to note that the quantile causal relations we find are quite robust to different sample periods and different model specifications. Indeed, the inclusion of the squares of lagged returns in the model may weaken the quantile causal effects of volume on return but does not affect the causality per se. Thus, lagged volumes carry information that is not contained in lagged returns and their squares, as argued by Blume, Easley, and O'Hara (1994). Our results also confirm that non-causality in mean bears no implication on non-causality in distribution (quantiles). A conventional test may find no causality in mean because the positive and negative quantile causal effects cancel out each other in least-squares estimation, as demonstrated in our study. It is therefore vulnerable to draw

a conclusion on causality solely based on a test of non-causality in mean.

This paper is organized as follows. We introduce the notion of Granger (non-)causality in quantiles in Section 2 and discuss the sup-Wald test of non-causality in quantiles in Section 3. The empirical results of different causal models are presented in Section 4. Section 5 concludes the paper.

## 2 Causality in Mean and Quantiles

Following Granger (1969, 1980), we say that the random variable  $x$  does not Granger cause the random variable  $y$  if

$$F_{y_t}(\eta | (\mathcal{Y}, \mathcal{X})_{t-1}) = F_{y_t}(\eta | \mathcal{Y}_{t-1}), \quad \forall \eta \in \mathbb{R}, \quad (1)$$

holds almost surely (a.s.), where  $F_{y_t}(\cdot | \mathcal{F})$  is the conditional distribution of  $y_t$ , and  $(\mathcal{Y}, \mathcal{X})_{t-1}$  is the information set generated by  $y_i$  and  $x_i$  up to time  $t - 1$ . That is, Granger non-causality requires that the past information of  $x$  does not alter the conditional distribution of  $y_t$ . The variable  $x$  is said to Granger cause  $y$  when (1) fails to hold. In what follows, Granger non-causality defined by (1) will be referred to as Granger non-causality in distribution.

As estimating and testing conditional distributions are practically cumbersome, it is more common to test a necessary condition of (1), namely,

$$\mathbb{E}[y_t | (\mathcal{Y}, \mathcal{X})_{t-1}] = \mathbb{E}(y_t | \mathcal{Y}_{t-1}), \quad a.s. \quad (2)$$

where  $\mathbb{E}(y_t | \mathcal{F})$  is the mean of  $F_{y_t}(\cdot | \mathcal{F})$ . We say that  $x$  does not Granger cause  $y$  in mean if (2) holds; otherwise,  $x$  Granger causes  $y$  in mean. Similarly, we may define non-causality in variance (Granger, Robins, and Engle, 1986; Cheung and Ng, 1996) and non-causality in other moments. Hong, Liu, and Wang (2008) consider “non-causality in risk,” a special case of (1) in which  $\eta$  is the negative of a VaR (Value at Risk). Note that these notions of non-causality are necessary for, but not equivalent to, Granger non-causality in distribution.

The hypothesis (2) is usually tested by evaluating a linear model of  $\mathbb{E}[y_t | (\mathcal{Y}, \mathcal{X})_{t-1}]$ :

$$\alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i} + \sum_{j=1}^q \beta_j x_{t-j},$$

which depends on the past information of  $y_{t-1}, \dots, y_{t-p}$  and  $x_{t-1}, \dots, x_{t-q}$ . Testing (2) now amounts to testing the null hypothesis that  $\beta_j = 0$ ,  $j = 1, \dots, q$ , in the postulated

model; that is, whether any lagged  $x$  has a significant impact on the conditional mean of  $y_t$ .<sup>1</sup> Rejecting this null hypothesis suggests that  $x$  Granger causes  $y$ . Yet, failing to reject the null is compatible with non-causality in mean but says nothing about causality in other moments or other distribution characteristics.

Given that a distribution is completely determined by its quantiles, Granger non-causality in distribution can also be expressed in terms of conditional quantiles. Letting  $Q_{y_t}(\tau|\mathcal{F})$  denote the  $\tau$ -th quantile of  $F_{y_t}(\cdot|\mathcal{F})$ , (1) is equivalent to

$$Q_{y_t}(\tau|(\mathcal{Y}, \mathcal{X})_{t-1}) = Q_{y_t}(\tau|\mathcal{Y}_{t-1}), \quad \forall \tau \in (0, 1), \quad a.s. \quad (3)$$

We say that  $x$  does not Granger cause  $y$  in all quantiles if (3) holds. We may also define Granger non-causality in the quantile range  $[a, b] \subset (0, 1)$  as

$$Q_{y_t}(\tau|(\mathcal{Y}, \mathcal{X})_{t-1}) = Q_{y_t}(\tau|\mathcal{Y}_{t-1}), \quad \forall \tau \in [a, b], \quad a.s. \quad (4)$$

Note that Lee and Yang (2006) considered only non-causality in a particular quantile, i.e., the equality in (3) holds for a given  $\tau$ .

### 3 Testing Non-Causality in Quantiles

This paper proposes to verify causal relations by testing (3), rather than testing non-causality in a moment (mean or variance) or non-causality in a given quantile. To this end, we postulate a model for  $Q_{y_t}(\tau | (\mathcal{Y}, \mathcal{X})_{t-1})$  and estimate this model by the quantile regression method of Koenker and Bassett (1978); see Koenker (2005) for a comprehensive study of quantile regression.

Letting  $\mathbf{y}_{t-1,p} = [y_{t-1}, \dots, y_{t-p}]'$ ,  $\mathbf{x}_{t-1,q} = [x_{t-1}, \dots, x_{t-q}]'$ , and  $\mathbf{z}_{t-1} = [1, \mathbf{y}'_{t-1,p}, \mathbf{x}'_{t-1,q}]'$ , we assume that the following model is correctly specified for the  $\tau$ -th conditional quantile function:

$$Q_{y_t}(\tau | \mathbf{z}_{t-1}) = a(\tau) + \mathbf{y}'_{t-1,p} \boldsymbol{\alpha}(\tau) + \mathbf{x}'_{t-1,q} \boldsymbol{\beta}(\tau) = \mathbf{z}'_{t-1} \boldsymbol{\theta}(\tau),$$

where  $\boldsymbol{\theta}(\tau) = [a(\tau), \boldsymbol{\alpha}(\tau)', \boldsymbol{\beta}(\tau)']'$  is the  $k$ -dimensional parameter vector with  $k = 1 + p + q$ . Note that the  $\tau$ -th conditional quantile of the error  $e_{t\tau} = y_t - \mathbf{z}'_{t-1} \boldsymbol{\theta}(\tau)$  is zero, a consequence of correct model specification. For a given  $\tau$ , the parameter vector  $\boldsymbol{\theta}(\tau)$  is estimated by minimizing asymmetrically weighted absolute deviations:

$$\min_{\boldsymbol{\theta}} \sum_{t=1}^T (\tau - \mathbf{1}_{\{y_t < \mathbf{z}'_{t-1} \boldsymbol{\theta}\}}) |y_t - \mathbf{z}'_{t-1} \boldsymbol{\theta}|,$$

---

<sup>1</sup>Clearly, this approach would be valid provided that the postulated linear model is correctly specified for the conditional mean function.

where  $\mathbf{1}_A$  is the indicator function of the event  $A$ . The solution to this problem, denoted as  $\hat{\boldsymbol{\theta}}_T(\tau)$ , can be computed using a linear programming algorithm.

In what follows, let  $\xrightarrow{D}$  denote convergence in distribution,  $\Rightarrow$  weak convergence (of associated probability measures), and  $\|\cdot\|$  the Euclidean norm. Under suitable regularity conditions,  $\hat{\boldsymbol{\theta}}_T(\tau)$  is consistent and asymptotically normally distributed such that

$$\sqrt{T} \left[ \hat{\boldsymbol{\theta}}_T(\tau) - \boldsymbol{\theta}(\tau) \right] \xrightarrow{D} [\tau(1-\tau)]^{1/2} \boldsymbol{\Omega}(\tau)^{1/2} \mathcal{N}(\mathbf{0}, \mathbf{I}_k),$$

where  $\boldsymbol{\Omega}(\tau) = \mathbf{D}(\tau)^{-1} \mathbf{M}_{zz} \mathbf{D}(\tau)^{-1}$ ,  $\mathbf{M}_{zz} := \lim_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T \mathbf{z}_{t-1} \mathbf{z}'_{t-1}$ , and

$$\mathbf{D}(\tau) := \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T f_{t-1}(F_{t-1}^{-1}(\tau)) \mathbf{z}_{t-1} \mathbf{z}'_{t-1},$$

with  $F_{t-1}$  and  $f_{t-1}$  being, respectively, the distribution and density functions of  $y_t$  conditional on  $\mathcal{Z}_{t-1}$ , the information set generated by  $\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \dots$ ; see Koenker (2005) and Koenker and Xiao (2006).<sup>2</sup>

Given a linear model for conditional quantiles, testing (3) amounts to testing

$$H_0 : \boldsymbol{\beta}(\tau) = \mathbf{0}, \quad \forall \tau \in (0, 1). \quad (5)$$

To this end, we must check significance of the entire parameter process  $\boldsymbol{\beta}(\cdot)$ . Letting  $\boldsymbol{\Psi}$  be a  $q \times k$  selection matrix such that  $\boldsymbol{\Psi} \boldsymbol{\theta}(\tau) = \boldsymbol{\beta}(\tau)$ , we have

$$\sqrt{T} \left[ \hat{\boldsymbol{\beta}}_T(\tau) - \boldsymbol{\beta}(\tau) \right] = \sqrt{T} \boldsymbol{\Psi} \left[ \hat{\boldsymbol{\theta}}_T(\tau) - \boldsymbol{\theta}(\tau) \right] \xrightarrow{D} [\tau(1-\tau)]^{1/2} [\boldsymbol{\Psi} \boldsymbol{\Omega}(\tau) \boldsymbol{\Psi}']^{1/2} \mathcal{N}(\mathbf{0}, \mathbf{I}_q). \quad (6)$$

For a given  $\tau$ , the Wald statistic of  $\boldsymbol{\beta}(\tau) = \mathbf{0}$  is

$$\mathcal{W}_T(\tau) := T \hat{\boldsymbol{\beta}}_T(\tau)' (\boldsymbol{\Psi} \hat{\boldsymbol{\Omega}}(\tau) \boldsymbol{\Psi}')^{-1} \hat{\boldsymbol{\beta}}_T(\tau) / [\tau(1-\tau)],$$

where  $\hat{\boldsymbol{\Omega}}(\tau)$  is a consistent estimator of  $\boldsymbol{\Omega}(\tau)$ . In the special case that  $f_t(\cdot) = f(\cdot)$ , the unconditional density of  $y_t$ ,  $\boldsymbol{\Omega}(\tau) = f(F^{-1}(\tau))^{-2} \mathbf{M}_{zz}^{-1}$ , and the Wald statistic becomes

$$\mathcal{W}_T(\tau) = T \hat{\boldsymbol{\beta}}_T(\tau)' (\boldsymbol{\Psi} \hat{\mathbf{M}}_{zz}^{-1} \boldsymbol{\Psi}')^{-1} \hat{\boldsymbol{\beta}}_T(\tau) \hat{f}^2 / [\tau(1-\tau)],$$

---

<sup>2</sup>Note that when  $\mathbb{E}[\tau - \mathbf{1}_{\{e_{t\tau} < 0\}} | \mathcal{Z}_{t-1}] = 0$ ,  $\mathbf{z}_{t-1}[\tau - \mathbf{1}_{\{e_{t\tau} < 0\}}]$  is a martingale difference sequence and hence obeys a central limit theorem:

$$T^{-1/2} \sum_{t=1}^T \mathbf{z}_{t-1}[\tau - \mathbf{1}_{\{e_{t\tau} < 0\}}] \xrightarrow{D} [\tau(1-\tau)]^{1/2} \mathbf{M}_{zz}^{1/2} \mathcal{N}(\mathbf{0}, \mathbf{I}_k).$$

The asymptotic normality of  $\hat{\boldsymbol{\theta}}_T(\tau)$  and the asymptotic covariance matrix  $\boldsymbol{\Omega}(\tau)$  readily follow from the Bahadur representation and this result. For some regularity conditions ensuring  $\mathbb{E}[\tau - \mathbf{1}_{\{e_{t\tau} < 0\}} | \mathcal{Z}_{t-1}] = 0$ , see Koenker and Xiao (2006).

where  $\widehat{\mathbf{M}}_{zz} = T^{-1} \sum_{t=1}^T \mathbf{z}_{t-1} \mathbf{z}'_{t-1}$ , and  $\hat{f}$  is a consistent estimator of  $f$ . To test (5), Koenker and Machado (1999) suggest using a sup-Wald test, i.e., the supremum of  $\mathcal{W}_T(\tau)$ .

Note that  $\mathbf{B}_q(\tau)$ , a vector of  $q$  independent Brownian bridges, equals  $[\tau(1-\tau)]^{1/2} \mathcal{N}(\mathbf{0}, \mathbf{I}_q)$  in distribution. Thus, (6) can be expressed as

$$\sqrt{T} \left[ \hat{\boldsymbol{\beta}}_T(\tau) - \boldsymbol{\beta}(\tau) \right] \xrightarrow{D} [\boldsymbol{\Psi} \boldsymbol{\Omega}(\tau) \boldsymbol{\Psi}']^{1/2} \mathbf{B}_q(\tau). \quad (7)$$

Under suitable conditions, (7) holds uniformly on a closed interval  $\mathcal{T} \subset (0, 1)$ , so that under the null hypothesis (5),

$$\mathcal{W}_T(\tau) \Rightarrow \left\| \frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1-\tau)}} \right\|^2, \quad \tau \in \mathcal{T},$$

where the weak limit is the sum of squares of  $q$  independent Bessel processes.<sup>3</sup> This immediately leads to the following result:

$$\sup_{\tau \in \mathcal{T}} \mathcal{W}_T(\tau) \xrightarrow{D} \sup_{\tau \in \mathcal{T}} \left\| \frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1-\tau)}} \right\|^2. \quad (8)$$

In practice, we may set  $\mathcal{T} = [\epsilon, 1 - \epsilon]$  for some small  $\epsilon$  in  $(0, 0.5)$  and choose  $n$  points ( $\epsilon = \tau_1 < \dots < \tau_n = 1 - \epsilon$ ). The sup-Wald test for (5) is computed as

$$\sup\text{-}\mathcal{W}_T = \sup_{i=1, \dots, n} \mathcal{W}_T(\tau_i).$$

When  $n$  is large, the right-hand side of (8) with  $\mathcal{T} = [\epsilon, 1 - \epsilon]$  ought to be a good approximation to the null limit of  $\sup\text{-}\mathcal{W}_T$ . See Koenker and Machado (1999) for some simulation results on the finite-sample performance of this test. Similarly, we may test the null:

$$H_0 : \boldsymbol{\beta}(\tau) = \mathbf{0}, \quad \forall \tau \in [a, b]. \quad (9)$$

by the supremum of  $\mathcal{W}_T(\tau_i)$  with  $a = \tau_1 < \dots < \tau_n = b$ . It is clear that the limit in (8) carries over to  $\mathcal{T} = [a, b]$ . The results of the sup-Wald test on various  $[a, b]$  may be used to identify the quantile range from which causality arises. For example, if the null hypothesis (5) is rejected but (9) is not rejected for some interval  $[a, b]$ , one may infer that causality mainly arises from the quantiles outside  $[a, b]$ .

**Remark:** The linear model considered here is convenient for model estimation and hypothesis testing. Yet, our approach to testing causality, the sup-Wald test in particular,

---

<sup>3</sup>Note that  $\|\mathbf{B}_q(\tau)/\sqrt{\tau(1-\tau)}\|$  tends to infinity when  $\tau \rightarrow 0$  or  $1$  (Andrews, 1993). Thus,  $\mathcal{W}_T(\tau)$ ,  $\tau \in \mathcal{T}$ , would not have a well defined limit unless  $\mathcal{T}$  is a closed interval in  $(0, 1)$ .

Table 1: The critical values of the sup-Wald test on [0.05, 0.95].

	$q = 1$	$q = 2$	$q = 3$
1%	13.01	16.30	19.21
5%	9.84	12.77	15.28
10%	8.19	11.05	13.49

*Note:*  $q$  is the dimension of the parameter vector being tested.

would be valid provided that the linear model is correctly specified for conditional quantile functions.

To determine the critical values for the sup-Wald test, we note that, for  $s = \tau/(1 - \tau)$ , the one-dimensional Bessel process  $B(\tau)/\sqrt{\tau(1 - \tau)}$  and the normalized, one-dimensional Brownian motion  $W(s)/\sqrt{s}$  are equal in distribution. It follows that

$$\mathbb{P} \left\{ \sup_{\tau \in [a, b]} \left\| \frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1 - \tau)}} \right\|^2 < c \right\} = \mathbb{P} \left\{ \sup_{s \in [1, s_2/s_1]} \left\| \frac{\mathbf{W}_q(s)}{\sqrt{s}} \right\|^2 < c \right\},$$

with  $s_1 = a/(1 - a)$ ,  $s_2 = b/(1 - b)$ , and  $\mathbf{W}_q$  a vector of  $q$  independent Brownian motions. That is, the critical values  $c$  are determined by the sum of squared normalized Brownian motions. The critical values for some  $q$  and  $s_2/s_1$  have been tabulated in DeLong (1981) and Andrews (1993); other critical values can be easily computed via simulations. The simulated critical values of the sup-Wald test (with  $q = 1, 2, 3$ ) on [0.05, 0.95] are summarized in Table 1.<sup>4</sup>

## 4 Empirical Study

Our empirical study of return-volume relations focuses on 3 stock market indices: NYSE, S&P 500 and FTSE 100. The daily data from the beginning of 1990 (Jan. 2 or Jan. 4) to June 30, 2006 are taken from *Datastream* database, and there are 4135, 4161 and 4166 observations for NYSE, S&P 500 and FTSE 100, respectively. As will be shown in Section 4.4, our results are quite robust to different sample periods.

Returns are calculated as  $r_t = 100 \times (\ln(p_t) - \ln(p_{t-1}))$ , where  $p_t$  is index at time  $t$ ; volumes  $v_t$  are the traded share volumes of these indices. Their summary statistics are

---

<sup>4</sup>Our simulation approximates the standard Brownian motion using a Gaussian random walk with 3000 i.i.d.  $\mathcal{N}(0, 1)$  innovations; the number of replications is 20,000.

Table 2: Summary statistics for stock returns  $r_t$  and volume  $v_t$ .

	NYSE		S&P 500		FTSE 100	
	$r_t$	$v_t$	$r_t$	$v_t$	$r_t$	$v_t$
mean	0.03	769.43	0.04	863.18	0.02	778.10
st. deviation	0.89	541.27	1.02	761.37	1.02	662.77
median	0.05	609.31	0.05	494.88	0.04	426.80
skewness	-0.23	0.53	-0.09	0.61	-0.11	0.94
kurtosis	4.15	-0.96	3.74	-1.07	3.11	-0.07
minimum	-6.79	31.64	-7.25	2.08	-5.89	26.36
maximum	5.18	2767.75	5.61	3345.21	5.90	4461.01

*Note:* Volumes here are traded share volumes times  $10^{-6}$ .

collected in Table 2. It can be seen that the mean and median returns are all close to zero and their standard deviations are close to one. Also, the return series behave similarly to what we usually observe in the literature: they fluctuate around their respective mean levels and exhibit volatility clustering and excess kurtosis. For each volume, the mean and median are quite different, and its kurtosis coefficient is small.

There are pronounced trending patterns in the volume series. Following Gallant, Rossi, and Tauchen (1992), we consider log volume series and remove their trends by regressing  $\ln v_t$  on a constant,  $t/T$  and  $(t/T)^2$ ; see also Chen, Firth, Rui (2000) and Lee and Rui (2002). To conserve space, we plot only the log volume series and their detrended residuals in Figure 1. It can be seen that there is no trend in these residual series. Our subsequent analysis of return-volume relations is thus based on  $r_t$  and  $\ln v_t$  while controlling the time trend effects.<sup>5</sup>

#### 4.1 Causal Effects of Volume on Return: Model without $r_{t-j}^2$

We first consider the following model for return and estimate this model using the least-squares (LS) and quantile regression (QR) methods:

$$r_t = a(\tau) + b(\tau) \frac{t}{T} + c(\tau) \left( \frac{t}{T} \right)^2 + \sum_{j=1}^q \alpha_j(\tau) r_{t-j} + \sum_{j=1}^q \beta_j(\tau) \ln v_{t-j} + e_t, \quad (10)$$

<sup>5</sup>We also considered the causal relations between return and the growth rate of volume and found that the latter does not Granger causes the former in quantiles. This agrees with the finding of Su and White (2007) which is based on a test at the distribution level.

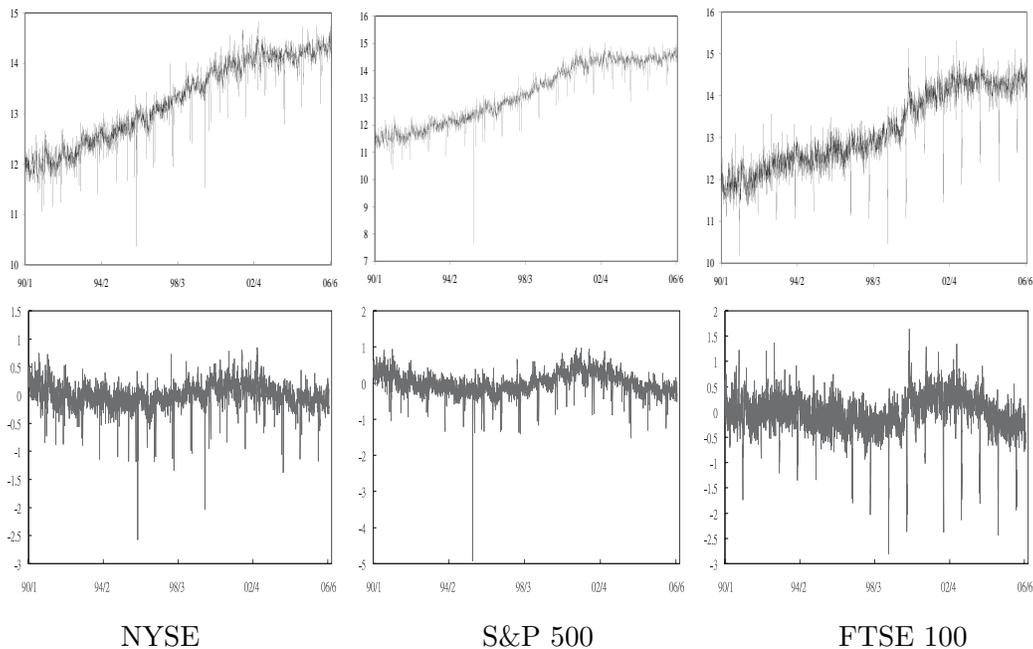


Figure 1: The series of log volume (upper panel) and detrended residuals (lower panel).

where  $T$  is the sample size and  $q \geq 1$ ; this model will be referred to as a lag- $q$  model. In the light of Figure 1, we include  $t/T$  and  $(t/T)^2$  as regressors in the model so as to control the trending effect in  $\ln v_t$ . We do not report the results of the model with detrended  $\ln v_t$  (i.e., the residuals of regressing  $\ln v_t$  on  $t/T$  and  $(t/T)^2$ ) as regressors because, as far as causality is concerned, all regressors should be in the information set so that the model involves no future information.<sup>6</sup> Although we may specify different models for the conditional mean and quantile functions, we estimate the same model (10) in our study so that the LS and QR estimates can be compared directly.

We apply the sup-Wald test to determine an appropriate lag order  $q^*$ . If the null of  $\beta_q(\tau) = 0$  for  $\tau$  in  $[0.05, 0.95]$  is not rejected for the lag- $q$  model but the null of  $\beta_{q-1}(\tau) = 0$  for  $\tau$  in  $[0.05, 0.95]$  is rejected for the lag- $(q - 1)$  model, we infer that  $\ln v_{t-q}$  does not Granger cause  $r_t$  in quantiles but  $\ln v_{t-q+1}$  does. The desired lag order is then set as  $q^* = q - 1$ . For simplicity, we do not consider the model that includes  $r_{t-j}$  and  $\ln v_{t-j}$  with different lag orders. For NYSE, the sup-Wald test of  $\beta_4(\tau)$  in the lag-4 model is 11.813 and that of  $\beta_3(\tau)$  in the lag-3 model is 18.261. The latter is significant at 1% level, but the former is not; see the critical values in Table 1 (under  $q = 1$ ). For S&P 500,

<sup>6</sup>Nonetheless, we find that the QR estimates of (10) are very close to those of the model with lagged  $r_t$  and lagged detrended  $\ln v_t$  as regressors.

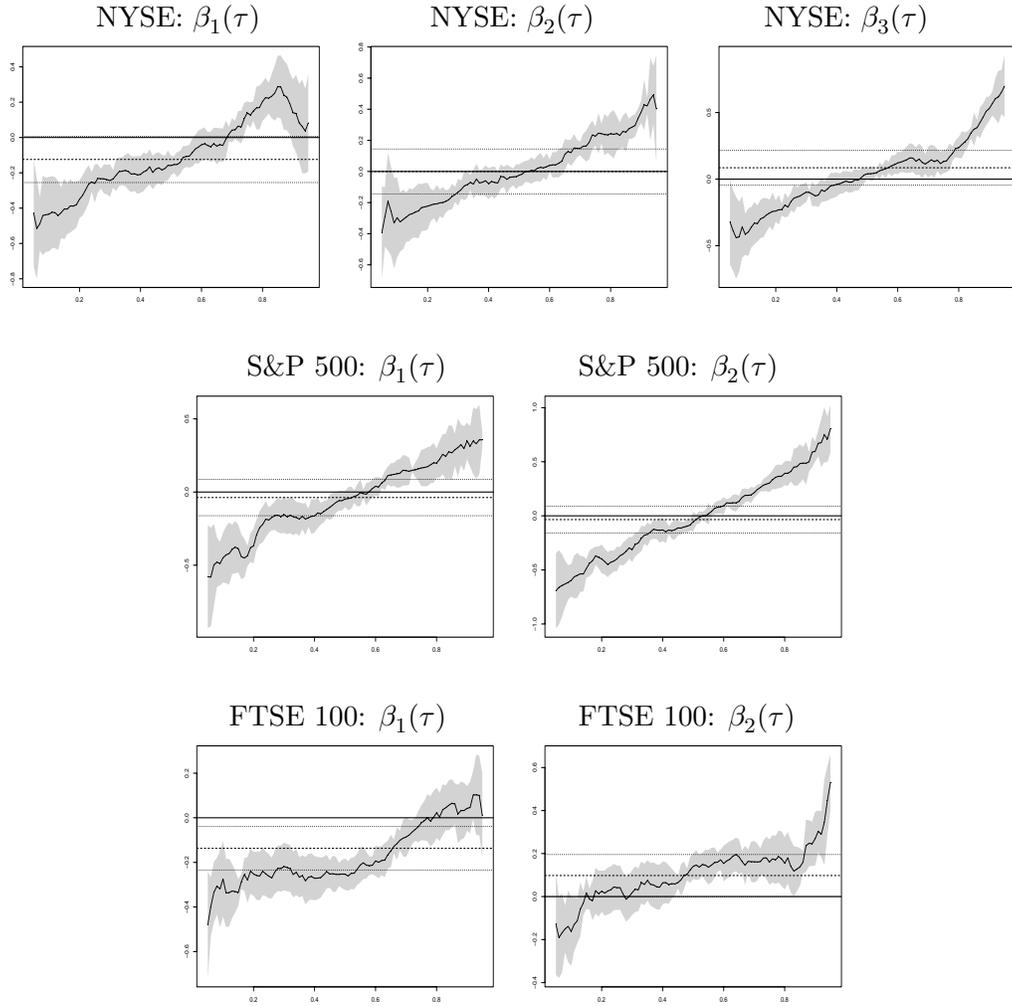


Figure 2: QR and LS estimates of the causal effects of log volume on return: Model without  $r_{t-j}^2$ .

the sup-Wald test of  $\beta_3(\tau)$  in the lag-3 model is 12.421 which is insignificant at 1% level, but that of  $\beta_2(\tau)$  in the lag-2 model is 25.227 which is significant. For FTSE 100, the sup-Wald test of  $\beta_3(\tau)$  in the lag-3 model is 7.7 which is insignificant even at 10% level, and that of  $\beta_2(\tau)$  in the lag-2 model is 13.567 which is significant at 1% level. Thus, we set  $q^* = 3$  for NYSE and  $q^* = 2$  for S&P 500 and FTSE 100.<sup>7</sup> For each lag- $q^*$  model (10), 91 quantile regressions (with  $\tau = 0.05, 0.06, \dots, 0.95$ ) are estimated using the R program (version 2.4.0) with the “quantreg” package (version 4.01) written by R. Koenker.<sup>8</sup>

<sup>7</sup>At 5% level, we find  $q^* = 5$  for NYSE,  $q^* = 5$  for S&P 500, and  $q^* = 2$  for FTSE 100. To ease our illustration, we choose 1% level and deal with simpler models.

<sup>8</sup>These programs are available from the CRAN website: <http://cran.r-project.org/>.

In Figure 2, we plot against  $\tau$  the QR estimates of  $\beta_j(\tau)$  (solid line) and their 95% confidence intervals (in shaded area), together with the LS estimate (dashed line) and its 95% confidence interval (dotted lines). It can be seen that, for NYSE and S&P 500, the LS estimates of  $\beta_j$ , the mean causal effects of log volumes, are all negative but insignificantly different from zero. This suggests no causality in mean in these 2 series. Yet, the QR estimates of  $\beta_j(\tau)$  vary with quantiles and exhibit an interesting pattern. First, the QR estimates are negative at lower quantiles and positive at upper quantiles. Second, the magnitude of these estimates increases as  $\tau$  moves toward 0 and 1. Third, these estimates are, in general, significant at tail quantiles.<sup>9</sup> Thus, lagged log volume exerts opposite and heterogeneous quantile causal effects on the two sides of the return distribution, and such effects are stronger at more extreme quantiles.

The estimation results for FTSE 100 are quite different. The LS estimate of  $\beta_1$  is significantly negative at 5% level, but that of  $\beta_2(\tau)$  is insignificant. This shows that there is causality in mean in FTSE 100. The QR estimates of  $\beta_j(\tau)$  are also heterogeneous across  $\tau$ . The QR estimates of  $\beta_1(\tau)$  are significantly negative at lower quantiles but insignificant at upper quantiles, and the QR estimates of  $\beta_2(\tau)$  are significantly positive at most upper quantiles.<sup>10</sup>

To be sure, we apply the sup-Wald test to check joint significance of all coefficients of lagged log volumes. The null hypothesis for NYSE is  $\beta_1(\tau) = \beta_2(\tau) = \beta_3(\tau) = 0$  on  $[0.05, 0.95]$ , and the null for S&P 500 and FTSE 100 is  $\beta_1(\tau) = \beta_2(\tau) = 0$  on  $[0.05, 0.95]$ . As shown in Table 3, these statistics overwhelmingly reject the null of non-causality at 1% level, suggesting causality in quantiles in these indices. We also test  $\beta_i(\tau) = 0$  on the ranges of quantiles at which the estimates of  $\beta_i(\tau)$  are found insignificant individually. As shown in Table 3, none of these null hypotheses can be rejected at 5% level. Thus, we conclude that, for NYSE and S&P 500, the quantile causal effects are mainly due to the tail quantiles outside the interquartile range (except that of  $\ln v_{t-1}$  for NYSE). Our results are in contrast with many existing findings of non-causality that are based on a test for linear causality in mean (e.g., Kocagil and Shachmurove, 1998; Chen, Firth, and Rui, 2001; Lee and Rui, 2002).

Following Buchinsky (1998), we test whether the pairwise causal effects at the  $\tau$ -th

---

<sup>9</sup>For NYSE, we obtain insignificant QR estimates of  $\beta_1(\tau)$  for  $\tau$  in  $[0.53, 0.79]$  and  $[0.87, 0.95]$ , insignificant estimates of  $\beta_2(\tau)$  for  $\tau$  in  $[0.30, 0.72]$ , and insignificant estimates of  $\beta_3(\tau)$  for  $\tau$  in  $[0.26, 0.62]$  and  $[0.68, 0.76]$ . For S&P 500, there are insignificant QR estimates of  $\beta_1(\tau)$  for  $\tau$  in  $[0.24, 0.64]$  and insignificant estimates of  $\beta_2(\tau)$  for  $\tau$  in  $[0.34, 0.65]$ .

<sup>10</sup>For FTSE 100, the quantiles range of insignificant QR estimates are  $[0.67, 0.95]$  for  $\beta_1(\tau)$  and  $[0.08, 0.49]$  and  $[0.83, 0.85]$  for  $\beta_2(\tau)$ .

Table 3: The sup-Wald tests of non-causality in different quantile ranges.

Index	$\beta_i(\tau) = 0, i = 1, 2, 3$	$\beta_1(\tau) = 0$	$\beta_2(\tau) = 0$	$\beta_3(\tau) = 0$
NYSE	[0.05, 0.95] 79.12**	[0.53, 0.79] 3.48	[0.87, 0.95] 3.11	[0.3, 0.72] 2.65
	$\beta_i(\tau) = 0, i = 1, 2$	$\beta_1(\tau) = 0$	$\beta_2(\tau) = 0$	
S&P 500	[0.05, 0.95] 134.27**	[0.24, 0.64] 5.80	[0.34, 0.65] 5.71	
FTSE 100	[0.05, 0.95] 27.53**	[0.67, 0.95] 2.18	[0.08, 0.49] 2.98	[0.83, 0.85] 2.78

*Note:* Each interval in the square bracket is the quantile range on which the null hypothesis holds; the entry below each interval is the sup-Wald statistic. \*\* and \* denote significance at 1% and 5% levels, respectively. The critical values for the tests on [0.05, 0.95] are in Table 1; the other critical values are obtained by simulations.

and  $(1 - \tau)$ -th quantiles are symmetric about the median, i.e.,  $\beta_i(\tau) + \beta_i(1 - \tau) = 2\beta_i(0.5)$  with  $i = 1, 2, 3$  for NYSE and  $i = 1, 2$  for both S&P 500 and FTSE 100. This amounts to checking whether

$$\hat{\delta}_{i,T}(\tau) = \hat{\beta}_{i,T}(\tau) + \hat{\beta}_{i,T}(1 - \tau) - 2\hat{\beta}_{i,T}(0.5)$$

is sufficiently close to zero. To this end, we conduct a  $\chi^2(1)$  test based on the square of the normalized  $\hat{\delta}_T(\tau)$  for the  $\tau$  pairs: (0.05, 0.95), (0.1, 0.9), ..., (0.45, 0.55), where the standard error of  $\hat{\delta}_T(\tau)$  is computed via design matrix bootstrap. We may also conduct a joint test to check if  $\hat{\delta}_{i,T}(\tau)$ ,  $i = 1, \dots, k$ , are close to zero. For NYSE, it is a  $\chi^2(3)$  test; for S&P 500 and FTSE 100, it is a  $\chi^2(2)$  test. The testing results of all indices are summarized in Table 4.

Table 4 shows that, for NYSE and S&P 500, the null of symmetric causal effects can not be rejected at 5% for all  $\tau$  pairs we considered. This is so for both individual test and joint test. For FTSE 100, these effects are not symmetric for some middle  $\tau$  pairs of  $\beta_1(\tau)$ . These symmetry results are somewhat different from those of Hutson, Kearney, and Lynch (2008). The symmetry of these quantile causal effects helps to explain why the conventional methods, such as correlation coefficient and LS estimation, usually yield an insignificant estimate of the causal effect of volume, as the positive and negative effects at corresponding upper and lower quantiles tend to cancel out each other in “averaging.”

Table 4: Testing symmetry of quantile causal effects: Models without  $r_{t-j}^2$ .

$\tau$ pair $(\tau, 1 - \tau)$	NYSE				S&P 500			FTSE 100		
	$\beta_1$	$\beta_2$	$\beta_3$	joint	$\beta_1$	$\beta_2$	joint	$\beta_1$	$\beta_2$	joint
0.05	0.027	0.139	2.692	4.325	0.335	0.760	0.724	0.042	0.447	0.874
0.10	0.023	0.699	0.056	1.695	0.000	0.384	0.608	3.615	1.477	3.857
0.15	1.665	0.029	0.001	3.134	0.020	0.452	0.566	4.253*	1.119	4.597
0.20	2.160	0.338	0.658	3.967	0.329	0.612	0.629	8.339**	1.082	10.236**
0.25	2.931	0.598	1.432	5.259	0.349	0.013	0.740	7.150**	0.883	8.749*
0.30	1.484	0.495	0.234	3.120	0.790	0.009	1.487	6.224*	1.434	7.040*
0.35	0.929	1.162	0.002	4.301	0.120	0.601	1.571	1.815	0.106	2.567
0.40	1.025	0.008	0.006	1.463	0.234	0.867	0.871	0.697	1.311	1.280
0.45	0.327	0.135	0.526	0.873	0.062	0.095	0.100	1.296	3.508	3.408

*Note:* Each entry is a test statistic for the hypothesis that the quantile causal effects are symmetric about the median. \*\* and \* denote significance at 1% and 5% levels, respectively; the corresponding critical values are 6.63 and 3.84 for  $\chi^2(1)$ , 9.21 and 5.99 for  $\chi^2(2)$ , and 11.34 and 7.81 for  $\chi^2(3)$ .

The estimation and testing results for NYSE and S&P 500 lead to a vivid pattern of quantile causal effects. By putting lagged log volume on the vertical axis and return on the horizontal axis, the quantile causal effects of log volume on return exhibit a spectrum of symmetric  $V$ -shape relations, in which the  $V$ 's at more extreme quantiles have wider opening. Thus, an increase in lagged log volume results in a larger return in either sign, and such effect is stronger for returns with larger magnitude. This dynamic  $V$ -shape pattern complements the findings of Karpoff (1987), Gallant, Rossi, and Tauchen (1992), and Blume, Easley, and O'Hara (1994). These  $V$ -shape relations also imply that the dispersion of return increases with lagged volume, so that return volatility depends positively on lagged volume, analogous to the results in the context of conditional variance, e.g., Lamoureux and Lastrapes (1990), Gallant, Rossi, and Tauchen (1992), Moosa and Al-Loughani (1995), Kocagil and Shachmurove (1998), Chen, Firth, and Rui (2001), and Xu, Chen, and Wu (2006).

#### 4.2 Causal Effects of Volume on Return: Model with $r_{t-j}^2$

From the preceding subsection we find that the dispersion (volatility) of return changes with lagged log volume. To see if the quantile causal effects of log volume are robust,

we take squared return as a proxy for return volatility and consider an extension of (10) which includes lagged  $r_{t-j}^2$  as additional regressors:

$$r_t = a(\tau) + b(\tau) \frac{t}{T} + c(\tau) \left( \frac{t}{T} \right)^2 + \sum_{j=1}^q \alpha_j(\tau) r_{t-j} + \sum_{j=1}^q \beta_j(\tau) \ln v_{t-j} + \sum_{j=1}^q \gamma_j(\tau) r_{t-j}^2 + e_t. \quad (11)$$

This allows us to examine whether log volume still Granger causes return in the presence of  $r_{t-j}^2$ . The model (11) carries the flavor of an ARCH-in-mean model and is also able to capture some nonlinearity in lagged return.

We again apply the sup-Wald test to determine an appropriate lag order  $q^*$ . For NYSE, the sup-Wald test of  $\beta_3(\tau)$  in the lag-3 model is 12.402 which is significant at 1% level and that of  $\beta_2(\tau)$  in the lag-2 model is 21.158 which is significant. For S&P 500, the sup-Wald test of  $\beta_3(\tau)$  in the lag-3 model is 11.362, and that of  $\beta_2(\tau)$  in the lag-2 model is 20.554; the latter is significant at 1% level but the former is not. Therefore, we estimate (11) with  $q^* = 2$  for both NYSE and S&P 500. For FTSE 100,  $q^* = 1$  because the sup-Wald test of  $\beta_2(\tau)$  in the lag-2 model is 5.27 which is insignificant at 10% level and that of  $\beta_1(\tau)$  in the lag-1 model is 20.513 which is significant at 1% level. Thus, the desired lag order  $q^*$  may be affected when  $r_{t-j}^2$  are included in the model. For each lag- $q^*$  model, we also estimate 91 quantile regressions. The resulting QR and LS estimates and their 95% confidence intervals are plotted in Figure 3.

For NYSE and S&P 500, we observe that the LS estimates of mean causal effect are insignificantly different from zero, except that the estimate of  $\beta_1$  for NYSE is significantly negative at 5% level (but still insignificant at 1% level). Thus, one may still conclude that there is no causality in mean in NYSE and S&P 500. On the other hand, their quantile causal effects are similar to those in Figure 2. For NYSE, the QR estimates of  $\beta_1(\tau)$  at upper quantiles are mostly insignificant, and those of  $\beta_2(\tau)$  are negative (positive) at lower (upper) quantiles and significant at tail quantiles. For S&P 500, the QR estimates for each  $\beta_j(\tau)$  also have opposite signs at two sides of the return distribution and are significant at tail quantiles.<sup>11</sup> Moreover, we find that the magnitude of the QR estimates at tail quantiles are weaker than the corresponding estimates in Figure 2. For FTSE 100, the LS estimate of  $\beta_1$  is significantly negative, and the QR estimates are significantly negative at

---

<sup>11</sup>For NYSE, the QR estimates of  $\beta_1(\tau)$  are insignificant for  $\tau$  in  $[0.55, 0.95]$ , and those of  $\beta_2(\tau)$  are insignificant for  $\tau$  in  $[0.31, 0.67]$ . For S&P 500, the QR estimates of  $\beta_1(\tau)$  are insignificant at  $[0.22, 0.29]$  and  $[0.44, 0.79]$ , and those of  $\beta_2(\tau)$  are insignificant at  $[0.34, 0.72]$ .

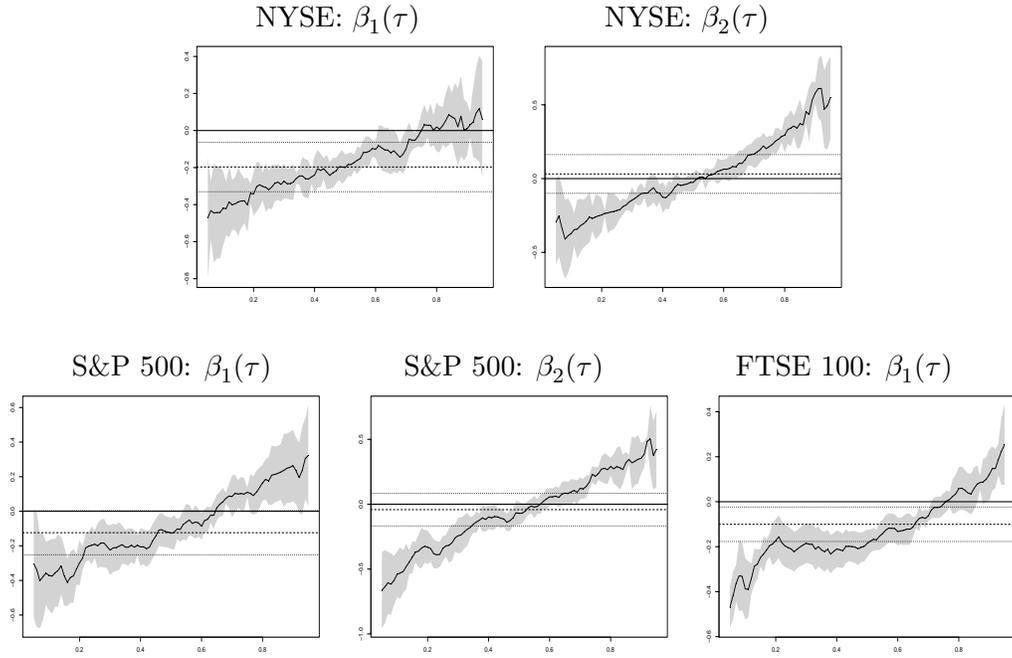


Figure 3: QR and LS estimates of the causal effects of log volume on return: Model with  $r_{t-j}^2$ .

middle and lower quantiles and significantly positive at right tail quantiles (except for  $\tau$  in  $[0.66, 0.89]$ ). These are somewhat similar to the results of FTSE 100 in Figure 2.

The sup-Wald test of non-causality again significantly rejects the null of  $\beta_i(\tau) = 0$  on  $[0.05, 0.95]$  for these indices. The results of the symmetry test are collected in Table 5, which are similar to those in Table 4. For NYSE and S&P 500, the quantile causal effects are symmetric about the median for all  $\tau$  pairs. For FTSE 100, the causal effects are symmetric, except for some pairs of middle quantiles. To summarize, the presence of  $r_{t-j}^2$  in the model may reduce the strength of quantile causal effects of log volume but does not affect causality in quantiles per se. For S&P 500, these quantile causal effects exhibit a spectrum of “smaller”, symmetric V-shape relations. This is also the pattern of the quantile causal effects of  $\ln v_{t-2}$  on the return of NYSE.

We also examine the effects of  $r_{t-j}^2$  on  $r_t$  i.e., the estimates of  $\gamma_j(\tau)$  in (11). These estimates are plotted in Figure 4. It is quite interesting to see that the heterogeneity of the estimated  $\gamma_j(\tau)$  is somewhat similar to that of the estimated  $\beta_j(\tau)$ . In particular, the quantile causal effects increase with  $\tau$ . The estimated  $\gamma_2(\tau)$  for NYSE and S&P 500 and the estimated  $\gamma_1(\tau)$  for FTSE 100 have opposite signs at the two side of the return distribution. Putting  $r_{t-j}^2$  on the vertical axis and  $r_t$  on the horizontal axis, we would also

Table 5: Testing symmetry of quantile causal effects: Models with  $r_{t-j}^2$ .

$\tau$ pair $(\tau, 1 - \tau)$	NYSE			S&P 500			FTSE 100
	$\beta_1$	$\beta_2$	joint	$\beta_1$	$\beta_2$	joint	$\beta_1$
0.05	0.005	1.732	2.003	1.256	0.105	1.409	0.629
0.10	0.003	2.520	2.885	0.612	0.061	0.748	0.660
0.15	0.259	1.093	2.576	0.395	0.123	1.299	3.390
0.20	0.426	0.811	2.422	0.751	0.489	2.921	8.367**
0.25	0.617	0.341	1.584	1.703	0.042	3.221	3.969*
0.30	0.063	0.801	1.519	1.357	0.031	2.408	2.820
0.35	0.217	0.554	1.322	0.513	0.581	2.569	0.349
0.40	0.953	0.044	0.962	0.202	1.374	1.544	0.157
0.45	0.017	0.481	0.696	0.745	0.000	1.274	0.270

*Note:* Each entry is a test statistic for the hypothesis that the quantile causal effects are symmetric about the median. \*\* and \* denote significance at 1% and 5% levels, respectively; the corresponding critical values are 6.63 and 3.84 for  $\chi^2(1)$  and 9.21 and 5.99 for  $\chi^2(2)$ .

obtain a spectrum of V-shape relations based on these estimates. This result, together with the quantile causal effects in Figures 2 and 3, confirms that both  $\ln v_{t-j}$  and  $r_{t-j}^2$  are able to account for distribution dispersion (volatility) in a similar manner.

### 4.3 Causal Effects of Return on Volume

To see if there is two-way causality between return and log volume, we now consider the following models for  $\ln v_t$ :

$$\begin{aligned} \ln v_t &= a^\dagger(\tau) + b^\dagger(\tau) \frac{t}{T} + c^\dagger(\tau) \left(\frac{t}{T}\right)^2 + \sum_{j=1}^q \alpha_j^\dagger(\tau) \ln v_{t-j} + \sum_{j=1}^q \beta_j^\dagger(\tau) r_{t-j} + e_t; \\ \ln v_t &= a^\dagger(\tau) + b^\dagger(\tau) \frac{t}{T} + c^\dagger(\tau) \left(\frac{t}{T}\right)^2 + \sum_{j=1}^q \alpha_j^\dagger(\tau) \ln v_{t-j} + \sum_{j=1}^q \beta_j^\dagger(\tau) r_{t-j} \\ &\quad + \sum_{j=1}^q \gamma_j^\dagger(\tau) r_{t-j}^2 + e_t. \end{aligned}$$

The first model is common in empirical studies; the second one extends the first by including  $r_{t-j}^2$  as regressors and is compatible with the model (11).

As in the preceding subsections, we first determine an appropriate lag order  $q^*$  by the

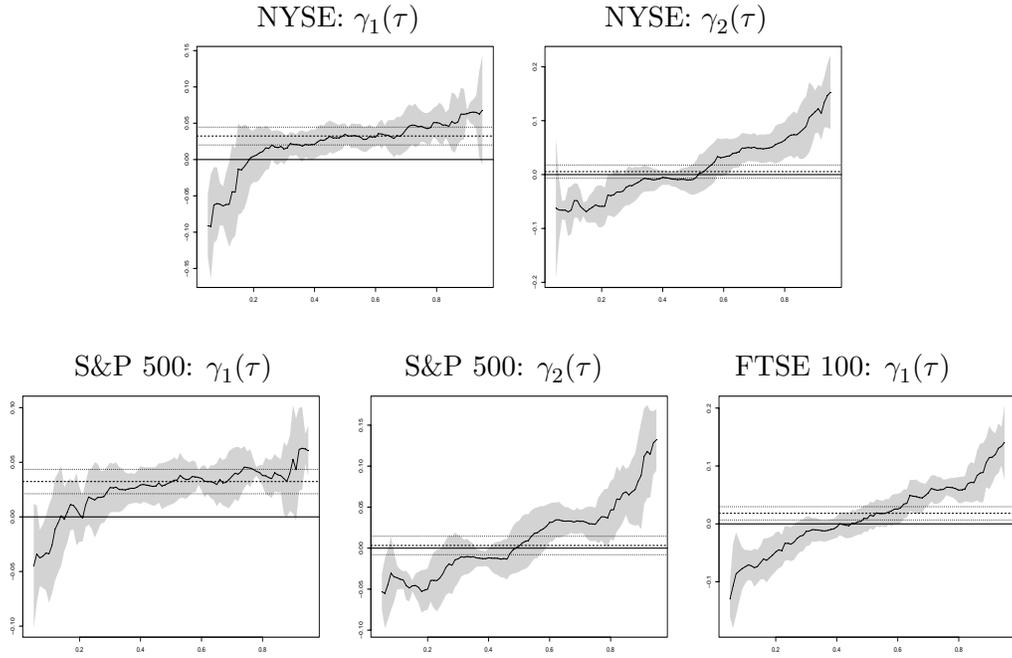


Figure 4: QR and LS estimates of the causal effects of  $r_{t-j}^2$  on  $r_t$ .

sup-Wald test. For NYSE, the sup-Wald test of  $\beta_2^\dagger(\tau)$  in the lag-2 model without  $r_{t-j}^2$  is 11.138 which is insignificant at 1% level and that of  $\beta_1^\dagger(\tau)$  in the lag-1 model without  $r_{t-j}^2$  is 16.298 which is significant. For S&P 500, the sup-Wald test of  $\beta_3^\dagger(\tau)$  in the lag-3 model without  $r_{t-j}^2$  is 10.107 and that of  $\beta_2^\dagger(\tau)$  in the lag-2 model is 16.636. The latter is significant at 1% level and the former is not. Therefore, for models without  $r_{t-j}^2$ , we set  $q^* = 1$  for NYSE and  $q^* = 2$  for S&P 500. We also find that, for models with  $r_{t-j}^2$ ,  $q^* = 1$  for NYSE and S&P 500.<sup>12</sup> On the other hand, the lagged return does not Granger cause log volume in FTSE 100 because the sup-Wald test of  $\beta_1^\dagger(\tau)$  in the lag-1 model without and with  $r_{t-j}^2$  yields 6.161 and 7.767 which are insignificant even at 10% level.

We now focus on NYSE and S&P 500 and summarize the LS and QR estimates of  $\beta_j^\dagger(\tau)$  in the models without and with  $r_{t-j}^2$  in Figures 5 and 6. We observe that the LS estimates in these models are all significantly negative. The QR estimates are also significantly negative at most quantiles and stay within the confidence interval of the corresponding LS estimate. Thus, the causal effects of return on log volume are relatively stable (ho-

<sup>12</sup>For NYSE, the sup-Wald test of  $\beta_2(\tau)$  in the lag-2 model is 6.808 and that of  $\beta_1(\tau)$  in the lag-1 model is 13.512. For S&P 500, the sup-Wald test of  $\beta_2(\tau)$  in the lag-2 model is 12.003 and that of  $\beta_1(\tau)$  in the lag-1 model is 20.564.

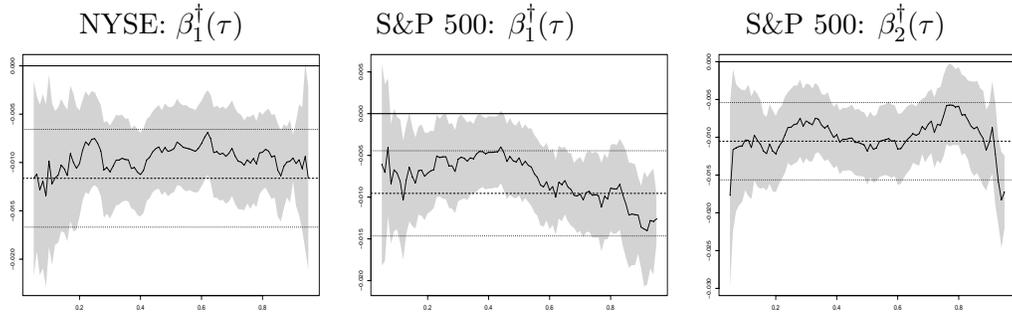


Figure 5: QR and LS estimates of the causal effects of return on log volume: Model without  $r_{t-j}^2$ .

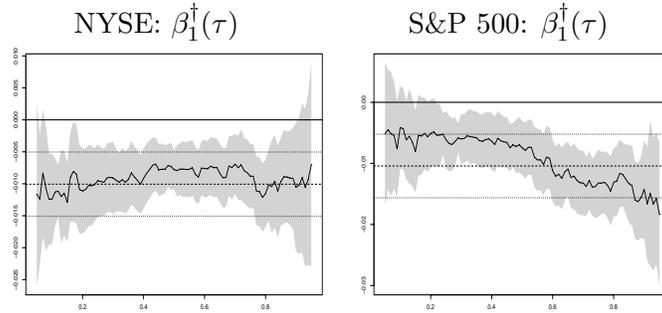


Figure 6: QR and LS estimates of the causal effects of return on log volume: Model with  $r_{t-j}^2$ .

mogeneous) across quantiles for NYSE and S&P 500. These results are consistent with the existing findings, such as Moosa and Al-Loughani (1995), Silvapulle and Choi (1999), Chen, Firth, and Rui (2001), and Lee and Rui (2002). We also note that the quantile causal effects of  $r_{t-1}^2$  on log volume are all significantly positive.

To summarize, we find from these subsections that there are two-way quantile causal relations between return and log volume for NYSE and S&P 500 but only one-way causality in quantiles from log volume to return for FTSE 100. The quantile causal effects of a lagged log volume on return still exhibit symmetric  $V$  shapes in general, yet the causal effects of a lagged return on log volume are negative. Thus, lagged log volume carries important information that is not contained in the past returns and past squared returns. Similarly, lagged return also carries information that is not contained in the past volumes and past squared returns. This is in line with Blume, Easley, and O'Hara (1994).

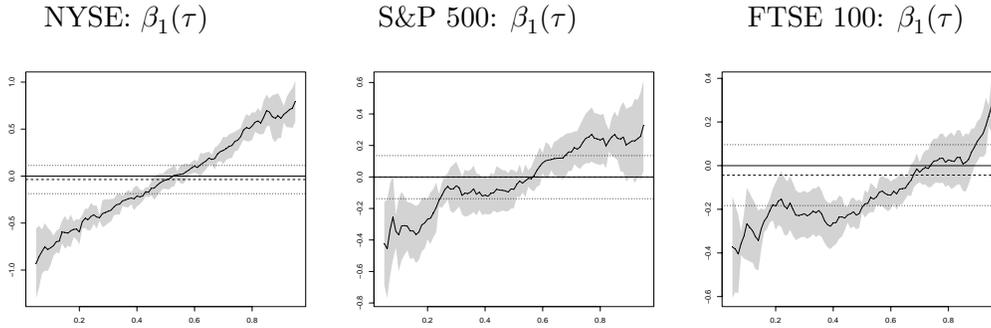


Figure 7: QR and LS estimates of the causal effects of log volume on return: 1995–2006, Model without  $r_{t-j}^2$ .

#### 4.4 Robustness Check

As our sample extends a fairly long time (17 years), we check the robustness of our results by evaluating the causal relations between return and volume in different sub-samples. Specifically, we conduct the same causality analysis on two sample periods: Jan. 1995 to June 2006 and Jan. 2000 to June 2006. The choice of these sub-samples is arbitrary.

We briefly summarize our estimation results here; the detailed statistics and estimation results are available upon request. At 1% level, the sup-Wald test suggests  $q^* = 1$  in the model (10) for each index in each sub-sample considered, except for S&P 500 in 1995–2006 the sup-Wald test of  $\beta_1(\tau) = 0$  in the lag-1 model is 9.69 which is almost significant at 5% level. To ease our comparison, we estimate all models with  $q^* = 1$ . The resulting estimates of  $\beta_1(\tau)$  based on the sample of 1995–2006 are plotted in Figure 7, and those based on the sample of 2000–2006 are plotted in Figure 8. It is readily seen that the LS estimates in these plots are all insignificant at 5% level while the quantile causal patterns of these estimates are qualitatively similar to those in Figure 2. Indeed, we observe V-shape causal relations for NYSE and S&P 500 in these sub-samples. Estimating the model (11) with  $r_{t-j}^2$  also yields similar results. To conserve space, the plots of these parameter estimates are not presented here.

## 5 Concluding Remarks

In this paper we estimate quantile causal effects and test Granger non-causality in different quantile ranges based on the quantile regressions of return (log volume). We find that there are quantile causal relations between return and log volume. More importantly, our results indicate that the causal relations may be far more complicated than what can be

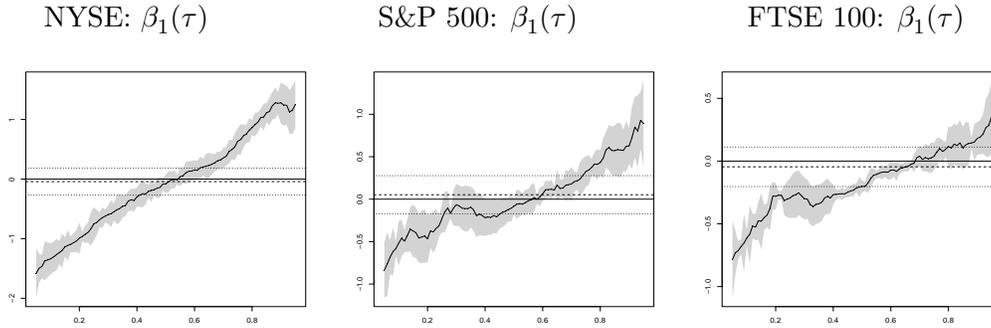


Figure 8: QR and LS estimates of the causal effects of log volume on return: 2000–2006, Model without  $r_{t-j}^2$ .

described using least-squares regression. Indeed, the causal effects may be heterogeneous across quantiles and that the causal effects at tail quantiles may be much different from those at middle quantiles and at the mean. Thus, the conclusion on non-causality based solely on a conventional test on the mean relation may be misleading.

The empirical results of causality in quantiles, however, can not be explained by existing equilibrium models (e.g., Campbell, Grossman, and Wang, 1993). These models typically yield implications on the conditional mean but say little about the behaviors of conditional quantiles. Therefore, different models are needed to account for the quantile causal patterns found in this paper. It is also interesting to note that quantile causal relations provide detailed information about distribution dispersion and hence can complement conventional volatility measures, such as conditional variance. How to incorporate such information to improve on the evaluation of volatility and related assets (e.g., VIX option) is an interesting topic and currently being investigated.

## References

- Andrews, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point, *Econometrica*, **61**, 821–856.
- Blume, L., D. Easley and M. O'Hara (1994). Market statistics and technical analysis: The role of volume, *Journal of Finance*, **XLIX**, 153–181.
- Buchinsky, M. (1998). Recent advances in quantile regression models: A practical guideline for empirical research, *Journal of Human Resources*, **33**, 88–126.
- Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading volume and serial correlation in stock returns, *Quarterly Journal of Economics*, **108**, 905–939.
- Chen, G., M. Firth and O. Rui (2001). The dynamic relation between stock returns, trading volume and volatility, *The Financial Review*, **38**, 153–174.
- Cheung, Y.-W. and L. Ng (1996). A causality-in-variance test and its application to financial market prices, *Journal of Econometrics*, **72**, 33–48.
- Ciner, C. (2002). The stock price-volume linkage on the Toronto stock exchange: Before and after automation, *Review of Quantitative Finance and Accounting*, **19**, 335–349.
- Clark, P. (1973). A subordinated stochastic process model with finite variance for speculative prices, *Econometrica*, **41**, 135–155.
- Copeland, T. (1976). A model of asset trading under the assumption of sequential information arrival, *Journal of Finance*, **XXXI**, 1149–1168.
- DeLong, D. (1981). Crossing probabilities for a square root boundary by a Bessel process, *Communications in Statistics—Theory. Math.*, **21**, 2197–2213.
- Diks, C. and V. Panchenko (2005). A note on the Hiemstra-Jones test for Granger causality, *Studies in Nonlinear Dynamic & Econometrics*, **9**, Article 4.
- Epps, T. and M. L. Epps (1976). The stochastic dependence of security price changes and transaction volumes: Implication for the mixture-of-distributions hypothesis, *Econometrica*, **44**, 305–321.
- Fujihara, R. and M. Mougoué (1997). An examination of linear and nonlinear causal relationships between price variability and volume in petroleum futures markets, *Journal of Futures Markets*, **17**, 385–416.
- Gallant, E., P. Rossi and G. Tauchen (1992). Stock price and volume, *The Review of*

- Financial Studies*, **5**, 199–242.
- Granger, C. W. J. (1969). Investigating causal relations by econometric models and cross-spectral methods, *Econometrica*, **37**, 424–438.
- Granger, C. W. J. (1980). Testing for causality: A personal viewpoint, *Journal of Economic Dynamic and Control*, **2**, 329–352.
- Granger, C. W. J. and O. Morgenstern (1963). Spectral analysis of New York stock market prices, *Kyklos*, **XVI**, 1–27.
- Granger, C. W. J., R. Robins and R. Engle (1986). Wholesale and retail prices: Bivariate time-series modeling with forecastable error variances in D. Belsley and E. Kuh, eds., *Model Reliability*, Cambridge: MIT Press.
- Harris, M. and A. Raviv (1993). Differences of Opinion make a horse race, *The Review of Financial Studies*, **6**, 473–506.
- Hiemstra, C. and J. Jones (1994). Testing for linear and nonlinear Granger causality in the stock price-volume relation, *Journal of Finance*, **XLIX**, 1639–1664.
- Hong, Y., Y. Liu and S. Wang (2008). Granger causality in risk and detection of extreme risk spillover between financial markets, *Journal of Econometrics*, forthcoming.
- Hutson, E., C. Kearney, and M. Lynch (2008). Volume and skewness in international equity markets, *Journal of Banking and Finance*, **32**, 1255–1268.
- Jennings, R. and C. Barry (1983). Information dissemination and portfolio choice, *Journal of Financial and Quantitative Analysis*, **18**, 1–19.
- Jennings, R., L. Starks and J. Fellingham (1981). An Equilibrium model of asset trading with sequential information arrival, *Journal of Finance*, **XXXVI**, 143–161.
- Karpoff, J. M. (1987). The relation between price changes and trading volume: A survey, *Journal of Financial and Quantitative Analysis*, **22**, 109–126.
- Kocagil, A. and Y. Shachmurove (1998). Return-volume dynamic in futures markets, *Journal of Futures Markets*, **18**, 399–426.
- Koenker, R. (2005). *Quantile Regression*, Cambridge: Cambridge University Press.
- Koenker, R. and G. Bassett (1978). Regression quantile, *Econometrica*, **46**, 33–50.
- Koenker, R. and J. Machado (1999). Goodness of fit and related inference processes for quantile regression, *Journal of the American Statistical Association*, **94**, 1296–1310.

- Koenker, R. and Z. Xiao (2006). Quantile autoregression, *Journal of the American Statistical Association*, **101**, 980–990.
- Lamoureux, C. and W. Lastrapes (1990). Heteroskedasticity in stock return data: Volume versus GARCH effects, *Journal of Finance*, **XLV**, 221–229.
- Lee, B.-S. and O. Rui (2002). The dynamic relationship between stock return and trading volume: Domestic and cross-country evidence, *Journal of Banking and Finance*, **26**, 51–78.
- Lee, T.-H. and W. Yang (2006). Money-income Granger-causality in quantiles, Working paper, UC Riverside.
- Moosa, I. and N. Al-Loughani (1995). Testing the price-volume relation in emerging Asian stock markets, *Journal of Asian Economics*, **6**, 407–422.
- Silvapulle, P. and J.-S. Choi (1999). Testing for linear and nonlinear Granger causality in the stock price-volume relation: Korean evidence, *Quarterly Review of Economics and Finance*, **39**, 59–76.
- Su, L. and H. White (2007). A consistent characteristic function-based test for conditional independence, *Journal of Econometrics*, **141**, 807–834.
- Suominen, M. (2001). Trading volume and information revelation in stock market, *Journal of Financial and Quantitative Analysis*, **36**, 545–565.
- Tauchen, G. and M. Pitts (1983). The price variability–volume relationship on speculative markets, *Econometrica*, **51**, 485–505.
- Wang, J. (1994). A model of competitive stock trading volume, *Journal of Political Economy*, **102**, 127–168.
- Xu, X. E., P. Chen, and C. Wu (2006). Time and dynamic volume-volatility relation, *Journal of Banking and Finance*, **30**, 1535–1558.
- Ying, C. (1966). Stock market prices and volumes of sales, *Econometrica*, **34**, 676–685.