TEACHING STATEMENT

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1. Introduction

I am very passionate about mathematics and I enjoy sharing the things I like. In particular, teaching mathematics has always been a very enjoyable experience for me, from my first teaching experience as a tutor in 2018 until my first experience as a main lecturer in 2024.

My teaching style has been very influenced by some of the great teachers that I have had during my career. I would describe my teaching style as a combination of the following two principles:

- **Conceptual understanding.** To master a topic, it is essential to have a deep understanding of its basic concepts and how they are related to one another. Understanding not only the definitions, but also the reasons behind them. Being able to explain a mathematical concept using plain words is a good sign of understanding it well-enough, so a catchphrase that I like to keep in mind is

  Say it with words!

  Moreover, since I am biased towards geometry, I also like to provide plenty of geometric intuition whenever it is relevant.

- **Correctness and rigor.** Having a good intuitive and conceptual understanding of a mathematical subject should not come at the cost of rigor. Saying things in plain words and visualizing concepts with pictures is great, but it is only helpful if things can be written down in proper mathematical language afterwards. For this reason, I always emphasize the importance of writing things down correctly. For instance, one point where I have noticed that students struggle particularly is with the use of quantifiers, so I pay especial attention to that during my lectures.

In Section 3, I describe some concrete teaching examples that illustrate these two principles. In addition to these two general principles, I also have a concrete rule during my lectures:

- **Avoid using the words "obvious" and "trivial".** I do believe that it is important for students to be mature enough to fill in the gaps in the arguments given in the lecture. And I understand that many lecturers use the words "obvious" or "trivial" to indicate to their students that they should go ahead and try to fill in that gap themselves. But at the same time I believe that the usage of such words has the potential danger of demotivating some students, especially if they find themselves struggling to prove some statement that was supposed to be obvious. For this reason, I try to avoid using these words, and instead I try to say things like "this follows directly from the definitions", or "this holds by a straight-forward computation", or "this is a small exercise".

My first experience as a teaching assistant was shortly after the COVID-19 pandemic. Online learning was an essential part of university lectures back then, and nowadays it still plays a very important role, as it has become a very convenient platform to organize lecture materials and communicate with students, especially when it comes to big lectures. In Section 4, I describe some of my experiences and approaches to online learning.

2. Teaching experience

- *Calculus for Economics* (main instructor). Series of 4 courses (2 per semester) with around 150 students each taught at the National Taiwan University during the academic year 2024/2025. My duties included preparing and teaching the theory lectures and coordinating the five teaching assistants that I had.
- *Elementary Geometry* (teaching assistant). Course with around 60 students taught at the University of Freiburg during the summer term 2023. My duties included creating the exercise sheets and offering a "question class" to the students, in which they could ask questions about the lecture.
- *Riemann Surfaces* (teaching assistant). Course with 14 students taught at the University of Freiburg during the winter term 2022/2023. My duties included creating the exercise sheets, grading the students' weekly homework and presenting the solutions in the exercise class.
- *Elementary Geometry* (teaching assistant). Course with around 60 students taught at the University of Freiburg during the summer term 2022. My duties included creating the exercise sheets and offering a "question class" to the students, in which they could ask questions about the lecture.
- *Algebra and Number Theory* (teaching assistant). Course with around 120 students taught at the University of Freiburg during the winter term 2021/2022. My duties included creating the exercise sheets and offering a "question class" to the students, in which they could ask questions about the lecture.
- *Introduction to Algebra* (tutor). Course taught at the University of Bonn during the winter term 2018/2019. My duties included grading the

weekly homework of a group of around 15 students and presenting the solutions in an exercise class.

- *Linear Algebra for Informatics and prospective teachers* (tutor). Course taught at the University of Bonn during the summer term 2018. My duties included grading the weekly homework of a group of around 15 students and presenting the solutions in an exercise class.

3. Concrete teaching examples

3.1. **Conceptual understanding.** During the academic year 2021/2022 and the summer term 2023, I taught various "question class" lectures. In these lectures, the students would come and ask freely questions about parts of the lecture that they did not understand. During these classes I was focused entirely on making sure that the students understood things properly, giving plenty of motivation and geometric intuition. It was also a particularly good chance to put my catchphrase into practice: I would often explain something with plain words, and I would also tell the students explicitly to try to say something in plain words every now and then, as a way for them to make sure that they have understood the concept properly. Here are a couple examples:

- What does "linearly independent set of vectors" mean? With plain words, it means that the only way to obtain the zero vector as a linear combination of these vectors is with the trivial linear combination.

- Let $P(x) \in \mathbb{Z}[x]$ be a monic polynomial and suppose that its image $\bar{P}(x) \in \mathbb{F}_p[x]$ is irreducible in $\mathbb{F}_p[x]$ for some prime number p. Can you explain with words why is $P(x)$ irreducible in $\mathbb{Z}[x]$ as well?

A more recent example from my lectures that combines geometric intuition and saying things in plain words is the following, which explains why the *Method of Langrange Multipliers* works:

- Consider the problem of finding exreme values of a function $f(x, y)$ subject to a constraint of the form $g(x, y) = k$ for some $k \in \mathbb{R}$, where $f(x, y)$ and $g(x, y)$ are smooth real-valued functions. Suppose that $P = (x_0, y_0)$ is a point in the curve $g(x, y) = k$ and let $c := f(P)$. Picture the level curves of f in the plane. If the level curve $f(x, y) = c$ is not tangent to the curve $g(x, y) = k$ at the point P, then we should be able to increase (resp. decrease) the value of f along the curve $g(x, y) = k$, by following the curve $g(x, y) = k$ in a direction in which f increases (resp. decreases). Recall that the gradient of a function at a point is perpendicular to the level curve of the function at that point. So if $c = f(P)$ is an extreme value of $f(x, y)$ subject to $g(x, y) = k$, then the gradients of f and g have to be parallel at P, i.e., assuming $\nabla g(x_0, y_0) \neq 0$, we must have $\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$ for some $\lambda \in \mathbb{R}$.

This strategy of saying things with plain words is not restricted to basic undergraduate mathematics. But, much like translating mathematical statements into formal first-order logic statements, this is a recursive process in which you gradually expand your vocabulary of plain words. Once you know how to explain a mathematical concept with plain words, you can add this concept to your vocabulary of plain words. So if a non-mathematician asks you what this concept means, you should be able to explain it to them, assuming that you are given enough time and enoguh patience on their part. For instance, some more abstract examples would be:

- A *scheme* is a geometric object that locally looks like the spectrum of a ring. (Here I am cheating only slightly: one has to know that "looks like" means "is isomorphic as locally ringed space".)
- An *effective Cartier divisor* is locally the zero locus of a non-zero divisor.

3.2. **Correctness and rigor.** Johan Commelin, a world-leading expert in formalization of mathematics using Lean, was based in Freiburg while I was studying my PhD there. Taking advantage of Lean being a hot topic in Freiburg at the time and basic abstract algebra being particularly suitable for this, I proposed several formalization exercises in the exercise sheets of the *Algebra and Number Theory* lecture. The students were invited to formalize their solutions to some of the exercises, to make sure that they had a fully detailed proof without any gaps. The exercise sheets also included some progressive indications on how to do this. For example, in the first exercise sheet, students are encouraged to formalize in Lean the solution to the following:

Exercise. Let $f\colon G \to G'$ be a group homomorphism. Show that $f(1_G) = 1_{G'}$, and that $f(a^{-1}) = f(a)^{-1}$ for all $a \in G$.

4. Online learning platforms

The *Algebra and Number Theory* lecture was taught shortly after the COVID-19 pandemic, so the course was taught in a hybrid format. For this reason, the usage of technology was essential to ensure that all students could follow the course properly. One of my favorite tools among the ones that were used at the time were the *quick tests.* Every week, the students would get to attempt an online quick test with a couple of very easy questions. This tests were a feature of the online learning platform that we were using at the time (*ILIAS*), and they also allowed us to give automatic feedback to the students right after submitting their solutions. For example, a question on such a quick test could be: Is it true or false that every abelian group is cyclic?

The online platform at National Taiwan University (*NTU COOL*) also allowed for the creation of such quick tests, so I continued using them during my time as a main instructor at National Taiwan University. Students have always participated actively in these quick tests, and they seem to appreciate this feature very much.