1. Introduction

1.1. Definition

- Tax (稅): 政府以其公權力對「經濟個體」或「經濟活動」所徵收之金錢
- Taxation (課稅): 政府徵稅之作爲
- Tax base (稅基): 政府徵稅之標的

1.2. Classification

- According to “tax base”:
  - Direct tax: levied on persons (eg: income tax)
    - Tax-inclusive base: tax base includes the amount of tax
  - Indirect tax: levied on goods/services (eg: sales tax)
    - Tax-exclusive base: tax base excludes the amount of tax

- According to “economic activities”:
  - Lump-sum/poll/head tax: fixed amount on person
  - Factor input tax: labor, capital, land
– **Sales tax**: retail sales tax, excise tax

– **Value-added tax (VAT)**: levied on increased value of goods

– **Selective v. general tax**

1.3. **Tax Incidence (稅負歸宿)**

- **Def**: Bearer of tax burden

- **Tax shifting (稅負轉嫁)**:
  - **Statutory incidence**: people **legally** responsible for tax
  - **Economic incidence**: people **actually** bearing burden

- **Considerations**:
  - **Scope of analysis**:
    * **Partial equilibrium**: focus on **taxed** market only
    * **General equilibrium**: **affected** markets also considered
  - **Market structure**: competition, monopoly, ...
  - **Time span**: SR v. LR
## 2. Partial Equilibrium Analysis

### 2.1. Commodity Tax (Perfect Competition)

1. Pre-tax: supply $S(P_S)$, demand $D(P_D)$:

   \[
P_D = P_S = P_0
   \]

   ▶ $P_D \equiv$ price paid by consumers
   ▶ $P_S \equiv$ price received by producers

2. Quantity (unit) tax: fixed tax $t$ per unit

   - Tax wedge: $P_D = P_S + t$

   - Market equilibrium:

     \[
     D(P_D) = S(P_S)
     \]

     ▶ Both sides split tax:

     \[
     Q \downarrow, \quad P_S \leq P_0 \leq P_D
     \]

   - Graphic interpretation:

     (Consumer) Supply $S \uparrow$ by $t$, equilibrium is $(P_D, Q)$ $\square$ Fig. 1a
     (Producer) Demand $D \downarrow$ by $t$, equilibrium is $(P_S, Q)$ $\square$ Fig. 1b

   - Tax incidence is independent of whom tax is levied on

     ▶ Only size (tax wedge) matters
Fig 1
Quantity tax

Fig 1a
Consumer viewpoint

Fig 1b
Producer viewpoint
• Demand/supply elasticity: less burden for more elastic agent

- $E^S$ perfectly inelastic (vertical): tax all on seller\(^1\)  \[\text{Fig. 2a}\]
- $E^S$ perfectly elastic (horizontal): tax all on buyer  \[\text{Fig. 2b}\]

▷ Bargaining: 《不賣走人》v. 《不買拉倒》

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\(^1\)For example: antiques, closing sale.
3. **Price (ad valorem) tax**: fixed proportion $\tau$ of price

- **Calculation:**
  \[
  p_S[1 + \tau] = p_D, \quad \tau = \frac{p_D - p_S}{p_S}
  \]

- **Pricing**: tax-exclusive (USA) v. tax-inclusive (Taiwan)

- **Equilibrium**: $D \downarrow$ or $S \uparrow$ proportionally
  \[
  P_D = [1 + \tau]P_S
  \]
  \[
  P_S \leq P_0 \leq P_D, \quad Q \downarrow
  \]

▷ Both sides split tax!

- Incidence independent of whom tax is levied on.
- Elasticity: less burden for more elastic agent.
- Tax evasion: under-reporting transaction price

  - [E] 3% charge by Yahoo auction
  - [E] Used-car sales tax
4. Equivalence of $t$ and $\tau$:

- Identical effects:
  - Government tax revenues (R)
  - Market surplus: CS/PS/DWL

- From $\tau$ to $t$:
  \[ P_D = (1 + \tau)P_S \Rightarrow (P_S, P_D) \Rightarrow t = P_D - P_s \]

- From $t$ to $\tau$:
  \[ P_D = P_S + t \Rightarrow (P_S, P_D) \Rightarrow \tau = \frac{P_D}{P_s} - 1 \]
2.2. Commodity Tax (Monopoly)

- Firm production cost: \( C(D) \)
  Demand: \( D(P_D) \)

- Monopolist goal:
  \[
  \max_D P_S(D)D - C(D)
  \]

- Quantity tax: \( P_D = P_S + t \)
  \[
  \max_D \quad P_S(D)D - C(D) \\
  = [P_D(D) - t]D - C(D) \\
  = P_D(D)D - [C(D) + tD]
  \]

foc: \( MR(D)=MC(D) \)

\[
P_D(D) + P_D'(D)D = C'(D) + t
\]

▷ Same effect as firm MC ↑ by \( t \)
• Price tax: \( P_D = P_S[1 + \tau] \)

\[
\max_D \frac{P_D(D)D}{1 + \tau} - C(D) \sim \max_D P_D(D)D - [1 + \tau]C(D)
\]

▷ Same effect as firm MC ↑ by proportion \(1 + \tau\).

• Equilibrium: \( P_S < P_0 < P_D \), quantity \( Q \downarrow \), firm profit \( \pi \downarrow \)

▷ Monopolist bears some tax!

• Constant MC case: depend on slope/shape of demand

  – Linear demand: \( D = a - bP_D \)

  \[
  \Delta P_D = \frac{t}{2}
  \]

  – Constant elasticity demand: \( P_D = \mu MR (\mu > 1) \)

  \[
  \Delta P_D > t
  \]

• Comparing \( t \) and \( \tau \): price tax is superior\(^2\)

  – Government revenues fixed, market quantity \( Q \) is higher.

  \[
  R_\tau = R_t \Rightarrow Q_\tau > Q_t
  \]

  – With lowered market \( Q \) fixed, government revenues higher.

  \[
  Q_\tau = Q_t \Rightarrow R_\tau > R_t
  \]

2.3. Tax on Production Factors

1. Payroll tax (工資課稅)
   - Labor demand $D \downarrow$, or labor supply $S \uparrow$
   - Statutory distinctn b/w boss and worker irrelevant!
   - Inelastic labor supply: workers bear all tax!
   - Backward-bending labor supply: $\Delta w > t$

2. Capital Tax (資本課税)
   - Closed economy: same analysis as before
   - Open economy: capital mobile, $S$ perfectly elastic
     - Capital demanders bear all tax!
2.4. Profit Tax

- Short-run:

\[
\max_Q \pi' = [1 - \tau] \pi
\]

▷ (P, Q) same, firms bear whole tax.

- Long-run:

  - Competition: firms may enter/exit
    ▷ LR firm \( \pi = 0 \), zero tax revenue
  
  - Monopoly: same as SR, post-tax profit reduced

\[
\max_Q \pi' = [1 - \tau] \pi
\]

▷ Monopolist bears full burden, no tax shifting
2.5. Land Tax

- Commodity properties:
  - Durable: rent stream $R_t \ (t = 0, 1, 2, \ldots, T)$
  - Fixed in supply
  - Competitive market

- Tax stream $\tau_t \ (t = 0, 1, 2, \ldots, T)$:
  - Pre-tax market value:
    $$V = \sum_{t=0}^{T} \frac{R_t}{[1 + r]^t}$$
  - Post-tax market value (buyer WTP):
    $$V' = \sum_{t=0}^{T} \frac{R_t - \tau_t}{[1 + r]^t}$$
  - Land price fall:
    $$\Delta V = V - V' = \sum_{t=0}^{T} \frac{\tau_t}{[1 + r]^t}$$

- Current landowners bear full tax
  Future landlords bear none

- Land tax capitalization
3. General Equilibrium Analysis

- Analyze both “taxed” and “affected” markets:
  - Taxed market: price/quantity change
  - Affected markets:
    1. Output substitution effect
    2. Input mobility effect

- Example: input mobility between 2 sectors \((H, M)\)
  - Tax on capital \((K^H)\) in housing \((H)\): \(K^H \downarrow, \ r^H_D > r^H_S\)
  - Capital flows from \(H\) to \(M\):
    \[K^M_S = K - K^H_S \uparrow\]
    - Overall capital return goes down: \(r^M_D = r^M_S \downarrow\)
    \[r^H_D - t = r^H_S = r^M_S = r^M_D\]
    - All capital suppliers bear tax burden!

\[\begin{array}{c}
\text{(Housing)} \\
\text{(Manufacturing)}
\end{array}\]

\[\begin{array}{c}
\text{S} \\
\text{D} \\
\text{S'} \\
\text{D}
\end{array}\]

- 2 inputs: capital \((K)\), labor \((L)\)

- 2 goods/sectors:
  - Food: \(F(K, L)\)
  - Manufacturing: \(M(K, L)\)

- 9 possible tax choices:
  - Income tax: \(t_I\)
  - Commodity tax: \(t^F, t^M\)
  - Production factor tax: \(t_K, t_L\)
  - Partial production factor tax: \(t^F_K, t^F_L, t^M_K, t^M_L\)

3.2. Assumptions

- Production factors: \((K, L)\)
  - both fixed in supply
  - perfectly mobile across sectors \(F\) and \(M\)

- Competitive markets: \((K, L)\) fully utilized and paid at MP

\[
r = MP_K, \quad w = MP_L
\]

- Homogeneous consumers: identical preferences

- CRTS production technology: different \(K/L\)-ratio
  - capital-intensive v. labor-intensive
3.3. Need to know:

- Output: changes in price/quantity

\[ (F, P_D^F \geq P_S^F) \]
\[ (M, P_D^M \geq P_S^M) \]

- Input: \( K \) and \( L \) fixed

\[ r_D^F \geq r_S^F = r_S^M \leq r_D^M \]
\[ w_D^F \geq w_S^F = w_S^M \leq w_D^M \]
3.4. **Income tax** \( t_I \)

- No shifting
- Consumers bear all burden!

3.5. **Labor tax** \( t_L \)

- Total supply fixed
- No shifting; workers bear all burden!

3.6. **Commodity tax on Food** \( t^F \)

- Consider both “output” and “input”

- Direct market effect:

  \[
  F \downarrow, \quad P_S^F \downarrow, \quad P_D^F \uparrow
  \]
• Output substitution:\(^3\)

\[ M \uparrow, \quad P_S^M = P_D^M \uparrow \]

• Input mobility: some \((L_F, K_F)\) released from \(F\), and flow to \(M\)
  
  – Capital return \(r \downarrow\) (if \(F\) is more K-intensive)
  
  – Labor return \(w \downarrow\) (if \(F\) is more L-intensive)

• Tax on L-intensive \(F\) lowers relative price of major input used in \(F\):

\[
\begin{align*}
  r_D^F &= r_S^F = r_M^M = r_D^M \uparrow \\
  w_D^F &= w_S^F = w_M^M = w_D^M \downarrow
\end{align*}
\]

\(^3\)Equilibrium conditions include:

– Market clearing:

\[
\begin{align*}
  D_F(P_D^F, P_D^M, I) &= S_F(P_S^F) \\
  D_M(P_D^F, P_D^M, I) &= S_M(P_S^M)
\end{align*}
\]

– Price equations:

\[
\begin{align*}
  P_D^F &= P_S^F + t \\
  P_D^M &= P_S^M
\end{align*}
\]
3.7. Partial production factor tax $t_K^M$

- Consider both “input” and “output”

- Direct effect:

$$r_S^M \downarrow, \quad r_D^M \uparrow$$

- (Input mobility) $K$ in $M$ flows to $F$: capital return lowered

$$r_S^F = r_D^F = r_S^M \downarrow$$

- (Output substitution)

$$P_M \uparrow, \quad M \downarrow$$

$$P_F \uparrow, \quad F \uparrow$$
• (Input mobility, again)
  
  – $M$ more K-intensive: $r \downarrow$ ⇒ Combined effect $r \downarrow \downarrow$

\[
  r_D^M > r_S^M = r_S^F = r_D^F \downarrow
\]

– $M$ more L-intensive: $r \uparrow$ ⇒ Combined effect $r$ ?

• Analysis steps:
  
  – Output:

\[
(F, P_D^F, P_S^F)
\]

\[
(M, P_D^M, P_S^M)
\]

  – Input: $K$ and $L$ fixed

\[
r_D^F \geq r_S^F = r_S^M \leq r_D^M
\]

\[
w_D^F \geq w_S^F = w_S^M \leq w_D^M
\]
3.8. Modification of Harberger Model

• Non-homogeneous consumers:

\[ \text{Laborers consume more F} \]

- Labor tax \( t_L \), born entirely by laborers
  \[ \Rightarrow \text{Demand for } F \downarrow \]
  \[ \Rightarrow \text{Some } (K, L) \text{ flows away from } F \text{ to } M \]
  \[ \Rightarrow r \downarrow (\because F \text{ more K-intensive}) \]
  \[ \Rightarrow \text{Capital bears some tax burden.} \]

• Factor immobility between sectors:

  - Factors taxed bear whole burden, no escape by migrating
  - Factor prices \((r,w)\) unchanged in untaxed sectors

• LR factor supply change:

\[ \text{Capital tax: } t_K \]

\[ \Rightarrow \text{LR supply } K \downarrow \]
\[ \Rightarrow \text{Economy K/L-ratio } \downarrow, \text{ wage } w \downarrow \]
\[ \Rightarrow \text{Workers hurt!} \]