Barter and Monetary Exchange under private Information

Williamson and Wright (1994 AER)

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Introduction

- Qualitative uncertainty and its impediments on exchange is used to motivate the role of a medium of exchange.

- Money
  - solves the double-coincident-of-wants-problem
  - reduces the information cost (e.g. Alchian 1977, Brunner and Meltzer 1971)

Adding in private info and see how the info frictions affect production decision (moral hazard problem) and trading strategy.

Study how money mitigates the private info problem to improve welfare.
Model: Production and preferences

• a continuum of infinitely lived agents $\in [0, 1]$

• one good \( \begin{cases} 
H: \text{production cost } \gamma, \text{utility } u. \\
L: \text{production cost } 0, \text{utility } 0.
\end{cases} \)

• consuming one’s own good: \( u = 0 \)
  $\rightarrow$ generate motive for trade.
Model: money and trade

- All objects are indivisible, freely disposable and storable at zero cost but only one unit of good or money at a time.
- One-for-one swap of inventories. No bargaining.
Model: Information

- Money is always identifiable.
- \( \text{Prob(} \text{an agent can recognize the quality of a commodity trader’s good)} = \theta \leq 1 \), independent across agents.
- Trading history and the fact if other agents recognize my good is private information.
- When two agents meet, they simply inspect each others’ inventories and simultaneously announce whether or not to trade.
Equilibrium

- Agents choose production, consumption, disposal, and trading strategies to max expected discounted utility of consumption net of production costs.

- Stationary Nash equilibrium where the probability of meeting agents and strategies are time invariant and expectation are rational.
Complete Information $\theta = 1$

- $P$: fraction of commodity traders holding $H$.
- Let $W = \max(V_g - \gamma, V_b)$.
- For a nondegenerate equilibrium, $V_g - \gamma \geq V_b \iff P > 0$.
- Suppose a nondegenerate eqm exists, $V_g - \gamma > 0$
  $\Rightarrow$ agents trade with $H$ but reject $L$.
  $\Rightarrow$ $L$ cannot trade and thus is never produced.
  $\Rightarrow P = 1$ in equilibrium.
Complete Information $\theta = 1$

- To check this is an equilibrium, use unimprovability criterion (agents can not improve payoff by a one-stage deviation)

\[
\begin{align*}
    rV_g &= u - \gamma \\
    rV_b &= V_g - \gamma - V_b
\end{align*}
\]

\[\Rightarrow V_g - \gamma \geq V_b\]

iff

\[\frac{u}{1 + r} \geq \gamma\]
Complete Information $\theta = 1$

- When others are producing only $H$, the indiv’s strategy of producing only $H$ is unimprovable (a best response) iff
  \[ \frac{u}{1+r} \geq \gamma. \]

Now, adding in money ...

- Whenever a nondegenerate monetary eqm exists, so does the nondegenerate nonmonetary eqm, but the latter is Pareto superior.
Private Info $\theta < 1$: strategies

- Consider Nonmonetary eqm.

- $\Sigma$: prob. a random agent with $H$ accepts unrecognized good
  $\sigma$: individual’s best response.

- Always trade $H$, always reject $L$.
  $\rightarrow$ the relevant decision: $(\Sigma, P)$

- In equilibrium, $\Sigma = \sigma$

- $\Sigma > 0$ in a nondegenerate eqm.
Private Info $\theta < 1$: Bellman equations

\[
\begin{align*}
rV_g &= \theta P[\theta + (1 - \theta)\Sigma](u + W - V_g) \\
&\quad + (1 - \theta) \max_{\sigma} \{ P[\theta + (1 - \theta)\Sigma](u + W - V_g) \\
&\quad + (1 - P)(W - V_g) \} \\
\end{align*}
\]

\[
\begin{align*}
rV_b &= P(1 - \theta)\Sigma(u + W - V_b).
\end{align*}
\]
Equilibrium $a$: $P = \Sigma = 1$

- need to check
  \[ V_b - \gamma \geq V_b; \quad P[\theta + (1 - \theta)\Sigma](u - \gamma) + (1 - P)(-\gamma) \geq 0 \]
- $P = \Sigma = 1 \iff \theta > \theta_1$
Equilibrium $b$: $P \in (0, 1), \Sigma = 1$

- $V_b - \gamma = V_b \sim$ solve for $P$
  then check the condition $0 < P < 1 \iff \theta > \theta_1$
- $\sigma = 1$ is a best response iff $Pu - \gamma \geq 0 \Rightarrow \theta < \theta_2$
Equilibrium \( c: \ P \in (0, 1), \Sigma \in (0, 1) \)

- \( P \in (0, 1), \Sigma \in (0, 1) \iff \theta_3 < \theta < \theta_2 \)
Results: existence of Equilibria

- \( \theta \) close 1 \( \sim \) “the first best.”
- \( \theta \) close 0 \( \sim \) No nondegenerate equilibrium exists.
- \( \exists \theta \) s.t. multiple equilibria exist.
- When \( \theta \) is low, eqm c has the greatest chance of surviving because \( \Sigma < 1 \) imposes the greatest discipline.
Results: welfare

- Equilibria are Pareto rankable.
  Welfare $z_j$: $z_c < z_b < z_a$

- The multiplicity of Pareto-ranked eqm is due to the strategic complementarity:

\[
\begin{align*}
(1) & \quad P \uparrow \quad \rightarrow \quad \text{value of producing } H \uparrow > L \uparrow \quad \rightarrow \quad P \uparrow. \\
(2) & \quad P \uparrow \quad \rightarrow \quad \Sigma \uparrow \rightarrow \quad P \downarrow
\end{align*}
\]

when $\theta$ is in the proper range, $(1) \succ (2) \Rightarrow$ multiplicity.
Private Information: monetary equilibrium

- $P = 1$: whenever a monetary eqm exists, there also exists a nonmonetary eqm which yields greater welfare.
- If money is to have a welfare-improving role, it can not completely alleviate the private information problem by driving out all $L$. 
Monetary equilibrium: strategies

- $\Omega$: prob. a random money trader accepts a unrecognized good
- $\omega$: individual’s best response.

- in equilibrium $\Omega = \omega$
Equilibrium \((\Sigma, \Omega) = (0, \phi)\)

- Check the best response conditions (27)-(30), satisfied as long as \(\gamma\) is not too big \(\forall M \in (\underline{M}, \overline{M})\).

- Low \(\theta \Rightarrow\)
  \(\text{Eqm } (\Sigma, \Omega) = (0, \phi)\) is the unique monetary eqm, which Pareto dominates the only degenerate nonmonetary eqm.
Equilibrium \((\Sigma, \Omega) = (0, \phi)\)

- \((\Sigma, \Omega) = (0, \phi)\) imposes the greatest amount of discipline on the producers of \(L\).

- \(\Sigma = 0 \implies L\) can never trade with \(H\) directly
  \(L \rightarrow\) Money \(\rightarrow\) \(H\).

- CIA constraint on \(L\) producers \(\rightarrow\) socially desirable.
How is \((\Sigma, \Omega) = (0, \phi)\) also individual optimal?

- \(\text{prob(money trades for } H) = P\)

- \(\text{prob(commodity trades for a unrecognized good which is } H) = p[\theta + (1 - \theta)\Sigma]\)

- Because money is always recognizable, you have a higher chance of getting \(H\) when you offer money rather than commodities.