Search, Bargaining, Money and Prices

Trejos and Wright (1995 *JPE*)

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Motivation

- Introduce bargaining into Kiyotaki and Wright (1991, 1993) in order to determine the price level endogenously.
Model: agents and meeting technology

- $M \in (0, 1)$: buyers; indivisible money
  $1 - M$: sellers; divisible good
- Agents do not consume their own output.
- Traders meet according to Poisson arrival rates that are proportional to the number of agents on the other side of market.
- $x = \text{prob. (the trade partner likes my output)}$, but both can produce each others’ consumption goods with prob. 0.
  $\Rightarrow$ No double coincidence of wants, so all trades involve a buyer paying with cash.
Model: production and preferences

• $u(q)$: utility of consuming $q$ units of good
  $c(q)$: disutility of producing $q$ units of good

• Assume

\[
\begin{align*}
  u(0) &= c(0), \quad u'(0) > c'(0) = 0 \\
  u'(q) &> 0, \quad c'(q) > 0 \\
  u'' &\leq 0, \quad c'' \geq 0 \text{ for } q > 0 \\
\end{align*}
\]

with at least one of the weak inequalities strict.

• $\exists \, \hat{q} > 0$ s.t. $u(\hat{q}) = c(\hat{q})$
Monetary theory

• If $q = Q$ is taken as given, then

\[ rV_s = \alpha Mx[V_b - V_s - c(Q)] \]  \hspace{1cm} (1)

\[ rV_b = \alpha (1 - M)x[u(Q) + V_s - V_b] \]  \hspace{1cm} (2)

can be solved for $V_b = V_b(Q)$ and $V_s = V_s(Q)$

• In the following, normalize $\alpha x = 1$. 
Price theory

Strategic bargaining game:

\[
\begin{array}{c}
1/2 \\
\uparrow \\
\downarrow \\
\text{accept} \\
\text{reject} \\
\uparrow \\
\text{walk away (threat)} \\
\downarrow \\
\text{wait } \triangle \\
\downarrow \\
\text{(no outside option)}
\end{array}
\]
Strategic bargaining game

- A unique subgame perfect equilibrium to the bargaining game ($V_b$ and $V_s$ taken as given).
- This equilibrium has the property that a seller always proposes $q_s = q_s(\Delta)$, a buyer always proposes $q_b = q_b(\Delta)$ and these proposals are always accepted.
- As $\Delta \to 0$, $q_s \to q$ and $q_b \to q$ where $q$ is a Nash bargaining solution.
Bargaining solution

\[ q_s : \quad V_s + u(q_s) = \frac{1}{1 + r\Delta} [V_s + \frac{1}{2} u(q_s) + \frac{1}{2} u(q_b)] \]

\[ q_b : \quad V_b - c(q_b) = \frac{1}{1 + r\Delta} [V_b - \frac{1}{2} c(q_s) - \frac{1}{2} c(q_b)] \]

As \( \Delta \to 0 \), \( q_s \to q \), and \( q_b \to q \); i.e.

\[ \frac{V_s + u(q)}{V_b - c(q)} = \frac{u'(q)}{c'(q)} \]

The solution to the symmetric Nash bargaining game without threat points:

\[ q = \arg \max [u(q) + V_s(q)][V_b(q) - c(q)] \]
Bargaining solution with outside options

\[ q = \arg \max \left[ V_s + u(q) - V_b \right] \left[ V_b - c(q) - V_s \right] \]
Definition of equilibrium

- When bargaining over $q$, agents take $V_b(Q)$ and $V_s(Q)$ as given. In eqm, $q = Q$.
- A steady-state equilibrium is a list $(Q, V_s, V_b)$ satisfying:
  (i) $q = Q$ solves the bargaining problem

$$q = \arg \max \left[ u(q) + V_s(q) \right] \left[ V_b(q) - c(q) \right]$$

s.t. $u(q) + V_s(Q) \geq V_b(Q)$ and $V_b(Q) - c(q) \geq V_s(Q)$

participation conditions

taking $V_b(Q)$ and $V_s(Q)$ as given; and (ii) $V_s$ and $V_b$ satisfy (1) and (2), taking $Q$ as given.
Equilibrium

- Taking $V_b = V_b(Q)$ and $V_s = V_s(Q)$ as given, FOC:

$$\frac{c'(q)}{u'(q)} = \frac{V_b(Q) - c(q)}{u(q) + V_s(Q)}$$

- Eq. (3) defines a function $q = e(Q)$ [or, $T(q) = 0$] goes though the origin and intersect the 45$^\circ$ line at a unique point $q^e$

- Show $T(0) = 0$, $T'(0) > 0$, $T(q_1) < 0$. By continuity, there exists a $q \in (0, q_1)$ s.t. $T(q) = 0$
Proposition 1

For any $r > 0$ and $M \in (0, 1)$, there exists a nonmonetary steady state equilibrium and a unique monetary steady state equilibrium. The monetary equilibrium is unconstrained, and it satisfies $u'(q) > c'(q)$. 
• CE: $u'(q^*) = c'(q^*)$

• ex ante welfare criterion:

$$W = MV_b + (1 - M) V_s$$
$$= M(1 - M)[u(q) - c(q)]$$

$$u'(q^e) > c'(q^e) \Rightarrow q^e < q^*$$
Monetary equilibrium is socially inefficient

- Uniqueness of monetary equilibrium (due to assumption of no barter)

- \( u'(q) > c'(q) \Rightarrow q < q^* \)
  monetary equilibrium is socially inefficient.

- Note that \( q \) is bilaterally efficient.
  → It is on the frontier of the payoff space in the bargaining game, given \( V_b, V_s \).
Pareto inefficiency

- Monetary equilibrium is inefficient according to the ex post Pareto criterion.

- Let $q_s^*, q_b^*$ denote the value of $q$, if imposed on all agents, maximize $V_s$ and $V_b$.

- Any $q \in [q_s^*, q_b^*]$ is Pareto efficient.
  - $q < q_s^*$ in equilibrium.
  - Even sellers would prefer a lower price (higher $q$) if that price could be imposed on all agents.
Other results of Proposition 1

• $q \to q^*$ as $r \to 0$, $q \to q^*$ as $\beta \to \infty$
  $\frac{\partial q}{\partial r} < 0$ and $\frac{\partial q}{\partial \beta} > 0$

• $\exists \mu = \mu(r)$ s.t. $\frac{\partial q}{\partial M} \leq 0$ iff $M \geq \mu$, $\mu < \frac{1}{2}$