On Money as a Medium of Exchange

Kiyotaki and Wright (1989 JPE)

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Motivations

- Capture the essential feature of money – a medium of exchange.
- Money emerges endogenously.
- Which goods are used as money depends on intrinsic properties (storability, recognizability...etc ) and extrinsic belief.
- How fiat money has value (extrinsic belief, preference, technology).
Model: production and consumption

- Main features: Specialization (in consumption and production) → a Double-coincidence-of-wants problem.
- A continuum of agents with measure 1
- 3 type of agents, 3 goods:
  Agent $i$ consumes good $i$ (utility $u_i$) and produces $i + 1 \pmod{3}$
Model: storability of goods

• Goods are indivisible, come in size one and storable at a cost $c_i$.

• Each agent can store only one unit of good at a time.

• Assume $c_3 > c_2 > c_1 \geq 0$: focus on storability
Model: matching and inventory distribution

- Meeting technology: bilateral meeting per period
- $P_{ij}(t)$: proportion of type $i$ with good $j$ at date $t$
  
  \[ P_{ii}(t) = 0 \quad \forall t \quad , \quad P_{ij} = 1 - P_{i,j+1} \]

- $P(t) = [P_{12}(t), P_{23}(t), P_{31}(t)]$ is all we need to know.
Trading strategy

- Each agent chooses a trading strategy to maximize his expected discounted utility from consumption net of production and storage costs, taking as given the strategies of other agents and $P(t)$.

- Environment is time-invariant, infinite horizon, steady state $\rightarrow P(t) = P, \forall t$, and we consider strategies for $i$ that depend on the good $j$ he has and the good $k$ his trading partner has.

- Trading strategy:

$$\tau_i(j, k) = \begin{cases} 
1 & \text{if } i \text{ trades } j \text{ for } k \\
0 & \text{otherwise}
\end{cases}$$
• $\tau_{j,i} = 1$: always trade for consumption good as long as $u_i$ is big enough.

• $\tau_{j,k} = 1 \Rightarrow \tau_{k,j} = 0$

• The only aspect of trading strategies to determine is whether type $i$ trades his production good $i + 1$ for good $i + 2$ as a medium of exchange.

• Agents trade iff it is mutually agreeable: 
$\tau_i(j, k) \cdot \tau_h(k, j) = 1$
Trading strategy (con’t)

- $V_{ij}$ - expected life-time discounted utility for type $i$ with good $j$ given that he follows a maximizing strategy.

- $\tau_i(j, k) = 1$ iff $V_{ik} > V_{ij}$.
  In equilibrium, agents of the same type will not trade.
Pure strategy profile: $S = (S_1, S_2, S_3)$

- Let $S_i = 1$ if $i$ trades good $i + 1$ for good $i + 2$, and 0, otherwise.

- A pure strategy profile is $S = (S_1, S_2, S_3)$.

- e.g. $S_2 = \Pr(\text{agent II trades 3 for 1})$  
  $\Rightarrow 1 - S_2 = \Pr(\text{agent II trades 1 for 3})$

- $\Delta_i \equiv V_{i,i+1} - V_{i,i+2} > 0 \iff S_i = 0$
Steady state inventory distribution

Let \( P_1 = P_{12}, P_2 = P_{23}, P_3 = P_{31}. \)

\[
\begin{align*}
&\Rightarrow \left\{ \\
&P_1 P_2 S_1 = (1 - P_1) P_3 \\
&P_2 P_3 S_2 = (1 - P_2) P_1 \\
&P_3 P_1 S_3 = (1 - P_3) P_2
\end{align*}
\]

solve for \( P^* = (P_1^*, P_2^*, P_3^*) \)
Definition of equilibrium

Definition

A steady state Nash equilibrium consists of \( \tau = (\tau_1, \tau_2, \tau_3) \) and \( P \) satisfying

- **Maximization**: Given other’s strategies and \( P, \tau_i \) maximizes expected utility for type \( i \).

- **Rational expectation**: Given \( \tau \), \( P \) is the resulting steady state inventory distribution.
Fundamental equilibrium: \( S = (0, 1, 0) \)

Given others’ strategies and \( P \),

\[
V_{12} = \left( \frac{1}{1 + r} \right) \{-C_2 + \frac{1}{3} [V_{12} + P_{21}(u_1 + V_{12}) + P_{23} \max(V_{12}, V_{13}) + V_{12}] \}
\]

\[
V_{13} = \left( \frac{1}{1 + r} \right) \{-C_3 + \frac{1}{3} [V_{13} + V_{13} + P_{31}(u_1 + V_{12}) + P_{32} \max(V_{12}, V_{13})] \}
\]
Fundamental equilibrium: \( S = (0, 1, 0) \)

\[
\begin{align*}
rV_{12} &= -C_2 + \frac{1}{3}[P_{21}u_1 + P_{23} \max(0, V_{13} - V_{12})] \\
rV_{13} &= -C_3 + \frac{1}{3}[P_{31}(u_1 + V_{12} - V_{13}) + P_{32} \max(V_{12} - V_{13}, 0)]
\end{align*}
\]

\[\Rightarrow C_3 - C_2 > \frac{1}{3}[P_{31} - P_{21}]u_1 \text{ then } V_{12} > V_{13}\]

\( V_{21} > V_{23} \) and \( V_{31} > V_{32} \) for all parameter values and \( P_{i,j} \).

Inventory distribution = \((1, \frac{1}{2}, 1)\)
Speculative equilibrium: \( S = (1, 1, 0) \)

\[ V_{13} > V_{12} \text{ iff } \]

\[ C_3 - C_2 < \frac{1}{3} \left( P_{31} - P_{21} \right) u_1 \]

\[ \sqrt{2} - 1 \]

⇒ \[ \begin{align*}
P_{21} &= 2 - \sqrt{2} \\
P_{12} &= \frac{\sqrt{2}}{2}
\end{align*} \]