CHAPTER 24
WARRANTS AND CONVERTIBLES

Answers to Concepts Review and Critical Thinking Questions

1. When a warrant is issued by the company, and when a warrant is exercised, the number of shares increases. A call option is a contract between investors and does not affect the number of shares of the firm.

2. a. If the stock price is less than the exercise price of the warrant at expiration, the warrant is worthless. Prior to expiration, however, the warrant will have value as long as there is some probability that the stock price will rise above the exercise price in the time remaining until expiration. Therefore, if the stock price is below the exercise price of the warrant, the lower bound on the price of the warrant is zero.

b. If the stock price is above the exercise price of the warrant, the warrant must be worth at least the difference between these two prices. If warrants were selling for less than the difference between the current stock price and the exercise price, an investor could earn an arbitrage profit (i.e. an immediate cash inflow) by purchasing warrants, exercising them immediately, and selling the stock.

c. If the warrant is selling for more than the stock, it would be cheaper to purchase the stock than to purchase the warrant, which gives its owner the right to buy the stock. Therefore, an upper bound on the price of any warrant is the firm’s current stock price.

3. An increase in the stock price volatility increases the bond price. If the stock price becomes more volatile, the conversion option on the stock becomes more valuable.

4. The two components of the value of a convertible bond are the straight bond value and the option value. An increase in interest rates decreases the straight value component of the convertible bond. Conversely, an increase in interest rates increases the value of the equity call option. Generally, the effect on the straight bond value will be much greater, so we would expect the bond value to fall, although not as much as the decrease in a comparable straight bond.

5. When warrants are exercised, however, the number of shares outstanding increases. This results in the value of the firm being spread out over a larger number of shares, often leading to a decrease in value of each individual share. The decrease in the per-share price of a company’s stock due to a greater number of shares outstanding is known as dilution.

6. In an efficient capital market the difference between the market value of a convertible bond and the value of straight bond is the fair price investors pay for the call option that the convertible or the warrant provides.

7. There are three potential reasons: 1) To match cash flows, that is, they issue securities whose cash flows match those of the firm. 2) To bypass assessing the risk of the company (risk synergy). For example, the risk of company start-ups is hard to evaluate. 3) To reduce agency costs associated with raising money by providing a package that reduces bondholder-stockholder conflicts.
8. Because the holder of the convertible has the option to wait and perhaps do better than what is implied by current stock prices.

9. Theoretically conversion should be forced as soon as the conversion value reaches the call price because other conversion policies will reduce shareholder value. If conversion is forced when conversion values are above the call price, bondholders will be allowed to exchange less valuable bonds for more valuable common stock. In the opposite situation, shareholders are giving bondholders the excess value.

10. No, the market price of the warrant will not equal zero. Since there is a chance that the market price of the stock will rise above the $21 per share exercise price before expiration, the warrant still has some value. Its market price will be greater than zero. As a practical matter, warrants that are far out-of-the-money may sell at 0, due to transaction costs.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The conversion price is the par value divided by the conversion ratio, or:
   
   \[
   \text{Conversion price} = \frac{\text{par value}}{\text{conversion ratio}} = \frac{\$1,000}{23.2} = \$43.10
   \]

2. The conversion ratio is the par value divided by the conversion price, or:
   
   \[
   \text{Conversion ratio} = \frac{\text{par value}}{\text{conversion price}} = \frac{\$1,000}{\$74.25} = 13.47
   \]

3. The conversion value is the number of shares that the bond can be converted to times the stock price. So, the conversion value for this bond is:
   
   \[
   \text{Conversion value} = \times \text{number of shares} \times \text{stock price} = 62(10.5) = \$651
   \]

4. First, we need to find the conversion price, which is the par value divided by the conversion ratio, or:
   
   \[
   \text{Conversion price} = \frac{\text{par value}}{\text{conversion ratio}} = \frac{\$1,000}{14.50} = \$68.97
   \]

   The conversion premium is the necessary increase in stock price to make the bond convertible. So, the conversion premium is:
   
   \[
   \text{Conversion premium} = \frac{(\$68.97 - 46.24)}{\$46.24} = 0.4915 \text{ or } 49.15\%
   \]
5. a. The conversion ratio is defined as the number of shares that will be issued upon conversion. Since each bond is convertible into 24.25 shares of Hannon’s common stock, the conversion ratio of the convertible bonds is 24.25.

b. The conversion price is defined as the face amount of a convertible bond that the holder must surrender in order to receive a single share. Since the conversion ratio indicates that each bond is convertible into 24.25 shares, the conversion price is:

\[
\text{Conversion price} = \frac{1,000}{24.25} = 41.24
\]

c. The conversion premium is defined as the percentage difference between the conversion price of the convertible bonds and the current stock price. So, the conversion premium is:

\[
\text{Conversion premium} = \frac{41.24 - 31.25}{31.25} = 0.3196 \text{ or } 31.96\%
\]

d. The conversion value is defined as the amount that each convertible bond would be worth if it were immediately converted into common stock. So, the conversion value is:

\[
\text{Conversion value} = 31.25 \times 24.25 = 757.81
\]

e. If the stock price increases by 2, the new conversion value will be:

\[
\text{Conversion value} = 33.25 \times 24.25 = 806.31
\]

6. The total exercise price of each warrant is shares each warrant can purchase times the exercise price, which in this case will be:

\[
\text{Exercise price} = 3(32) = 96
\]

Since the shares of stock are selling at 38, the value of three shares is:

\[
\text{Value of shares} = 3(38) = 114
\]

Therefore, the warrant effectively gives its owner the right to buy 114 worth of stock for 96. It follows that the minimum value of the warrant is the difference between these numbers, or:

\[
\text{Minimum warrant value} = 114 - 96 = 18
\]

If the warrant were selling for less than 18, an investor could earn an arbitrage profit by purchasing the warrant, exercising it immediately, and selling the stock. Here, the warrant holder pays less than 18 while receiving the 18 difference between the price of three shares and the exercise price.
7. Since a convertible bond gives its holder the right to a fixed payment plus the right to convert, it must be worth at least as much as its straight value. Therefore, if the market value of a convertible bond is less than its straight value, there is an opportunity to make an arbitrage profit by purchasing the bond and holding it until expiration. In Scenario A, the market value of the convertible bond is $1,000. Since this amount is greater than the convertible’s straight value ($900), Scenario A is feasible. In Scenario B, the market value of the convertible bond is $900. Since this amount is less than the convertible’s straight value ($950), Scenario B is not feasible.

8. a. Using the conversion price, we can determine the conversion ratio, which is:

\[
\text{Conversion ratio} = \frac{\text{Value of Bond}}{\text{Conversion Price}} = \frac{1,000}{25} = 40
\]

So, each bond can be exchanged for 40 shares of stock. This means the conversion price of the bond is:

\[
\text{Conversion price} = 40 \times 21 = 840
\]

Therefore, the minimum price the bond should sell for is $840. Since the bond price is higher than this price, the bond is selling at the straight value, plus a premium for the conversion feature.

b. A convertible bond gives its owner the right to convert his bond into a fixed number of shares. The market price of a convertible bond includes a premium over the value of immediate conversion that accounts for the possibility of increases in the price of the firm’s stock before the maturity of the bond. If the stock price rises, a convertible bondholder will convert and receive valuable shares of equity. If the stock price decreases, the convertible bondholder holds the bond and retains his right to a fixed interest and principal payments.

9. You can convert or tender the bond (i.e., surrender the bond in exchange for the call price). If you convert, you get stock worth 35 × $40 = $1,400. If you tender, you get $1,100 (110 percent of par). It’s a no-brainer: convert.

10. a. Since the stock price is currently below the exercise price of the warrant, the lower bound on the price of the warrant is zero. If there is only a small probability that the firm’s stock price will rise above the exercise price of the warrant, the warrant has little value. An upper bound on the price of the warrant is $33, the current price of the common stock. One would never pay more than $33 to receive the right to purchase a share of the company’s stock if the firm’s stock were only worth $33.

b. If the stock is trading for $39 per share, the lower bound on the price of the warrant is $4, the difference between the current stock price and the warrant’s exercise price. If warrants were selling for less than this amount, an investor could earn an arbitrage profit by purchasing warrants, exercising them immediately, and selling the stock. As always, the upper bound on the price of a warrant is the current stock price. In this case, one would never pay more than $39 for the right to buy a single share of stock when he could purchase a share outright for $39.
11. a. The minimum convertible bond value is the greater of the conversion price or the straight bond price. To find the conversion price of the bond, we need to determine the conversion ratio, which is:

\[
\text{Conversion ratio} = \frac{\text{₪1,000}}{\text{₪125}}
\]

Conversion ratio = 8

So, each bond can be exchanged for 8 shares of stock. This means the conversion price of the bond is:

\[
\text{Conversion price} = 8(\text{₪35})
\]

Conversion price = ₪280

And the straight bond value is:

\[
P = \text{₪35} \left( \frac{1 - \left[1/(1 + .06)\right]^{60}}{.06} \right) + \frac{\text{₪1,000}}{(1 + .06)^{60}}
\]

\[
P = \text{₪595.96}
\]

So, the minimum price of the bond is ₪595.96

c. If the stock price were growing by 15 percent per year forever, each share of its stock would be worth approximately ₪32(1.15)^t after t years. Since each bond is convertible into 8 shares, the conversion value of the bond equals (₪32)(8)(1.15)^t after t years. In order to calculate the number of years that it will take for the conversion value to equal ₪1,100, set up the following equation:

\[
(\text{₪32})(8)(1.15)^t = \text{₪1,100}
\]

\[
t = 10.43 \text{ years}
\]

12. a. The percentage of the company stock currently owned by the CEO is:

\[
\text{Percentage of stock} = \frac{500,000}{4,000,000}
\]

Percentage of stock = .1250 or 12.50%

b. The conversion price indicates that for every $25 of face value of convertible bonds outstanding, the company will be obligated to issue a new share upon conversion. So, the new number of shares the company must issue will be:

New shares issued = $20,000,000 / $25
New shares issued = 800,000

So, the new number of shares of company stock outstanding will be:

New total shares = 4,000,000 + 800,000
New total shares = 4,800,000
After the conversion, the percentage of company stock owned by the CEO will be:

New percentage of stock = 500,000 / 4,800,000
New percentage of stock = .1042 or 10.42%

13. a. Before the warrant was issued, the firm’s assets were worth:

Value of assets = 5 oz of platinum (€1,000 per oz)
Value of assets = €5,000

So, the price per share is:

Price per share = €5,000 / 3
Price per share = €1,666.67

b. When the warrant was issued, the firm received €1,000, increasing the total value of the firm’s assets to €6,000 (= €5,000 + 1,000). If the two shares of common stock were the only outstanding claims on the firm’s assets, each share would be worth €2,000 (= €6,000 / 3 shares). However, since the warrant gives warrant holder a claim on the firm’s assets worth €1,000, the value of the firm’s assets available to stockholders is only €5,000 (= €6,000 − 1,000). Since there are three shares outstanding, the value per share remains at €1,666.67 (= €5,000 / 3 shares) after the warrant issue. Note that the firm uses warrant price of €1,000 to purchase one more ounce of platinum.

c. If the price of platinum is €1,100 per ounce, the total value of the firm’s assets is €6,600 (= 6 oz of platinum × €1,100 per oz). If the warrant is not exercised, the value of the firm’s assets would remain at €6,600 and there would be three shares of common stock outstanding. If the warrant is exercised, the firm would receive the warrant’s €2,100 strike price and issue one share of stock. The total value of the firm’s assets would increase to €8,700 (= €6,600 + 2,100). Since there would now be 4 shares outstanding and no warrants, the price per share would be €2,175.00 (= €8,700 / 4 shares). Since the €2,200 value of the share that the warrant holder will receive is greater than the €2,100 exercise price of the warrant, investors will expect the warrant to be exercised. The firm’s stock price will reflect this information and will be priced at €2,175 per share on the warrant’s expiration date.

14. The value of the company’s assets is the combined value of the stock and the warrants. So, the value of the company’s assets before the warrants are exercised is:

Company value = 10,000,000($17) + 1,000,000($3)
Company value = $173,000,000

When the warrants are exercised, the value of the company will increase by the number of warrants times the exercise price, or:

Value increase = 1,000,000($15)
Value increase = $15,000,000
So, the new value of the company is:

New company value = $173,000,000 + 15,000,000
New company value = $188,000,000

This means the new stock price is:

New stock price = $188,000,000 / 11,000,000
New stock price = $17.09

Note that since the warrants were exercised when the price per warrant ($3) was above the exercise value of each warrant ($2 = $17 – 15), the stockholders gain and the warrant holders lose.

**Challenge**

15. The straight bond value today is:

Straight bond value = £68(PVIFA_{10\%,25}) + £1,000/1.10^{25}
Straight bond value = £709.53

And the conversion value of the bond today is:

Conversion value = £44.75(£1,000/£150)
Conversion value = £298.33

We expect the bond to be called when the conversion value increases to £1,300, so we need to find the number of periods it will take for the current conversion value to reach the expected value at which the bond will be converted. Doing so, we find:

\[ £298.33(1.12)^t = £1,300 \]
\[ t = 12.99 \text{ years.} \]

The bond will be called in 12.99 years.

The bond value is the present value of the expected cash flows. The cash flows will be the annual coupon payments plus the conversion price. The present value of these cash flows is:

Bond value = £68(PVIFA_{10\%,12.99}) + £1,300/1.10^{12.99} = £859.80

16. The value of a single warrant (W) equals:

\[ W = \left[ \frac{\#}{\# + \#_W} \right] \times \text{Call\{S = (V/\#), K = K_W\}} \]

where:

- \# = the number of shares of common stock outstanding
- \#_W = the number of warrants outstanding
- Call\{S, K\} = a call option on an underlying asset worth S with a strike price K
- V = the firm’s value net of debt
- K_W = the strike price of each warrant
Therefore, the value of a single warrant (W) equals:

\[
W = \left[ \frac{\#}{(\# + \#_W)} \right] \times \text{Call} \{ S = \frac{V}{\#}, K = K_w \} \\
= \left[ \frac{4,000,000}{(4,000,000 + 500,000)} \right] \times \text{Call} \{ S = \frac{\$88,000,000}{4,000,000}, K = \$20 \} \\
= (\frac{8}{9}) \times \text{Call} \{ S = 22, K = 20 \}
\]

In order to value the call option, use the Black-Scholes formula. Solving for \( d_1 \) and \( d_2 \), we find

\[
d_1 = \left[ \ln(S/K) + \left( R + \frac{1}{2}\sigma^2 \right)t \right] / (\sigma^2 t)^{1/2} \\
d_1 = \left[ \ln(22/20) + \{0.07 + \frac{1}{2}(0.04)\} \times (1) \right] / (0.04 \times 1)^{1/2} \\
d_1 = 0.9266
\]

\[
d_2 = d_1 - (\sigma^2 t)^{1/2} \\
d_2 = 0.9266 - (0.04 \times 1)^{1/2} \\
d_2 = 0.7266
\]

Next, we need to find \( N(d_1) \) and \( N(d_2) \), the area under the normal curve from negative infinity to \( d_1 \) and negative infinity to \( d_2 \), respectively.

\[
N(d_1) = N(0.9266) = 0.8229 \\
N(d_2) = N(0.7266) = 0.7663
\]

According to the Black-Scholes formula, the price of a European call option (C) on a non-dividend paying common stock is:

\[
C = SN(d_1) - Ke^{-Rt}N(d_2) \\
C = (22)(0.8229) - (20)e^{-0.07 \times 1} (0.7663) \\
C = 3.81
\]

Therefore, the price of a single warrant (W) equals:

\[
W = \left( \frac{8}{9} \right) \times \text{Call} \{ S = 22, K = 20 \} \\
W = (\frac{8}{9})(3.81) \\
W = 3.39
\]

17. To calculate the number of warrants that the company should issue in order to pay off Rs.10 million in six months, we can use the Black-Scholes model to find the price of a single warrant, then divide this amount into the present value of Rs.10 million to find the number of warrants to be issued. So, the value of the liability today is:

\[
\text{PV of liability} = \frac{10,000,000e^{-0.06 \times 1/12}}{e^{-0.06 \times 1/12}} \\
\text{PV of liability} = 9,704,455.34
\]

The company must raise this amount from the warrant issue.
The value of company’s assets will increase by the amount of the warrant issue after the issue, but this increase in value from the warrant issue is exactly offset by the bond issue. Since the cash inflow from the warrants offsets the firm’s debt, the value of the warrants will be exactly the same as if the cash from the warrants were used to immediately pay off the debt. We can use the market value of the company’s assets to find the current stock price, which is:

\[
\text{Stock price} = \frac{\text{Rs.}160,000,000}{1,500,000} = \text{Rs.}106.67
\]

The value of a single warrant (W) equals:

\[
W = \left[ \frac{#}{# + #W} \right] \times \text{Call(S, K)}
\]

\[
W = \left[ \frac{1,500,000}{1,500,000 + #W} \right] \times \text{Call(Rs.106.67, Rs.95)}
\]

Since the firm must raise Rs.9,704,455 as a result of the warrant issue, we know #W × W must equal Rs.9,704,455.

Therefore, it can be stated that:

\[
\text{Rs.9,704,455} = (#W)(W)
\]

\[
\text{Rs.9,704,455} = (#W)(\frac{1,500,000}{1,500,000 + #W}) \times \text{Call(Rs.106.67, Rs.95)}
\]

Using the Black-Scholes formula to value the warrant, which is a call option, we find:

\[
d_1 = \frac{\ln(S/K) + (R + \frac{1}{2}\sigma^2)(t)}{\sigma^2t^{1/2}}
\]

\[
d_1 = \frac{\ln(\text{Rs.}106.67/\text{Rs.95}) + \{.06 + \frac{1}{2}(.65^2)\}(6/12)}{(.65^2 \times 6/12)^{1/2}}
\]

\[
d_1 = 0.5471
\]

\[
d_2 = d_1 - (\sigma^2t)^{1/2}
\]

\[
d_2 = 0.5471 - (.65^2 \times 6/12)^{1/2}
\]

\[
d_2 = 0.0875
\]

Next, we need to find N(d_1) and N(d_2), the area under the normal curve from negative infinity to d_1 and negative infinity to d_2, respectively.

\[
N(d_1) = N(0.5471) = 0.7078
\]

\[
N(d_2) = N(0.0875) = 0.5349
\]

According to the Black-Scholes formula, the price of a European call option (C) on a non-dividend paying common stock is:

\[
C = SN(d_1) - Ke^{-Rt}N(d_2)
\]

\[
C = (\text{Rs.106.67})(0.7078) - (\text{Rs.95})e^{-0.06(6/12)}(0.5349)
\]

\[
C = \text{Rs.26.19}
\]

Using this value in the equation above, we find the number of warrants the company must sell is:

\[
\text{Rs.}9,704,455 = (#W)(\frac{1,500,000}{1,500,000 + #W}) \times \text{Call(Rs.106.67, Rs.95)}
\]

\[
\text{Rs.}9,704,455 = (#W)(\frac{1,500,000}{1,500,000 + #W}) \times \text{Rs.26.19}
\]

\[
#W = 492,006
\]