CHAPTER 23
OPTIONS AND CORPORATE FINANCE: EXTENSIONS AND APPLICATIONS

Answers to Concepts Review and Critical Thinking Questions

1. One of the purposes to give stock options to CEOs (instead of cash) is to tie the performance of the firm’s stock with the compensation of the CEO. In this way, the CEO has an incentive to increase shareholder value.

2. Most businesses have the option to abandon under bad conditions and the option to expand under good conditions.

3. Virtually all projects have embedded options, which are ignored in NPV calculations and likely leads to undervaluation.

4. As the volatility increases, the value of an option increases. As the volatility of coal and oil increases, the option to burn either increases. However, if the prices of coal and oil are highly correlated, the value of the option would decline. If coal and oil prices both increase at the same time, the option to switch becomes less valuable since the company will likely save less money.

5. The advantage is that the value of the land may increase if you wait. Additionally, if you wait, the best use of the land other than sale may become more valuable.

6. The company has an option to abandon the mine temporarily, which is an American put. If the option is exercised, which the company is doing by not operating the mine, it has an option to reopen the mine when it is profitable, which is an American call. Of course, if the company does reopen the mine, it has another option to abandon the mine again, which is an American put.

7. Your colleague is correct, but the fact that an increased volatility increases the value of an option is an important part of option valuation. All else the same, a call option on a venture that has a higher volatility will be worth more since the upside potential is greater. Even though the downside is also greater, with an option, the downside is irrelevant since the option will not be exercised and will expire worthless no matter how low the asset falls. With a put option, the reverse is true in that the option becomes more valuable the further the asset falls, and if the asset increases in value, the option is allowed to expire.

8. Real option analysis is not a technique that can be applied in isolation. The value of the asset in real option analysis is calculated using traditional cash flow techniques, and then real options are applied to the resulting cash flows.

9. Insurance is a put option. Consider your homeowner’s insurance. If your house were to burn down, you would receive the value of the policy from your insurer. In essence, you are selling your burned house (“putting”) to the insurance company for the value of the policy (the strike price).
10. In a market with competitors, you must realize that the competitors have real options as well. The decisions made by these competitors may often change the payoffs for your company’s options. For example, the first entrant into a market can often be rewarded with a larger market share because the name can become synonymous with the product (think of Q-tips and Kleenex). Thus, the option to become the first entrant can be valuable. However, we must also consider that it may be better to be a later entrant in the market. Either way, we must realize that the competitors’ actions will affect our options as well.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. a. The inputs to the Black-Scholes model are the current price of the underlying asset (S), the strike price of the option (K), the time to expiration of the option in fractions of a year (t), the variance of the underlying asset, and the continuously-compounded risk-free interest rate (R). Since these options were granted at-the-money, the strike price of each option is equal to the current value of one share, or €50. We can use Black-Scholes to solve for the option price. Doing so, we find:

\[
d_1 = \frac{\ln(S/K) + (R + \sigma^2/2)(t)}{(\sigma^2t)^{1/2}}
\]

\[
d_1 = \frac{\ln(€50/€50) + (.06 + .55^2/2) \times (4)}{(.55 \times \sqrt{4})} = .7682
\]

\[
d_2 = .7682 - (.55 \times \sqrt{4}) = -.3318
\]

Find N(d1) and N(d2), the area under the normal curve from negative infinity to d1 and negative infinity to d2, respectively. Doing so:

\[
N(d_1) = N(0.7682) = 0.7788
\]

\[
N(d_2) = N(-0.3318) = 0.3700
\]

Now we can find the value of each option, which will be:

\[
C = SN(d_1) - Ke^{-Rt}N(d_2)
\]

\[
C = €50(0.7788) - (€50e^{-0.06(4)})(0.3700)
\]

\[
C = €24.39
\]

Since the option grant is for 20,000 options, the value of the grant is:

Grant value = 20,000(€24.39)

Grant value = €487,747.66
b. Because he is risk-neutral, you should recommend the alternative with the highest net present value. Since the expected value of the stock option package is worth more than €450,000, he would prefer to be compensated with the options rather than with the immediate bonus.

c. If he is risk-averse, he may or may not prefer the stock option package to the immediate bonus. Even though the stock option package has a higher net present value, he may not prefer it because it is undiversified. The fact that he cannot sell his options prematurely makes it much more risky than the immediate bonus. Therefore, we cannot say which alternative he would prefer.

2. The total compensation package consists of an annual salary in addition to 10,000 at-the-money stock options. First, we will find the present value of the salary payments. Since the payments occur at the end of the year, the payments can be valued as a three-year annuity, which will be:

\[ PV(\text{Salary}) = Rs.400,000 \times (PVIFA_{9\%,3}) \]
\[ PV(\text{Salary}) = Rs.1,012,517.87 \]

Next, we can use the Black-Scholes model to determine the value of the stock options. Doing so, we find:

\[ d_1 = \frac{\ln(S/K) + (R + \sigma^2/2)(t)}{(\sigma^2 t)^{1/2}} \]
\[ d_1 = \frac{\ln(Rs.40/Rs.40) + (.05 + .68^2/2) \times (3)}{(1.68 \times \sqrt{3})} = .7163 \]
\[ d_2 = .7163 - (1.68 \times \sqrt{3}) = -.4615 \]

Find \( N(d_1) \) and \( N(d_2) \), the area under the normal curve from negative infinity to \( d_1 \) and negative infinity to \( d_2 \), respectively. Doing so:

\[ N(d_1) = N(0.7163) = 0.7631 \]
\[ N(d_2) = N(-0.4615) = 0.3222 \]

Now we can find the value of each option, which will be:

\[ C = SN(d_1) - Ke^{-Rt}N(d_2) \]
\[ C = Rs.40(0.7631) - (Rs.40e^{-0.05(3)})(0.3222) \]
\[ C = Rs.19.43 \]

Since the option grant is for 10,000 options, the value of the grant is:

Grant value = 10,000(Rs.19.43)
Grant value = Rs.194,303.49

The total value of the contract is the sum of the present value of the salary, plus the option value, or:

Contract value = Rs.1,012,517.87 + 194,303.19
Contract value = Rs.1,206,821.05
3. Since the contract is to sell up to 5 million gallons, it is a call option, so we need to value the contract accordingly. Using the binomial mode, we will find the value of \( u \) and \( d \), which are:

\[
\begin{align*}
\mu &= e^{\sigma \sqrt{n}} \\
\mu &= e^{46 \sqrt{12/3}} \\
\mu &= 1.26 \\
\sigma &= 1 / \mu \\
\sigma &= 1 / 1.26 \\
\sigma &= 0.79
\end{align*}
\]

This implies the percentage increase if gasoline increases will be 26 percent, and the percentage decrease if prices fall will be 21 percent. So, the price in three months with an up or down move will be:

\[
\begin{align*}
P_{\text{Up}} &= $1.65(1.26) \\
P_{\text{Up}} &= $2.08 \\
P_{\text{Down}} &= $1.65(0.79) \\
P_{\text{Down}} &= $1.31
\end{align*}
\]

The option is worthless if the price decreases. If the price increases, the value of the option per gallon is:

\[
\begin{align*}
\text{Value with price increase} &= $2.08 - 1.85 \\
\text{Value with price increase} &= $0.23
\end{align*}
\]

Next, we need to find the risk neutral probability of a price increase or decrease, which will be:

\[
\begin{align*}
0.06 / (12/3) &= 0.26(\text{Probability of rise}) + -0.21(1 - \text{Probability of rise}) \\
\text{Probability of rise} &= 0.4751 \\
\text{Probability of decrease} &= 1 - 0.4751 \\
\text{Probability of decrease} &= 0.5249
\end{align*}
\]

The contract will not be exercised if gasoline prices fall, so the value of the contract with a price decrease is zero. So, the value per gallon of the call option contract will be:

\[
\begin{align*}
C &= [0.4751(0.23) + 0.5249(0)] / [1 + 0.06/(12 / 3)] \\
C &= $0.106
\end{align*}
\]

This means the value of the entire contract is:

\[
\begin{align*}
\text{Value of contract} &= $0.106(5,000,000) \\
\text{Value of contract} &= $530,516.17
\end{align*}
\]
4. When solving a question dealing with real options, begin by identifying the option-like features of the situation. First, since the company will exercise its option to build if the value of an office building rises, the right to build the office building is similar to a call option. Second, an office building would be worth Ca$10 million today. This amount can be viewed as the current price of the underlying asset (S). Third, it will cost Ca$10.5 million to construct such an office building. This amount can be viewed as the strike price of a call option (K), since it is the amount that the firm must pay in order to exercise its right to erect an office building. Finally, since the firm’s right to build on the land lasts only 1 year, the time to expiration (t) of the real option is one year. We can use the two-state model to value the option to build on the land. First, we need to find the return of the land if the value rises or falls. The return will be:

\[
\begin{align*}
R_{\text{rise}} &= (\text{Ca$16,800,000} - 13,500,000) / \text{Ca$13,500,000} \\
&= .2444 \text{ or } 24.44\% \\
R_{\text{fall}} &= (\text{Ca$11,800,000} - 13,500,000) / \text{Ca$13,500,000} \\
&= -.1259 \text{ or } -12.59\%
\end{align*}
\]

Now we can find the risk-neutral probability of a rise in the value of the building as:

\[
\text{Risk-free rate} = (\text{Probability}_{\text{rise}})(R_{\text{rise}}) + (1 - \text{Probability}_{\text{rise}})(R_{\text{fall}})
\]

\[
0.05 = (\text{Probability}_{\text{rise}})(0.2444) + (1 - \text{Probability}_{\text{rise}})(-0.1259)
\]

\[
\text{Probability}_{\text{rise}} = 0.4696
\]

So, a probability of a fall is:

\[
\text{Probability}_{\text{fall}} = 1 - \text{Probability}_{\text{rise}}
\]

\[
\text{Probability}_{\text{fall}} = 1 - 0.4696 = 0.5304
\]

Using these risk-neutral probabilities, we can determine the expected payoff of the real option at expiration.

\[
\text{Expected payoff at expiration} = (0.4696)(\text{Ca$1,800,000}) + (0.50)(\text{Ca$0})
\]

\[
\text{Expected payoff at expiration} = \text{Ca$845,280}
\]
Since this payoff will occur 1 year from now, it must be discounted at the risk-free rate in order to find its present value, which is:

\[ PV = \left( \frac{\text{Ca$845,280}}{1.05} \right) \]
\[ PV = \text{Ca$806,564.89} \]

Therefore, the right to build an office building over the next year is worth Ca$806,564.89 today. Since the offer to purchase the land is less than the value of the real option to build, the company should not accept the offer.

5. When solving a question dealing with real options, begin by identifying the option-like features of the situation. First, since the company will only choose to drill and excavate if the price of oil rises, the right to drill on the land can be viewed as a call option. Second, since the land contains 125,000 barrels of oil and the current price of oil is $55 per barrel, the current price of the underlying asset (S) to be used in the Black-Scholes model is:

“Stock” price = 125,000($55)
“Stock” price = $6,875,000

Third, since the company will not drill unless the price of oil in one year will compensate its excavation costs, these costs can be viewed as the real option’s strike price (K). Finally, since the winner of the auction has the right to drill for oil in one year, the real option can be viewed as having a time to expiration (t) of one year. Using the Black-Scholes model to determine the value of the option, we find:

\[ d_1 = \frac{\ln(S/K) + (R + \sigma^2/2)(t)}{(\sigma^2t)^{1/2}} \]
\[ d_1 = \frac{\ln($6,875,000/$10,000,000) + (.065 + .50^2/2) \times (1)}{(.50 \times \sqrt{1})} = -.3694 \]

\[ d_2 = -.3694 - (.50 \times \sqrt{1}) = -.8694 \]

Find N(d_1) and N(d_2), the area under the normal curve from negative infinity to d_1 and negative infinity to d_2, respectively. Doing so:

\[ N(d_1) = N(-.3694) = 0.3559 \]
\[ N(d_2) = N(-.8694) = 0.1923 \]

Now we can find the value of call option, which will be:

\[ C = SN(d_1) - Ke^{-Rt}N(d_2) \]
\[ C = $6,875,000(0.3559) - ($10,000,000e^{-0.055})(0.1923) \]
\[ C = $644,800.53 \]

This is the maximum bid the company should be willing to make at auction.
6. When solving a question dealing with real options, begin by identifying the option-like features of the situation. First, since Bjeorn Smelters will only choose to manufacture the steel rods if the price of steel falls, the lease, which gives the firm the ability to manufacture steel, can be viewed as a put option. Second, since the firm will receive a fixed amount of money if it chooses to manufacture the rods:

\[
\text{Amount received} = 4,800 \text{ steel rods}(€360 – 120)
\]
\[
\text{Amount received} = €1,152,000
\]

The amount received can be viewed as the put option’s strike price (K). Third, since the project requires Bjeorn Smelters to purchase 400 tons of steel and the current price of steel is €3,600 per ton, the current price of the underlying asset (S) to be used in the Black-Scholes formula is:

\[
\text{“Stock” price} = 400 \text{ tons}(€3,600 \text{ per ton})
\]
\[
\text{“Stock” price} = €1,440,000
\]

Finally, since Bjeorn Smelters must decide whether to purchase the steel or not in six months, the firm’s real option to manufacture steel rods can be viewed as having a time to expiration (t) of six months. In order to calculate the value of this real put option, we can use the Black-Scholes model to determine the value of an otherwise identical call option then infer the value of the put using put-call parity. Using the Black-Scholes model to determine the value of the option, we find:

\[
d_1 = \left[ \ln\left(\frac{S}{K}\right) + \left( R + \frac{\sigma^2}{2}\right) t \right] / \left( \sigma \sqrt{t} \right)
\]
\[
d_1 = \left[ \ln\left(\frac{€1,440,000}{€1,152,000}\right) + \left(0.045 + \frac{.45^2}{2} \times (6/12) \right) \right] / \left( .45 \times \sqrt{6/12} \right) = .9311
\]
\[
d_2 = .9311 – (.45 \times \sqrt{6/12} ) = .6129
\]

Find \(N(d_1)\) and \(N(d_2)\), the area under the normal curve from negative infinity to \(d_1\) and negative infinity to \(d_2\), respectively. Doing so:

\[
N(d_1) = N(0.9311) = 0.8241
\]
\[
N(d_2) = N(0.6129) = 0.7300
\]

Now we can find the value of call option, which will be:

\[
C = SN(d_1) – Ke^{-Rt}N(d_2)
\]
\[
C = €1,440,000(0.8241) – (€1,152,000e^{-0.045(6/12)})(0.7300)
\]
\[
C = €364,419.87
\]

Now we can use put-call parity to find the price of the put option, which is:

\[
C = P + S – Ke^{-Rt}
\]
\[
€364,419.87 = P + €1,440,000 – €1,152,000e^{-0.045(6/12)}
\]
\[
P = €50,789.29
\]

This is the most the company should be willing to pay for the lease.
7. In one year, the company will abandon the technology if the demand is low since the value of abandonment is higher than the value of continuing operations. Since the company is selling the technology in this case, the option is a put option. The value of the put option in one year if demand is low will be:

Value of put with low demand = $7,000,000 − 6,000,000
Value of put with low demand = $1,000,000

Of course, if demand is high, the company will not sell the technology, so the put will expire worthless. We can value the put with the binomial model. In one year, the percentage gain on the project if the demand is high will be:

Percentage increase with high demand = ($10,500,000 − 9,100,000) / $9,100,000
Percentage increase with high demand = .1538 or 15.38%

And the percentage decrease in the value of the technology with low demand is:

Percentage decrease with high demand = ($6,000,000 − 9,100,000) / $9,100,000
Percentage decrease with high demand = −.3407 or −34.07%

Now we can find the risk-neutral probability of a rise in the value of the technology as:

Risk-free rate = (Probability_Rise)(Return_Rise) + (Probability_Fall)(Return_Fall)
Risk-free rate = (Probability_Rise)(0.1538) + (1 − Probability_Rise)(−.3407)
0.06 = (Probability_Rise)(0.1538) + (1 − Probability_Rise)(−.3407)
Probability_Rise = 0.8102

So, a probability of a fall is:

Probability_Fall = 1 − Probability_Rise
Probability_Fall = 1 − 0.8102
Probability_Fall = 0.1898

Using these risk-neutral probabilities, we can determine the expected payoff of the real option at expiration. With high demand, the option is worthless since the technology will not be sold, and the value of the technology with low demand is the $1 million we calculated previously. So, the value of the option to abandon is:

Value of option to abandon = [(.8102)(0) + (.1898)($1,000,000)] / (1 + .06)
Value of option to abandon = $179,035.64
8. Using the binomial mode, we will find the value of $u$ and $d$, which are:

\[
\begin{align*}
  u &= e^{\sigma \sqrt{n}} \\
  u &= e^{0.65/\sqrt{12}} \\
  u &= 1.21 \\
  \end{align*}
\]

\[
\begin{align*}
  d &= 1 / u \\
  d &= 1 / 1.21 \\
  d &= 0.83 \\
\end{align*}
\]

This implies the percentage increase is if the stock price increases will be 21 percent, and the percentage decrease if the stock price falls will be 17 percent. The monthly interest rate is:

Monthly interest rate = 0.05/12
Monthly interest rate = 0.0042

Next, we need to find the risk neutral probability of a price increase or decrease, which will be:

\[0.0042 = 0.21(\text{Probability of rise}) + \text{–0.17}(1 – \text{Probability of rise})\]

Probability of rise = 0.4643

And the probability of a price decrease is:

Probability of decrease = 1 – 0.4643
Probability of decrease = 0.5357

The following figure shows the stock price and put price for each possible move over the next two months:

| Stock price (D) | £ 91.69 |
| Put price | £ - |
| Stock price (B) | £ 76.00 |
| Put price | £ 3.73 |
| Stock price (A) | £ 63.00 |
| Put price | £ 11.21 |
| Stock price (E) | £ 63.00 |
| Put price | £ 7.00 |
| Stock price (C) | £ 52.22 |
| Put price | £ 17.78 |
| Stock price (F) | £ 43.29 |
| Put price | £ 26.71 |
The stock price at node (A) is the current stock price. The stock price at node (B) is from an up move, which means:

\[
\text{Stock price (B)} = £63(1.2064) \\
\text{Stock price (B)} = £76.00
\]

And the stock price at node (D) is two up moves, or:

\[
\text{Stock price (D)} = £63(1.2064)(1.2064) \\
\text{Stock price (D)} = £91.69
\]

The stock price at node (C) is from a down move, or:

\[
\text{Stock price (C)} = £63(0.8289) \\
\text{Stock price (C)} = £52.22
\]

And the stock price at node (F) is two down moves, or:

\[
\text{Stock price (D)} = £63(0.8289)(0.8289) \\
\text{Stock price (D)} = £43.29
\]

Finally, the stock price at node (E) is from an up move followed by a down move, or a down move followed by an up move. Since the binomial tree recombines, both calculations yield the same result, which is:

\[
\text{Stock price (E)} = £63(1.2064)(0.8289) = £63(0.8289)(1.2064) \\
\text{Stock price (E)} = £63.00
\]

Now we can value the put option at the expiration nodes, namely (D), (E), and (F). The value of the put option at these nodes is the maximum of the strike price minus the stock price, or zero. So:

\[
\text{Put value (D)} = \text{Max}(£70 – 91.69, £0) \\
\text{Put value (D)} = £0
\]

\[
\text{Put value (E)} = \text{Max}(£70 – 63, £0) \\
\text{Put value (E)} = £7
\]

\[
\text{Put value (F)} = \text{Max}(£70 – 43.29, £0) \\
\text{Put value (F)} = £26.71
\]

The value of the put at node (B) is the present value of the expected value. We find the expected value by using the value of the put at nodes (D) and (E) since those are the only two possible stock prices after node (B). So, the value of the put at node (B) is:

\[
\text{Put value (B)} = \frac{.4643(£0) + .5357(£7)}{1.0042} \\
\text{Put value (B)} = £3.73
\]
Similarly, the value of the put at node (C) is the present value of the expected value of the put at nodes (E) and (F) since those are the only two possible stock prices after node (C). So, the value of the put at node (C) is:

\[
\text{Put value (C)} = \frac{0.4643(£7) + 0.5357(£26.71)}{1.0042} = £17.49
\]

Notice, however, that the put option is an American option. Because it is an American option, it can be exercised any time prior to expiration. If the stock price falls next month, the value of the put option if exercised is:

\[
\text{Value if exercised} = £70 - 52.22 = £17.78
\]

This is greater than the present value of waiting one month, so the option will be exercised early in one month if the stock price falls. This is the value of the put option at node (C). Using this put value, we can now find the value of the put today, which is:

\[
\text{Put value (A)} = \frac{0.4643(£3.73) + 0.5357(£17.78)}{1.0042} = £11.21
\]

9. Using the binomial model, we will find the value of \( u \) and \( d \), which are:

\[
\begin{align*}
  u &= e^{r \sqrt{n}} \\
  u &= e^{0.25 / 1/2} \\
  u &= 1.19 \\
  d &= 1 / u \\
  d &= 1 / 1.19 \\
  d &= 0.84
\end{align*}
\]

This implies the percentage increase is if the stock price increases will be 19 percent, and the percentage decrease if the stock price falls will be 16 percent. The six month interest rate is:

\[
\text{Six month interest rate} = 0.08/2 = 0.04
\]

Next, we need to find the risk neutral probability of a price increase or decrease, which will be:

\[
0.04 = 0.19(\text{Probability of rise}) + -0.16(1 - \text{Probability of rise})
\]

Probability of rise = 0.5685

And the probability of a price decrease is:

\[
\begin{align*}
  \text{Probability of decrease} &= 1 - 0.5685 \\
  \text{Probability of decrease} &= 0.4315
\end{align*}
\]
The following figure shows the stock price and call price for each possible move over the each of the six month steps:

<table>
<thead>
<tr>
<th>(values in billions)</th>
<th>Value (D)</th>
<th>¥64,085.356</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>¥17,085.356</td>
<td></td>
</tr>
</tbody>
</table>

Value pre-payment ¥53,701.406
Value post-payment (B) ¥53,201.406
Call price ¥9,338.963

<table>
<thead>
<tr>
<th>Value (E)</th>
<th>¥44,581.017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>¥0</td>
</tr>
</tbody>
</table>

Stock price(A) ¥45,000.00
Call price ¥5,104.736

<table>
<thead>
<tr>
<th>Value (F)</th>
<th>¥44,403.318</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>¥0</td>
</tr>
</tbody>
</table>

Value pre-payment ¥37,708.510
Value post-payment (C) ¥37,208.510
Call price ¥0

<table>
<thead>
<tr>
<th>Value (G)</th>
<th>¥31,179.499</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call price</td>
<td>¥0</td>
</tr>
</tbody>
</table>

First, we need to find the building value at every step along the binomial tree. The building value at node (A) is the current building value. The building value at node (B) is from an up move, which means:

Building value (B) = ¥45,000,000,000(1.1934)
Building value (B) = ¥53,701,406,000

At node (B), the accrued rent payment will be made, so the value of the building after the payment will be reduced by the amount of the payment, which means the building value at node (B) is:

Building value (B) after payment = ¥53,701,406,000 – 500,000,000
Building value (B) after payment = ¥53,201,406,000
To find the building value at node (D), we multiply the after-payment building value at node (B) by the up move, or:

Building value (D) = ¥53,201,406,000(1.1934)
Building value (D) = ¥64,085,356,000

To find the building value at node (E), we multiply the after-payment building value at node (B) by the down move, or:

Building value (E) = ¥53,201,406,000(0.8380)
Building value (E) = ¥44,581,017,000

The building value at node (C) is from a down move, which means the building value will be:

Building value (E) = ¥45,000,000,000(0.8380)
Building value (E) = ¥37,708,510,000

At node (C), the accrued rent payment will be made, so the value of the building after the payment will be reduced by the amount of the payment, which means the building value at node (C) is:

Building value (C) after payment = ¥37,708,510,000 – 500,000,000
Building value (C) after payment = ¥37,208,510,000

To find the building value at node (F), we multiply the after-payment building value at node (C) by the down move, or:

Building value (F) = ¥37,208,510,000(1.1934)
Building value (F) = ¥44,403,318,000

Finally, the building value at node (G) is from a down move from node (C), so the building value is:

Building value (G) = ¥37,208,510,000(0.8380)
Building value (G) = ¥31,179,499,000

Note that because of the accrued rent payment in six months, the binomial tree does not recombine during the next step. This occurs whenever a fixed payment is made during a binomial tree. For example, when using a binomial tree for a stock option, a fixed dividend payment will mean that the tree does not recombine. With the expiration values, we can value the call option at the expiration nodes, namely (D), (E), (F), and (G). The value of the call option at these nodes is the maximum of the building value minus the strike price, or zero. We do not need to account for the value of the building after the accrued rent payments in this case since if the option is exercised, you will receive the rent payment. So:

Call value (D) = Max(¥64,085,356,000 – 47,000,000,000, ¥0)
Call value (D) = ¥17,085,356,000

Call value (E) = Max(¥44,581,017,000 – 47,000,000,000, ¥0)
Call value (E) = ¥0
Call value (F) = Max(¥44,403,318,000 – 47,000,000,000, ¥0)
Call value (F) = ¥0

Call value (G) = Max(¥31,179,499,000 – 47,000,000,000, ¥0)
Call value (G) = ¥0

The value of the call at node (B) is the present value of the expected value. We find the expected value by using the value of the call at nodes (D) and (E) since those are the only two possible building values after node (B). So, the value of the call at node (B) is:

Call value (B) = [.5685(¥17,085,356,000) + .4315(¥0)] / 1.04
Call value (B) = ¥9,338,963,000

Note that you would not want to exercise the option early at node (B). The value of the option at node (B) is exercised if the value of the building including the accrued rent payment minus the strike price, or:

Option value at node (B) if exercised = ¥53,701,406,000 – 45,000,000,000
Option value at node (B) if exercised = ¥8,701,406,000

Since this is less than the value of the option if it left “alive”, the option will not be exercised. With a call option, unless a large cash payment (dividend) is made, it is generally not valuable to exercise the call option early. The reason is that the potential gain is unlimited. In contrast, the potential gain on a put option is limited by the strike price, so it may be valuable to exercise an American put option early if it is deep in the money.

We can value the call at node (C), which will be the present value of the expected value of the call at nodes (F) and (G) since those are the only two possible building values after node (C). Since neither node has a value greater than zero, obviously the value of the option at node (C) will also be zero. Now we need to find the value of the option today, which is:

Call value (A) = [.5685(¥9,338,963,000) + .4315(¥0)] / 1.04
Call value (A) = ¥5,104,736,000