Answers to Practice Questions

1. a. \[ u = e^{0.24 \sqrt{0.5}} = 1.185 \; \; d = 1/u = 0.844 \]

\[
\begin{align*}
$45 & \\
& \quad \begin{cases}
\text{u} = 1.185 & \\
\text{d} = 0.844 &
\end{cases}
\end{align*}
\]

\[
\begin{align*}
$45 & \\
& \quad \begin{cases}
\text{u} = 1.127 & \\
\text{d} = 0.887 &
\end{cases}
\end{align*}
\]
b. \[ u = e^{0.3 \sqrt{0.5}} = 1.236 \text{, } d = 1/u = 0.809 \]

\[ u = e^{0.3 \sqrt{0.25}} = 1.162 \text{, } d = 1/u = 0.861 \]
2.  a.  The diagram on the left below displays the possible stock prices. The diagram on the right displays the corresponding call option values.

\[
\begin{array}{c|c|c}
5000 & 260.68 \\
6000 & 4167 & 500 \\
0 & 1333
\end{array}
\]

\[u - 1 = (¥6000/¥5000) - 1 = 0.200\]
\[d - 1 = (¥4167/¥5000) - 1 = -0.167\]

Let \(p\) equal the probability of a rise in the stock price. Then, if investors are risk-neutral:

\[(p \times 0.20) + (1 - p) \times (-0.167) = 0.03\]
\[p = 0.537\]

The value of the call is:

\[
\frac{(0.537 \times 500) + (0.463 \times 0)}{1.03} = 260.68
\]

The possible stock prices and the corresponding put option values are shown in the diagrams below:

\[
\begin{array}{c|c|c}
5000 & 599.20 \\
6000 & 4167 & 0 \\
0 & 1333
\end{array}
\]

The value of the put is:

\[
\frac{(0.537 \times 0) + (0.463 \times 1333)}{1.03} = 599.20
\]

b. Let \(X\) equal the break-even exercise price. Then the following must be true:

\[X - ¥5000 = [(p)(¥0) + (1 - p)(X - ¥4167)]/1.03\]

That is, the value of the put if exercised immediately equals the value of the put if it is held to next period. Solving for \(X\), we find that the break-even exercise price is: ¥5,680.21

At any higher exercise price, the put should be exercised immediately.
c. Again, let X equal the break-even exercise price. Then the following must be true:

\[ X - ¥5000 = [(p)(¥0) + (1 - p)(X - ¥4167)]/1.14 \]

Solving for X, we find that the break-even exercise price is: ¥5,569.69

At any higher exercise price, the put should be exercised immediately.

d. The diagram on the left below displays the possible stock prices. The diagram on the right displays the corresponding call option values.

\[
\begin{array}{c}
5000 \\
6000 \\
4167 \\
319.79 \\
0 \\
166.73
\end{array}
\]

The value of the call is:

\[
\frac{(0.537 \times 319.79) + (0.463 \times 0)}{1.03} = 166.73
\]

3. a. The future stock prices of Moria Mining are:

\[
\begin{array}{c}
\text{With dividend} \\
80 \\
125 \\
\text{Ex-dividend} \\
60 \\
48 \\
75 \\
105 \\
84 \\
131.25
\end{array}
\]

Let p equal the probability of a rise in the stock price. Then, if investors are risk-neutral:

\[
(p \times 0.25) + (1 - p)(-0.20) = 0.10
\]

\[ p = 0.67 \]

Now, calculate the expected value of the call in month 6.
If stock price decreases to $80 in month 6, then the call is worthless. If stock price increases to $125, then, if it is exercised at that time, it has a value of ($125 – $80) = $45. If the call is not exercised, then its value is:

\[
\frac{(0.67 \times $51.25) + (0.33 \times $4)}{1.10} = $32.42
\]

Therefore, it is preferable to exercise the call.

The value of the call in month 0 is:

\[
\frac{(0.67 \times $45) + (0.33 \times $0)}{1.10} = $27.41
\]

b. The future stock prices of Moria Mining are:

With dividend

Ex-dividend

Let \( p \) equal the probability of a rise in the price of the stock. Then, if investors are risk-neutral:

\[
(p \times 0.25) + (1 – p) \times (–0.20) = 0.10
\]

\( p = 0.67 \)

Now, calculate the expected value of the call in month 6.

If stock price decreases to $80 in month 6, then the call is worthless. If stock price increases to $125, then, if it is exercised at that time, it has a value of ($125 – $80) = $45. If the call is not exercised, then its value is:

\[
\frac{(0.67 \times $45) + (0.33 \times $0)}{1.10} = $27.41
\]

Therefore, it is preferable to exercise the call.

The value of the call in month 0 is:

\[
\frac{(0.67 \times $45) + (0.33 \times $0)}{1.10} = $27.41
\]
4. a. The possible prices of CH₄ Trading stock and the associated call option values (shown in parentheses) are:

\[
\begin{array}{c}
110 \\
(?) \\
88 \\
(?) \\
70.40 \\
(0) \\
137.50 \\
(?) \\
110 \\
(20) \\
171.88 \\
(81.88)
\end{array}
\]

Let \( p \) equal the probability of a rise in the stock price. Then, if investors are risk-neutral:

\[
p (0.25) + (1 – p)(–0.20) = 0.03
\]

\[p = 0.51\]

If the stock price in month 3 is €88, then the option will not be exercised so that it will be worth:

\[
[(0.51 \times €20) + (0.49 \times €0)]/1.03 = €9.90
\]

Similarly, if the stock price is €137.50 in month 3, then, if it is exercised, it will be worth (€137.50 – €90) = €47.50. If the option is not exercised, it will be worth:

\[
[(0.51 \times €81.88) + (0.49 \times €20)]/1.03 = €50.06
\]

Therefore, the call option will not be exercised, so that its value today is:

\[
[(0.51 \times €50.06) + (0.49 \times €9.90)]/1.03 = €29.50
\]

b. (i) If the price rises to €137.50:

\[
\text{Delta} = \frac{81.88 - 20}{171.88 - 110} = 1.0
\]

(ii) If the price falls to €88:

\[
\text{Delta} = \frac{20 - 0}{110 - 70.40} = 0.505
\]
c. The option delta is 1.0 when the call is certain to be exercised and is zero when it is certain not to be exercised. If the call is certain to be exercised, it is equivalent to buying the stock with a partly deferred payment. So a one-dollar change in the stock price must be matched by a one-dollar change in the option price. At the other extreme, when the call is certain not to be exercised, it is valueless, regardless of the change in the stock price.

d. If the stock price is €88 at 3 months, the option delta is 0.505. Therefore, in order to replicate the stock, we buy 1.98 calls and lend, as follows:

<table>
<thead>
<tr>
<th>Initial Outlay</th>
<th>Stock Price = 70.40</th>
<th>Stock Price = 110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy 1.98 calls</td>
<td>-19.60</td>
<td>0.00</td>
</tr>
<tr>
<td>Lend PV(70.40)</td>
<td>-68.35</td>
<td>+70.40</td>
</tr>
<tr>
<td></td>
<td>-87.95</td>
<td>+70.40</td>
</tr>
</tbody>
</table>

This strategy is equivalent to:

Buy stock -88.00 +70.40 +110.00

5. a. Yes, it is rational to consider the early exercise of an American put option.

b. The possible prices of CH₄ Trading stock and the associated American put option values (shown in parentheses) are:

\[
\begin{array}{c}
110 \\
88 \\
70.40 \\
171.88 \\
137.50 \\
\end{array}
\]

Let \( p \) equal the probability of a rise in the stock price. Then, if investors are risk-neutral:

\[
p (0.25) + (1 - p)(-0.20) = 0.03
\]

\[
p = 0.51
\]

If the stock price in month 3 is €88, and if the American put option is not exercised, it will be worth:

\[
[(0.51 \times 0) + (0.49 \times 39.60)]/1.03 = €18.84
\]

On the other hand, if it is exercised after 3 months, it is worth €22. Thus, the investor should exercise the put early.
Similarly, if the stock price in month 3 is €137.50, and if the American put option is not exercised, it will be worth:

\[
\frac{(0.51 \times 0) + (0.6 \times 0)}{1.03} = 0
\]

On the other hand, if it is exercised after 3 months, it will cost the investor €27.50. The investor should not exercise early.

Finally, the value today of the American put option is:

\[
\frac{(0.51 \times 0) + (0.49 \times 22)}{1.03} = 10.47
\]

c. Unlike the American put in part (b), the European put can not be exercised prior to expiration. We noted in part (b) that, if the stock price in month 3 is €88, the American put would be exercised because its value if exercised (i.e., €22) is greater than its value if not exercised (i.e., €18.84). For the European put, however, the value at that point is €18.84 because the European put can not be exercised early. Therefore, the value of the European put is:

\[
\frac{(0.51 \times 0) + (0.49 \times 18.84)}{1.03} = 8.96
\]

6. a. The following tree shows stock prices, with option values in parentheses:

```
  110 (24.55)
 /     \
|      |
With dividend 88.00 137.50
 /     \
|      |
Ex-dividend 75.50(2.17) 125.00 (47.50)
 /     \
|      |
60.40 (0) 94.38 (4.38) 100.00 (10.00) 156.25 (66.25)
```

We calculate the option value as follows:

1. The option values in month 3, if the option is not exercised, are computed as follows:

\[
\frac{(0.51 \times 4.38) + (0.49 \times 0)}{1.03} = 2.17
\]

\[
\frac{(0.51 \times 66.25) + (0.49 \times 10)}{1.03} = 37.56
\]
If the stock price in month 3 is €88, then it would not pay to exercise the option. If the stock price in month 3 is €137.50, then the call is worth: $(€137.50 – €90.00) = €47.50$. Therefore, the option would be exercised at that time.

2. Working back to month 0, we find the option value as follows:

\[
\text{Option value} = \frac{(0.51 \times 47.50) + (0.49 \times 2.17)}{1.03} = 24.55
\]

b. If the option were European, it would not be possible to exercise early. Therefore, if the price rises to €137.50 at month 3, the value of the option is €37.56, not €47.50 as is the case for the American option. Therefore, in this case, the value of the European option is less than the value of the American option. The value of the European option is computed as follows:

\[
\text{Option value} = \frac{(0.51 \times 37.56) + (0.49 \times 2.17)}{1.03} = 19.63
\]

7. The following tree (see Practice Question 4) shows stock prices, with values for the option in parentheses:

\[
\begin{array}{c}
88 \quad (43.86) \\
70.40 \quad (0)
\end{array}
\quad
\begin{array}{c}
110 \\
171.88 \quad (89.88)
\end{array}
\quad
\begin{array}{c}
137.50 \quad (57.82) \\
110 \quad (28)
\end{array}
\]

The put option is worth €0 in month 3 if the stock price falls and €0 if the stock price rises. Thus, with a 3-month stock price of €88, the investor would not exercise the put since it would cost €6 to exercise. With a price in month 3 of €137.50, the investor would not exercise the put since it would cost €55.50 to exercise. The values for the option in month 3, if it is not exercised, are determined as follows:

\[
\frac{(0.51 \times 28) + (0.49 \times 0)}{1.03} = 13.86
\]

\[
\frac{(0.51 \times 89.88) + (0.49 \times 28)}{1.03} = 57.82
\]
Therefore, the month 0 value of the option is:

\[
\text{Option value} = \frac{(0.51 \times 57.82) + (0.49 \times 13.86)}{1.03} = 35.22
\]

8.  a. The following tree shows stock prices (with put option values in parentheses):

```
     100 (2.35)
       /  \
    90.0 (7.15) 111.1 (.55)
      /    \
  81.0 (21) 100 (2) 123.4 (0)
```

Let \( p \) equal the probability that the stock price will rise. Then, for a risk-neutral investor:

\[
(p \times 0.111) + (1 - p) \times (-0.10) = 0.05
\]

\[p = 0.71\]

If the stock price in month 6 is C$111.1, then the value of the European put is:

\[
\left(0.71 \times \text{C$0}\right) + \left(0.29 \times \text{C$2}\right) = \text{C$0.55}\]

If the stock price in month 6 is C$90.0, then the value of the put is:

\[
\left(0.71 \times \text{C$2}\right) + \left(0.29 \times \text{C$21}\right) = \text{C$7.15}\]

Since this is a European put, it can not be exercised at month 6.

The value of the put at month 0 is:

\[
\left(0.71 \times \text{C$0.55}\right) + \left(0.29 \times \text{C$7.15}\right) = \text{C$2.35}\]
b. Since the American put can be exercised at month 6, then, if the stock price is $90.0, the put is worth $(102 - 90) = 12$ if exercised, compared to $7.15$ if not exercised. Thus, the value of the American put in month 0 is:

\[
\frac{(0.71 \times 0.55) + (0.29 \times 12)}{1.05} = \$3.69
\]

9. a. \( P = 200 \quad EX = 180 \quad \sigma = 0.223 \quad t = 1.0 \quad r_f = 0.21 \)

\[
d_1 = \frac{\log(P/PV(EX)) + \sigma \sqrt{t} + \sigma \sqrt{t}/2}{\sigma \sqrt{t}} = \frac{\log(200/(180/1.21)) + (0.223 \times \sqrt{1}) + (0.223 \times \sqrt{1.0})/2}{0.223} = 1.4388
\]

\[
d_2 = d_1 - \sigma \sqrt{t} = 1.4388 - (0.223 \times \sqrt{1.0}) = 1.2158
\]

\[
N(d_1) = N(1.4388) = 0.9249
\]

\[
N(d_2) = N(1.2158) = 0.8880
\]

Call value = \[N(d_1) \times P] - [N(d_2) \times PV(EX)]

\[
= [0.9249 \times 200] - [0.8880 \times (180/1.21)] = \$52.88
\]

b. 1+ upside change = \( u = e^{\sigma \sqrt{t}} = e^{0.223 \times \sqrt{1.0}} = 1.2498 \)

1+ downside change = \( d = 1/u = 1/1.2498 = 0.8001 \)

Let \( p \) equal the probability that the stock price will rise. Then, for a risk-neutral investor:

\[
(p \times 0.25) + (1 - p) \times (-0.20) = 0.21
\]

\[
p = 0.91
\]

In one year, the stock price will be either $250 or $160, and the option values will be $70 or $0, respectively. Therefore, the value of the option is:

\[
\frac{(0.91 \times 70) + (0.09 \times 0)}{1.21} = \$52.64
\]
c. 

\[ 1 + \text{upside change} = u = e^{0.223 \cdot 0.5} = 1.1708 \]
\[ 1 + \text{downside change} = d = 1/u = 1/1.1708 = 0.8541 \]

Let \( p \) equal the probability that the stock price will rise. Then, for a risk-neutral investor:

\[ (p \times 0.171) + (1 - p) \times (-0.146) = 0.10 \]

\[ p = 0.776 \]

The following tree gives stock prices, with option values in parentheses:

```
      200 (52.63)
       /   \
 170.8 (14.11)  234.2 (70.53)
     /    \      /    \       /    \      /    \     
145.9 (0) 200.0 (20) 274.2 (94.2)
```

Option values are calculated as follows:

1. \[
\frac{(0.776 \times \$20) + (0.224 \times \$0)}{1.10} = \$14.11
\]
2. \[
\frac{(0.224 \times \$20) + (0.776 \times \$94.2)}{1.10} = \$70.53
\]
3. \[
\frac{(0.224 \times \$14.11) + (0.776 \times \$70.53)}{1.10} = \$52.63
\]
d. (i) Option delta = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}} = \frac{70.53 - 14.11}{234.2 - 170.8} = 0.89

To replicate a call, buy 0.89 shares and borrow:
\[ \frac{(0.89 \times 170.8) - 14.11}{1.10} = 125.37 \]

(ii) Option delta = \frac{94.2 - 20}{274.2 - 200} = 1.00

To replicate a call, buy one share and borrow:
\[ \frac{(1.0 \times 274.2) - 94.2}{1.10} = 163.64 \]

(iii) Option delta = \frac{20 - 0}{200 - 145.9} = 0.37

To replicate a call, buy 0.37 shares and borrow:
\[ \frac{(0.37 \times 200) - 20}{1.10} = 49.09 \]

10. To hold time to expiration constant, we will look at a simple one-period binomial problem with different starting stock prices. Here are the possible stock prices:

```
50  200  55  220
```

Now consider the effect on option delta:

<table>
<thead>
<tr>
<th>Option Deltas</th>
<th>Current Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-the-money (EX = 60)</td>
<td>140/150 = 0.93</td>
</tr>
<tr>
<td>At-the-money (EX = 100)</td>
<td>100/150 = 0.67</td>
</tr>
<tr>
<td>Out-of-the-money (EX = 140)</td>
<td>60/150 = 0.40</td>
</tr>
</tbody>
</table>

Note that, for a given difference in stock price, out-of-the-money options result in a larger change in the option delta. If you want to minimize the number of times you rebalance an option hedge, use in-the-money options.
11. a. The call option. (You would delay the exercise of the put until after the dividend has been paid and the stock price has dropped.)

   b. The put option. (You never exercise a call if the stock price is below exercise price.)

   c. The put when the interest rate is high. (You can invest the exercise price.)

12. a. When you exercise a call, you purchase the stock for the exercise price. Naturally, you want to maximize what you receive for this price, and so you would exercise on the with-dividend date in order to capture the dividend.

   b. When you exercise a put, your gain is the difference between the price of the stock and the amount you receive upon exercise, i.e., the exercise price. Therefore, in order to maximize your profit, you want to minimize the price of the stock and so you would exercise on the ex-dividend date.

13. Internet exercise; answers will vary.

14. a. The value of the alternative share = (V/N) where V is the total value of equity (common stock plus warrants) and N is the number of shares outstanding. For Electric Bassoon:

   \[
   \frac{V}{N} = \frac{20,000 + 5,000}{2,000} = 12.50
   \]

   When valuing the warrant, we use the standard deviation of this alternative share. This can be obtained from the following relationship (see chapter footnote 21):

   The proportion of the firm financed by equity (calculated before the issue of the warrant) times the standard deviation of stock returns (calculated before the issue of the warrant)

   *is equal to*

   the proportion of the firm financed by equity (calculated after the issue of the warrant) times the standard deviation of the alternative share.
b. The value of the warrant is equal to the value of \( 1/(1 + q) \) call options on the alternative share, where \( q \) is the number of warrants issued per share outstanding. For Electric Bassoon:

\[
q = \frac{1,000}{2,000} = 0.5
\]

Therefore:

\[
1/(1 + q) = 1/1.5 = 0.67
\]

The value of the warrant is: \( (0.67 \times $6) = $4 \)

At the current price of $5 the warrants are overvalued.

15. a. \( P = 24.47 \quad EX = 24.47 \quad \sigma = 0.42 \quad t = 7.0 \quad r_f = 0.039 \)

\[
d_1 = \frac{\log[P/PV(EX)]}{\sigma \sqrt{t}} + \frac{\sigma \sqrt{t}}{2}
\]

\[
= \frac{\log[24.47/(24.47/1.039)]}{(0.42 \times \sqrt{7.0})} + (0.42 \times \sqrt{7.0})/2 = 0.7966
\]

\[
d_2 = d_1 - \sigma \sqrt{t} = 0.7966 - (0.42 \times \sqrt{7.0}) = -0.3146
\]

\[
N(d_1) = N(0.7966) = 0.7872
\]

\[
N(d_2) = N(-0.3146) = 0.3765
\]

Call value = \( [N(d_1) \times P] - [N(d_2) \times PV(EX)] \)

\[
= [0.7872 \times 24.47] - [0.3765 \times (24.47/1.039^7)] = $12.21
\]

Price per option \( \times \) number of options = \( $12.21 \times 254 \text{ million} = $3.101 \text{ billion} \)

b. Answers will vary depending on the remaining time to expiration of the warrants and the current price of Kindred Healthcare common stock.

16. Individual exercise; answers will vary.
Challenge Questions

1. For the one-period binomial model, assume that the exercise price of the options (EX) is between $u$ and $d$. Then, the spread of possible option prices is:
   For the call: $[(u - EX) - 0]$
   For the put: $[(d - EX) - 0]$

   The option deltas are:
   Option delta(call) = $[(u - EX)/(u - d)]$
   Option delta(put) = $[(d - EX)/(u - d)]$

   Therefore:
   $[Option delta(call) - 1] = [(u - EX)/(u - d)] - 1$
   $= [(u - EX)/(u - d)] - [(u - d)/(u - d)]$
   $= [(u - EX) - (u - d)]/(u - d)$
   $= [d - EX]/(u - d) = Option delta(put)$

2. If the exercise price of a call is zero, then the option is equivalent to the stock, so that, in order to replicate the stock, you would buy one call option. Therefore, if the exercise price is zero, the option delta is one. If the exercise price of a call is indefinitely large, then the option value remains low even if there is a large percentage change in the price of the stock. Therefore, the dollar change in the value of the option will be much smaller than the dollar change in the price of the stock, so that the option delta is close to zero. Between these two extreme cases, the option delta varies between zero and one.

3. Both of these announcements may convey information about company prospects, and thereby affect the price of the stock. But, when the dividend is paid, stock price decreases by an amount approximately equal to the amount of the dividend. This price decrease reduces the value of the option. On the other hand, a stock repurchase at the market price does not affect the price of the stock. Therefore, you should hope that the board will decide to announce a stock repurchase program.
4. a. As the life of the call option increases, the present value of the exercise price becomes infinitesimal. Thus the only difference between the call option and the stock is that the option holder misses out on any dividends. If dividends are negligible, the value of the option approaches its upper bound, i.e., the stock price.

b. While it is true that the value of an option approaches the upper bound as maturity increases and dividend payments on the stock decrease, a stock that never pays dividends is valueless.