Problem Set 1 Answer Key
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1. Mankiw textbook, Chapter 3, Problems and Applications #1.

Ans:
   a. According to the neoclassical theory of distribution, the real wage equals the marginal product of labor. Because of diminishing returns to labor, an increase in the labor force causes the marginal product of labor to fall. Hence, the real wage falls.
   b. The real rental price equals the marginal product of capital. If an earthquake destroys some of the capital stock (yet miraculously does not kill anyone and lower the labor force), the marginal product of capital rises and, hence, the real rental price rises.
   c. If a technological advance improves the production function, this is likely to increase the marginal products of both capital and labor. Hence, the real wage and the real rental price both increase.

2. Mankiw textbook, Chapter 3, Problems and Applications #3.

Ans:
   a. A Cobb–Douglas production function has the form \( Y = AK^\alpha L^{1-\alpha} \). The text showed that the marginal products for the Cobb–Douglas production function are:
      \[
      MPL = (1 - \alpha)Y/L, \\
      MPK = \alpha Y/K.
      \]

      Competitive profit-maximizing firms hire labor until its marginal product equals the real wage, and hire capital until its marginal product equals the real rental rate. Using these facts and the above marginal products for the Cobb–Douglas production function, we find:
      \[
      W/P = MPL = (1 - \alpha)Y/L. \\
      R/P = MPK = \alpha Y/K.
      \]

      Rewriting this:
      \[
      (W/P)L = MPL \times L = (1 - \alpha)Y, \\
      (R/P)K = MPK \times K = \alpha Y.
      \]

      Note that the terms \((W/P)L\) and \((R/P)K\) are the wage bill and total return to capital, respectively. Given that the value of \(\alpha = 0.3\), then the above formulas indicate that labor receives 70 percent of total output, which is \((1 - 0.3)\), and capital receives 30 percent of total output.
b. To determine what happens to total output when the labor force increases by 10 percent, consider the formula for the Cobb–Douglas production function:

\[ Y = AK^\alpha L^{1-\alpha}. \]

Let \( Y_1 \) equal the initial value of output and \( Y_2 \) equal final output. We know that \( \alpha = 0.3 \). We also know that labor \( L \) increases by 10 percent:

\[ Y_1 = AK^{0.3}L^{0.7}. \]
\[ Y_2 = AK^{0.3}(1.1L)^{0.7}. \]

Note that we multiplied \( L \) by 1.1 to reflect the 10-percent increase in the labor force.

To calculate the percentage change in output, divide \( Y_2 \) by \( Y_1 \):

\[ \frac{Y_2}{Y_1} = \frac{AK^{0.3}(1.1L)^{0.7}}{AK^{0.3}L^{0.7}} = (1.1)^{0.7} = 1.069. \]

That is, output increases by 6.9 percent.

To determine how the increase in the labor force affects the rental price of capital, consider the formula for the real rental price of capital \( R/P \):

\[ R/P = MPK = \alpha AK^{\alpha -1}L^{1-\alpha}. \]

We know that \( \alpha = 0.3 \). We also know that labor \( L \) increases by 10 percent. Let \((R/P)_1\) equal the initial value of the rental price of capital, and \((R/P)_2\) equal the final rental price of capital after the labor force increases by 10 percent. To find \((R/P)_2\), multiply \( L \) by 1.1 to reflect the 10-percent increase in the labor force:

\[ (R/P)_1 = 0.3AK^{0.7}L^{0.3}. \]
\[ (R/P)_2 = 0.3AK^{0.7}(1.1L)^{0.3}. \]

The rental price increases by the ratio

\[ \frac{(R/P)_2}{(R/P)_1} = \frac{0.3AK^{0.7}(1.1L)^{0.3}}{0.3AK^{0.7}L^{0.3}} = (1.1)^{0.7} = 1.069. \]

So the rental price increases by 6.9 percent.

3. The definition of “capital share” of an economy is \((MPK \times K)/Y\) which measures the fraction of total output that goes to the capital owners as factor payments. Similar, the “labor share” is \((MPL \times L)/Y\). Show that for a Cobb-Douglas production function \( Y = F(K,L) = AK^\alpha L^\beta \), the capital share equals \( \alpha \) and the labor share equals \( \beta \).

\textbf{Ans:}

\[ MPK = \frac{\partial F(K,L)}{\partial K} = A\alpha K^{\alpha -1}L^\beta \]

Capital Share \( = \frac{MPK \cdot K}{Y} = \frac{A\alpha K^{\alpha -1}L^\beta \times K}{AK^\alpha L^\beta} = \alpha \).

Similarly, Labor Share \( = \beta \).