Note: Answers without explanation or calculation earn no point.

Problem 1 (15%) Answer the following questions.

1. Let $A$ be an event of the sample space (state space) $\Omega$ (that is, $A \subset \Omega$). Then show that $A$ and $\Omega$ are independent.

2. Given that $Z = aX + b$ and $W = cY + d$, where $X$, $Y$ are random variables, and $a$, $b$, $c$ and $d$ are positive constants. Show that the correlation coefficient between $Z$ and $W$ equals to the correlation coefficient between $X$ and $Y$. That is, $\rho_{ZW} = \rho_{XY}$.

3. Given that $X \sim N(0,1)$ with MGF

$$M_X(t) = e^{tx^2}.$$ 

Let $Y = \mu + \sigma X$, use MGF to show that

$$Y \sim N(\mu, \sigma^2).$$

Problem 2 (20%) Given that $X \sim U[-1,1]$. Let random variable $Y$ be defined as

$$Y = \begin{cases} -1 & \text{if } |X| < \frac{1}{2}, \\ 1 & \text{if } |X| \geq \frac{1}{2}. \end{cases}$$

1. Find $E(X) =$?

2. Find $E(Y) =$?

Problem 3 (40%) Let $Z \sim U(0,1)$, and $Y = -\ln(1 - Z)$.

1. Use CDF technique to identify the distribution of $Y$.

2. Suppose that $X \sim$ Poisson$(Y)$.

(a) Find $E(X|Y = y) =$?

(b) Find $E(X) =$?

(c) Find $Var(X) =$?
Problem 4 (10%)  Given that \{X, Y\} \sim \text{BTP}(2, \frac{4}{3})..

1. Find out the joint pmf \( f(x, y) \).

2. Show that
\[
\sum_{y \in \text{supp}(Y)} f(x, y) = f(x).
\]

Problem 5 (15%)  假定經濟系的學生有 25% 的人有閃光，而統計助教認為，在求學期間與異性發展友達以上的情感會影響學習。以該班期中考成績來看，有閃光的學生有 40% 的人不及格，沒有閃光的學生有 \( p \) 的比例不及格。令 \( F \) 代表隨機抽出一名學生，而該學生有閃光；\( A \) 代表隨機抽出一名學生，而該學生成績及格。

1. 不及格的比率如為 25%，請問 \( p \) 爲多少？

2. 承上，請問及格的學生當中，有多少比例的學生有閃光？

3. 承上，請問統計助教的想法是否正確？