Problem 1  State, with explanation, whether the following statements are TRUE or FALSE. You will NOT receive credit if you do not justify your answers.

1. (3%) If $A$ and $B$ are disjoint, then $P(A - B | A \cup B) = \frac{P(A)}{P(A) + P(B)}$.

2. (3%) $f(x) = \frac{1}{x-1}$ is the discrete pdf of some random variable, $X = \{3, 4, 5\}$.

3. Let $Z$ be a random variable which takes the value $z_1$ with probability 0.75 and the value $z_2$ with probability 0.25, where $z_1 \neq z_2$. Then
   
   (a) (3%) $E(z_1) = 0.75z_1 + 0.25z_2$
   
   (b) (3%) $Var(Z) = 0.75(z_1 - E(z_1))^2 + 0.25(z_2 - E(z_2))^2$.
   
   (c) (3%) $E(3^Z) = 0.75 \times 3^{z_1} + 0.25 \times 3^{z_2}$.

Problem 2 (15%) Exam is not accurate for measuring whether a student studies hard or not. Suppose a midterm exam has the following properties: If the student studies hard, 90% of the time the student gets high score. If the student does not study hard, 95% of the time the student gets low score. Finally, suppose that only 0.1% of all students taking exam, in fact, do not study hard.

Now, if the student receives low score in the exam, what is the probability that, in fact, the student truly does not study hard?
Problem 3  The following is the information about the returns on Blue chip stock (B) and the returns on Technology stock (T):

\[
E(R_B) = 0.10 \quad E(R_T) = 0.12 \\
Var(R_B) = 0.0324 \quad Var(R_T) = 0.0400 \\
Cov(R_B, R_T) = 0.0206
\]

The market portfolio (super-efficient portfolio) has expected returns of 0.110 and variance of returns of 0.0282. There exists a capital allocation line with slope equaling 0.7. The slope of the capital market line is 0.3.

1. (5%) Compute the expected returns and variance of returns of a portfolio which consists of 15% Blue chip stock and 85% Technology stock.

2. (5%) Is there a portfolio consisting Blue chip stock and Technology stock which exhibits lower absolute risk than both the portfolio consisting solely of Blue chip and the portfolio consisting solely of Technology? Why or why not?

3. (5%) Compute \( Cov(R_B, R_m) \) and \( Cov(R_T, R_m) \).

4. (5%) Which stock, Blue chip or Technology, exhibits greater absolute risk? Why?

5. (5%) Which stock, Blue chip or Technology, exhibits greater marginal risk? Why?
Problem 4  The joint distribution of the returns of eBay and the returns on the market portfolio (super-efficient portfolio) are presented in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$R_m$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>$k$</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>0.06</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>0.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. (5%) Find out the value of $k$.

2. (5%) What is the beta of eBay, $\beta_e$?

3. (5%) What is the risk-free rate of return?

Problem 5 (10%)  設豆瓣和運動的觀察值求得平均數為 56, 標準差為 9 後, 發現遺漏了 32 及 60 兩數, 試求加入該兩數後的平均數與標準差。Hint: $Var(X) = \frac{1}{n}(\sum_{i=1}^{n} x_i^2 - n\mu^2)$

Problem 6  一間斷機率密度函數如下所示，

$$f(x) = \begin{cases} 
  cx, & x = 1, 2 \\
  (x - 1)c, & x = 3, 4 \\
  (x - 4)c^2, & x = 5, 6 \\
  7c^2 + c, & x = 7, 8 \\
  0, & \text{otherwise}
\end{cases}$$

試求:

1. (5%) $c$ 之值

2. (5%) $P(X \geq 5)$

3. (5%) Find out the CDF of $X$ (denote it as $F(x)$)

4. (5%) Sketch the CDF of $X$