1. Suppose that the annual marginal benefit of crime prevention program that puts police walking the beat on the streets of Crimeville has been estimated as \( MB = 500,000 - 1,200q \), where \( q \) is the number of police employed in the program.
   a. What is the total benefit of the program when 200 police are employed?
   b. What is the change in total benefit if we increase the number of police from 200 to 250?
   c. If the marginal cost of employing a policeman is $80,000 per year, what is the optimal number of policemen to hire?
   d. If the optimal number of police officers are hired, what is the expected net benefit of the program?

Answers:
   a. At \( q = 200 \), \( MB = 500,000 - 1,200(200) = 260,000 \). Total benefit is the area under the MB curve from \( q = 0 \) out to \( q = 200 \), which is equal to \((\frac{1}{2})(500,000 - 260,000)(200) = 24 \text{ million}\).
   b. When \( q \) rises to \( q = 250 \), \( MB = 200,000 \) and total benefit is the area under the MB curve from \( q = 0 \) out to \( q = 250 \), which is equal to \((\frac{1}{2})(500,000 - 200,000)(250) = 37.5 \text{ million}\). The change in total benefit from added police is 13.5 million.
   c. Set \( MB = MC \)
      \[ 80,000 = 500,000 - 1,200q \]
      solve for \( q = 350 \).
   d. Net benefit at \( q = 350 \)
      total benefit is: \((\frac{1}{2})(500,000 - 80,000)(350) = 73.5 \text{ million}\)
      total cost is: \( (350)(80,000) = 28 \text{ million}\)
      net benefit = 73.5 - 28 = 45.5 million.

2. The total benefits of the COMMIT program which is designed to encourage school children to commit themselves to a life free of drug addiction has been estimated to be \( B = 800q - 4q^2 \), where \( q \) is the number of days the program is run in local schools. The total cost of running the program is $400 per day.
   a. What is the maximum number of days that the COMMIT program can be run before net benefits turn negative?
   b. What is the optimal number of days to run the program in order to maximize net benefits? (MB = 800 - 8q.)
   c. At the optimal number of days, what is the net benefit of the program?
   d. If the cost of running the program were to rise to 500 per day, how would that affect the optimal number of days the program should be run?

Answers:
a. Net benefit = B - C = 800q - 4q^2 - 400q. This equals zero at q=100.
b. Set MB = MC. 800 - 8q = 400, so q = 50.
c. At q = 50, B - C = 800(50) - 4(50)^2 - 400(50) = 10,000
d. Set MB = MC. 800 - 8q = 500, so q = 300/8 = 37.5.

3. Use the perpetuity formula to estimate the value of the following projects generating net benefits in perpetuity.
   a. Bi = 5,000, r = 0.06.
   b. Bi = 10,000, r = 0.06.
   c. Bi = 10,000, r = 0.05.
   d. Bi = 10,000, r = 0.04.
   e. Bi = 5,000, r = 0.035.

   Answers:
   a. 83,333.33
   b. 166,666.67
   c. 200,000.00
   d. 250,000.00
   e. 142,857.14

4. Sketch the conceptual outline of how you would conduct a cost-benefit analysis of the 55 mph speed limit law implemented in the U.S. during the 1970s and 80s.
   a. Identify benefits, both direct and indirect.
   b. Identify costs, both direct and indirect.
   c. Suggest an evaluation criterion and its application.
   d. Suppose that you had the option of determining the appropriate scale for a general speed limit law, i.e. you could select any speed limit, not just 55 mph. Explain how you would determine the most economical speed limit.

   Answers:
   a. Direct benefits: value of lives saved, value of serious injuries avoided, value of automobiles not damaged, lower automobile insurance rates for all drivers, better gas mileage and consequently lower prices for gasoline because of decreased demand, increased state revenue due to speed limit enforcement. Indirect benefits: increased sales of restaurant meals, snack foods, motel room rentals due to slower travel times and increased length of trips.
   b. Direct costs: increased travel and freight costs due to longer trip times, increased cost of speed limit enforcement. Indirect costs: increased costs associated with restaurant meals, snack foods, motel room rentals due to slower travel times and increased length of trips.
   c. Positive net benefits. The scale of the project, enforcement of a 55 mph speed limit, is already selected, so the evaluation criterion simply involves comparing the value of the various benefits, all properly discounted, to the value of the costs and proceeding if they are positive.
   d. If the scale can be chosen, then the evaluation criterion could be maximizing
net benefits. The speed limit could be set where the difference between
benefits and costs is greatest. Perhaps a 60 MPH limit or some other level
would provide greater net benefits than the 55MPH limit.

5. Analyze the Phillip Morris study of the public finance consequences of smoking
discussed in the Policy Study: The Benefits of Smoking. Do so by answering the
following questions.
   a. Identify the primary benefits and costs of smoking.
   b. Identify secondary benefits and costs of smoking.
   c. Should the cost savings due to smokers’ early deaths be included?
   d. Are there other costs that the study ignores?

   Answers:
   a. Primary benefits: pleasure derived from smoking, tax revenue collected on
      the sale of tobacco products, reduced health care costs due to early deaths of
      smokers. Primary costs: cost of tobacco products, premature deaths of
      smokers leading to loss of productive years of work; costs of public health
      care for smokers.
   b. Secondary benefits: income generated from smoking cessation products and
      programs. Secondary costs: costs of smoking cessation products and
      programs, cost of social programs for caring for families of dead smokers,
      costs of illnesses for nonsmokers caused by second-hand smoke.
   c. The cost savings should be included.
   d. The lost productivity of the smokers and the costs to their families and society
      of caring for their families.

6. A cost-benefit analysis of a new irrigation project indicates that the net benefits of the
project in each of the first four years will be -$2 million. That is, \( B_i - C_i = -2 \) million
for \( i = 1, 2, 3, 4 \). Thereafter, the project will yield positive net benefits of $750,000
each year for the next 21 years, i.e. \( B_i - C_i = .75 \) million for \( i = 5...25 \).

   a. Calculate the present value of net benefits when the social rate of discount is
      10%. (Assume that the net benefits in year \( i \) are received at the end of year \( i \)
in this and other problems.)
   b. Explain whether the program merits approval on the basis of this information.
   c. How is the present value of the net benefits changed if the social rate of
discount is 5%? Explain why.

   Answers:
   a. The present value of net benefits for this 25-year project with a social discount
      rate of 10 percent is -$1,909,349.96.

   Solution hints: Set up columns in a spreadsheet program like Excel. Find the
discounted value for Year 1, then multiply it and every succeeding cell by \( 1/(1 + \text{discount rate}) \). Sum the annual discounted values to find the present value of
the net benefits. (See the table at the conclusion of this solution) Or: Use the
NPV function within a spreadsheet program to find the net present value. You
may want to use both methods to confirm your answer.

b. The program does not merit approval. It has a negative present value of net benefits.

c. The present value of the net benefits of this project change to $819,094.54 when the social discount rate is 5 percent. The change from a negative to a positive net present value occurs because with the lower discount rate the present value of the years of positive benefits count for more than when the discount rate is higher.

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7. Calculate the present value of each of the following income streams using a 5% discount rate:

a. An income stream of $1,000 per year for 20 years.
b. An income stream of $1,000 per year for years 1-10 followed by an income stream of $500 per year for years 11-20.
c. A net income stream of -$1,000 per year for years 1-3 followed by an income stream of $2,000 per year for years 4-10.
d. A perpetual income stream of $1,000 per year.
e. A perpetual income stream of $10,000 per year.

Answers:
a. $12,462.21
b. $10,091.97
c. $7,273.73
d. $20,000
e. $200,000

8. Suppose that a proposed government irrigation project lowers the marginal cost of producing food and thereby lowers the market price of food. If the market demand for food is given by the estimated inverse demand equation \( P = 3 - 0.2Q \), where \( P \) is a food price index and \( Q \) is the quantity of food measured in millions of pounds per year, and the irrigation project reduces the price from 1.4 to 1.2, compute the benefit of the project reducing food prices that must be included in the cost-benefit study.

Answer:
If the food price index was 1.4 before the project, then the quantity of food produced must have been 8 million pounds. \( 0.2Q = 3 - 1.4; 0.2Q = 1.6; Q = 8 \) The project promises to reduce the index to 1.2, so the new value of \( Q \) must be 9 million pounds. \( 0.2Q = 3 - 1.2; 0.2Q = 1.8; Q = 9 \) The annual value of the project in reduced food prices is the increased consumer surplus (area under the demand curve, above the given price). Initially, the consumer surplus is \( CS^0 = (1/2)(3.0-1.4)(8) = 6.4 \). After the price reduction, the consumer surplus is \( CS^1 = (1/2)(3.0-1.2)(9) = 8.1 \). Hence the change in consumer surplus is \( CS^1 - CS^0 = 8.1 - 6.4 = 1.7 \) million dollars. (No currency unit is given for the food price index, but it could be assumed the index is stated in dollars.) This value will occur every year of the life of the project. Thus, it must be discounted by the appropriate rate of social discount, but once this is done, the sum of the value is part of the benefit of the project.

Optional Advanced Problem:

9. You work for the State Department of Labor and have been given the task of evaluating a state job-training program. In order to conduct the analysis suppose that the typical client for the program has the following utility function of income \( I \):

\[
u = (I + \alpha)^\theta
\]
where $\alpha \geq 0, 0 < \beta < 1$.

a. Sketch the typical client’s utility function and explain the person’s attitude toward risk.

b. Suppose that you must evaluate a proposed government job-training program that would have an unpredictable effect on the typical client’s income, which is currently $20,000 per year. The program will leave the person’s annual income unchanged with probability 0.3, or it will increase his income by $10,000 with probability 0.7. Calculate the benefit of the program to the typical client, assuming that $\beta$=5,000 and $\delta$=0.5.

Answers:

A.

The typical client’s utility function is concave, reflecting an aversion to risk. If the value of $\delta$ was 1, then the person would be risk neutral, and if the value was greater than 1, the person would be risk-loving. As it is, however, the marginal utility of additional dollars of income is declining. The effect is that the typical client is willing to forgo a chance at a larger income in return for certainty of a smaller income.

b. With $\delta = 0.5$, the typical client receives utility equal to the square root of Income plus $\beta$=5,000. The utility of the typical client when there is no program or no increase in income from the program is therefore $158.11$ (the
square root of $20,000 + $5,000). The utility to the typical client when the
program increases income by $10,000 is the square root of $30,000 + $5,000,
or $187.08. The benefit of the program to the typical client is the difference
in the expected value of utility caused by the program. The expected value of
the program is the weighted probability of the program not producing an
increase in income and producing an increase in income. The difference in
expected value is (0.3 x 158.11) + (0.7 x 187.08), or $178.39, minus the
expected value of utility without the program, $158.11, which is $20.28.