Volatility Smiles

Chapter 16
Put-Call Parity Arguments

- Put-call parity \( p + S_0 e^{-qT} = c + Ke^{-rT} \) holds regardless of the assumptions made about the stock price distribution.

- It follows that

\[ p_{\text{mkt}} - p_{\text{bs}} = c_{\text{mkt}} - c_{\text{bs}} \]
Implied Volatilities

- When $p_{bs}=p_{mkt}$, it must be true that $c_{bs}=c_{mkt}$
- It follows that the implied volatility calculated from a European call option should be the same as that calculated from a European put option when both have the same strike price and maturity.
- The same is approximately true of American options.
Volatility Smile

- A volatility smile shows the variation of the implied volatility with the strike price.
- The volatility smile should be the same whether calculated from call options or put options.
The Volatility Smile for Foreign Currency Options
(Figure 16.1, page 377)
Implied Distribution for Foreign Currency Options (Figure 16.2, page 377)

- Both tails are heavier than the lognormal distribution
- It is also “more peaked” than the lognormal distribution
The Volatility Smile for Equity Options (Figure 16.3, page 380)
Implied Distribution for Equity Options (Figure 16.4, page 380)

- The left tail is heavier and the right tail is less heavy than the lognormal distribution
Other Volatility Smiles?

What is the volatility smile if

- True distribution has a less heavy left tail and heavier right tail
- True distribution has both a less heavy left tail and a less heavy right tail
Possible Causes of Volatility Smile

- Asset price exhibiting jumps rather than continuous change
- Volatility for asset price being stochastic

(One reason for a stochastic volatility in the case of equities is the relationship between volatility and leverage)
Volatility Term Structure

- In addition to calculating a volatility smile, traders also calculate a volatility term structure.
- This shows the variation of implied volatility with the time to maturity of the option.
Volatility Term Structure

The volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low.
Example of a Volatility Surface
(Table 16.2, page 382)

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>0.90</th>
<th>0.95</th>
<th>1.00</th>
<th>1.05</th>
<th>1.10</th>
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<tr>
<td>1 month</td>
<td>14.2</td>
<td>13.0</td>
<td>12.0</td>
<td>13.1</td>
<td>14.5</td>
</tr>
<tr>
<td>3 month</td>
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<td>13.0</td>
<td>12.0</td>
<td>13.1</td>
<td>14.2</td>
</tr>
<tr>
<td>6 month</td>
<td>14.1</td>
<td>13.3</td>
<td>12.5</td>
<td>13.4</td>
<td>14.3</td>
</tr>
<tr>
<td>1 year</td>
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<td>14.0</td>
<td>13.5</td>
<td>14.0</td>
<td>14.8</td>
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<tr>
<td>2 year</td>
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<td>14.4</td>
<td>14.0</td>
<td>14.5</td>
<td>15.1</td>
</tr>
<tr>
<td>5 year</td>
<td>14.8</td>
<td>14.6</td>
<td>14.4</td>
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