Day Count Conventions in the U.S. (Page 129)

Treasury Bonds: Actual/Actual (in period)

Corporate Bonds: 30/360

Money Market Instruments: Actual/360

- The day count convention is used to calculate the interest earned between the two dates
Treasury Bond (>10yrs) Price Quotes in the U.S.

Cash price (dirty price)

= Quoted price (clean price) + Accrued Interest
The short side can deliver any government bond has more than 15 years to maturity on the first day of the delivery month and is not callable with 15 years from that day.
Cash price received by party with short position =
Most Recent Settlement Price ×
Conversion factor + Accrued interest
Example

- Quoted price of bond futures = 90.00
- Conversion factor = 1.3800
- Accrued interest on bond = 3.00
- Price received for bond is
  \[1.3800 \times 9.00 + 3.00 = $127.20\]
  (per $100 of principal)
Conversion Factor

The conversion factor for a bond is approximately equal to the value of the bond on the assumption that the yield curve is flat at 6% with semiannual compounding.
CBOT
T-Bonds & T-Notes

- Factors that affect the futures price:
  - Delivery can be made any time during the delivery month
  - Any of a range of eligible bonds can be delivered
  - The wild card play

- Above statements describe the delivery option in the Treasury bond futures contract
Eurodollar Futures (Page 137-142)

- A Eurodollar is a dollar deposited in a bank outside the United States.
- The Eurodollar interest rate is the rate of interest earned on Eurodollars deposited by one bank with another bank. It is essentially the same as LIBOR introduced before.
- Eurodollar futures are futures locks the 3-month Eurodollar deposit forward rate at the maturity date of the futures.
One contract is on the rate earned on $1 million

If $Z$ is the quoted price of a Eurodollar futures contract, the value of one contract is $10,000[100-0.25(100-Z)]$

A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25
Eurodollar Futures continued

- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month) $Z$ is set equal to 100 minus the 90 day Eurodollar interest rate (actual/360) and all contracts are closed out
Example

● Suppose you buy (take a long position in) a contract on November 1
● The contract expires on December 21
● The prices are as shown
● How much do you gain or lose a) on the first day, b) on the second day, c) over the whole time until expiration?
## Example

<table>
<thead>
<tr>
<th>Date</th>
<th>Quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov 1</td>
<td>97.12</td>
</tr>
<tr>
<td>Nov 2</td>
<td>97.23</td>
</tr>
<tr>
<td>Nov 3</td>
<td>96.98</td>
</tr>
<tr>
<td>.......</td>
<td>.....</td>
</tr>
<tr>
<td>Dec 21</td>
<td>97.42</td>
</tr>
</tbody>
</table>
Example continued

- If on Nov. 1 you know that you will have $1 million to invest on for three months on Dec 21, the contract locks in a rate of $100 - 97.12 = 2.88%.

- In the example you earn $100 – 97.42 = 2.58% on $1 million for three months ($6,450) and make a gain day by day on the futures contract of $30×$25 = $750.
Forward Rates and Eurodollar Futures (Page 139-142)

- Eurodollar futures contracts last as long as 10 years
- For Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate
Forward Rates and Eurodollar Futures continued

- There are two reasons
  - Futures is settled daily while forward is settled once
  - Futures is settled at the beginning of the underlying three-month period; forward is settled at the end of the underlying three-month period

- The variable underlying the Eurodollar futures contract is an interest rate and tends to be highly positively correlated to other interest rates

=> futures rate > forward rates
Forward Rates and Eurodollar Futures continued

A "convexity adjustment" often made is

Forward rate = Futures rate $- \frac{1}{2}\sigma^2 t_1 t_2$

where $t_1$ is the time to maturity of the futures contract, $t_2$ is the maturity of the rate underlying the futures contract (90 days later than $t_1$) and $\sigma$ is the standard deviation of the short rate changes per year (typically $\sigma$ is about 0.012)
Convexity Adjustment when $\sigma = 0.012$ (Table 6.3, page 141)

<table>
<thead>
<tr>
<th>Maturity of Futures</th>
<th>Convexity Adjustment (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.2</td>
</tr>
<tr>
<td>4</td>
<td>12.2</td>
</tr>
<tr>
<td>6</td>
<td>27.0</td>
</tr>
<tr>
<td>8</td>
<td>47.5</td>
</tr>
<tr>
<td>10</td>
<td>73.8</td>
</tr>
</tbody>
</table>
Extending the LIBOR Zero Curve

- LIBOR deposit rates define the LIBOR zero curve out to one year.
- Eurodollar futures can be used to determine forward rates and the forward rates can then be used to bootstrap the zero curve.
Example

\[
F_i = \frac{R_{i+1}T_{i+1} - R_i T_i}{T_{i+1} - T_i}
\]

so that

\[
R_{i+1} = \frac{F_i(T_{i+1} - T_i) + R_i T_i}{T_{i+1}}
\]

If the 400 day LIBOR rate has been calculated as 4.80% and the forward rate for the period between 400 and 491 days is 5.30 the 491 days rate is 4.893%
Duration Matching

- This involves hedging against interest rate risk by matching the durations of assets and liabilities.
- It provides protection against small parallel shifts in the zero curve.
Use of Eurodollar Futures

- One contract locks in an interest rate on $1 million for a future 3-month period.
- How many contracts are necessary to lock in an interest rate for a future six month period?
Duration-Based Hedge Ratio

\[ \frac{PD_P}{F_C D_F} \]

interest rate futures will be shorted for hedge

- \( F_C \) Contract price for interest rate futures
- \( D_F \) Duration of asset underlying futures at maturity
- \( P \) Value of portfolio being hedged
- \( D_P \) Duration of portfolio at hedge maturity
Example

- It is August. A fund manager has $10 million invested in a portfolio of government bonds with a duration of 6.80 years and wants to hedge against interest rate moves between August and December.
- The manager decides to use December T-bond futures. The futures price is 93-02 or 93.0625 and the duration of the cheapest to deliver bond is 9.2 years.
- The number of contracts that should be shorted is

\[
\frac{10,000,000}{93,062.50} \times \frac{6.80}{9.20} = 79
\]
Limitations of Duration-Based Hedging

- Assumes that only parallel shift in yield curve take place
- Assumes that yield curve changes are small
GAP Management (Business Snapshot 6.3)

This is a more sophisticated approach used by banks to hedge interest rate. It involves:

- Bucketing the zero curve
- Hedging exposure to situation where rates corresponding to one bucket change and all other rates stay the same.