Hedging Strategies Using Futures

Chapter 3
Long & Short Hedges

- A long futures hedge is appropriate when you know you will purchase an asset in the future and want to lock in the price.
- A short futures hedge is appropriate when you know you will sell an asset in the future & want to lock in the price.
Arguments in Favor of Hedging

- Companies should focus on the main business they are in and take steps to minimize risks arising from interest rates, exchange rates, and other market variables.
Arguments against Hedging

- Shareholders are usually well diversified and can make their own hedging decisions.
- It may increase risk to hedge when competitors do not.
- Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult.
Basis Risk

- Basis is the difference between spot & futures
- Basis risk arises because of the uncertainty about the basis when the hedge is closed out
Convergence of Futures to Spot
(Hedge initiated at time $t_1$ and closed out at time $t_2$)

<table>
<thead>
<tr>
<th>Time</th>
<th>Futures Price</th>
<th>Spot Price</th>
<th>Basis &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Futures Price</th>
<th>Spot Price</th>
<th>Basis &gt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Long Hedge

- Suppose that
  \[ F_1 : \text{Initial Futures Price} \]
  \[ F_2 : \text{Final Futures Price} \]
  \[ S_2 : \text{Final Asset Price} \]
- You hedge the future purchase of an asset by entering into a long futures contract
- Cost of Asset = \[ S_2 - (F_2 - F_1) = F_1 + \text{Basis} \]
Short Hedge

- Suppose that
  \( F_1 \): Initial Futures Price
  \( F_2 \): Final Futures Price
  \( S_2 \): Final Asset Price

- You hedge the future sale of an asset by entering into a short futures contract

- Price Realized = \( S_2 + (F_1 - F_2) = F_1 + \text{Basis} \)
Choice of Contract

- Choose a delivery month that is as close as possible to, but later than, the end of the life of the hedge.
- When there is no futures contract on the asset being hedged, choose the contract whose futures price is most highly correlated with the asset price. This is known as cross hedging.
Optimal Hedge Ratio

Proportion of the exposure that should optimally be hedged is

\[ \rho \frac{\sigma_S}{\sigma_F} \]

where

- \( \sigma_S \) is the standard deviation of \( \Delta S \), the change in the spot price during the hedging period,
- \( \sigma_F \) is the standard deviation of \( \Delta F \), the change in the futures price during the hedging period
- \( \rho \) is the coefficient of correlation between \( \Delta S \) and \( \Delta F \).
Optimal Hedge Ratio

\[
\begin{align*}
\min_h \text{var}(\Delta S - h \Delta F) &= \sigma_s^2 - 2h \rho \sigma_s \sigma_F + h^2 \sigma_F^2 \\
FOC \Rightarrow 2h \sigma_F^2 - 2 \rho \sigma_s \sigma_F &= 0 \\
\Rightarrow h &= \rho \frac{\sigma_s}{\sigma_F} \\
\text{配方法} \Rightarrow (h \sigma_F - \rho \sigma_s)^2 + \sigma_s^2 - \rho^2 \sigma_s^2 \\
\Rightarrow h &= \rho \frac{\sigma_s}{\sigma_F}
\end{align*}
\]
To hedge the risk in a portfolio the number of contracts that should be shorted is

\[ \beta \frac{P}{A} \]

where \( P \) is the value of the portfolio, \( \beta \) is its beta, and \( A \) is the value of the assets underlying one futures contract.
Reasons for Hedging an Equity Portfolio

- Desire to be out of the market for a short period of time. (Hedging may be cheaper than selling the portfolio and buying it back.)
- Desire to hedge systematic risk (Appropriate when you feel that you have picked stocks that will outperform the market.)
Example

Value of S&P 500 is 1,000
One futures is for delivery of $250 \times \Delta \text{Index}
Value of Portfolio is $5 \text{ million}
Beta of portfolio is 1.5

What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?
Changing Beta

- What position is necessary to reduce the beta of the portfolio to 0.75?
- What position is necessary to increase the beta of the portfolio to 2.0?
Hedging Price of an Individual Stock

- Similar to hedging a portfolio
- Does not work as well because only the systematic risk is hedged
- The unsystematic risk that is unique to the stock is not hedged
Why Hedge Equity Returns

- May want to be out of the market for a while. Hedging avoids the costs of selling and repurchasing the portfolio.
- Suppose stocks in your portfolio have an average beta of 1.0, but you feel they have been chosen well and will outperform the market in both good and bad times. Hedging ensures that the return you earn is the risk-free return plus the excess return of your portfolio over the market.
We can use a series of futures contracts to increase the life of a hedge.

Each time we switch from 1 futures contract to another we incur a type of basis risk.