Chapter 4
Background on Traded Instruments
Introduction

- **Market Risk**
  Arising from the possibility of losses resulting from unfavorable market movements

- **Financial Instruments**
  - Fixed income (Bonds)
  - Forward rate agreements
  - Stocks
  - Foreign exchange
  - Forwards and Futures
  - Swaps
  - Options

* Valuation of each instrument is important in risk measurement because risk is all about potential changes in value
Bonds Structure

- Maturity
  - > 5 years => bonds
  - 1 ~ 5 years => notes
  - < 1 year => bills or money market instruments

- Issuer Credit Ratings
  - Credit spread (Investment grade bonds must be rated BBB or better)
  - The capital requirement of different credit quality of the bonds suggested by the Basel committee (p.52 Table 4-1)

- Payment Structure
  - Fixed-rate vs. Floating-rate

- Currency
Bonds Valuation

- Value = \sum_{t} \frac{C_t}{(1+r_t)^t}, where \ r_t \ is the discount interest rate

- Yield Curve (Term Structure) (p. 55 Figure 4-1)
  “Bootstrap” p.56 Table 4-2 的例子

- Yield to Maturity (IRR of a bond)

  Market Price = \sum_{t} \frac{C_t}{(1+y)^t}

  y: 代表平均的 discount rate
Duration

- if \( B = \sum_{t} \frac{C_t}{(1 + r)^t} \)
  - Macauley Duration \( = \sum_{t} t \cdot \frac{C_t}{(1 + r)^t} \) (單位是年)，\( \frac{C_t}{B} \) is the weight of time
  - Modified Duration \( = -\frac{d}{dr} \left[ \frac{\%}{\%/yr} \right] \) (比較精準)

\[
\begin{align*}
\frac{dB}{dr} &= \left( \frac{\%}{\%/yr} \right) \\
&= \frac{1}{B} \left( -\frac{dB}{dr} \right) \\
&= \frac{1}{B} \left( -\sum_{t} \frac{(-t)C_t}{(1 + r)^{t+1}} \right) \\
&= \frac{1}{B} \left( \sum_{t} \frac{t \cdot C_t}{(1 + r)^t} \cdot \frac{1}{1 + r} \right) \\
&= \frac{1}{1 + r} \cdot \sum_{t} t \cdot \frac{(1 + r)^t}{B} = \frac{1}{1 + r} \cdot \text{Macauley Duration}
\end{align*}
\]
## Effective Interest Rates

<table>
<thead>
<tr>
<th>Period per year (m)</th>
<th>Final Sum</th>
<th>EAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.06</td>
<td>6.0000%</td>
</tr>
<tr>
<td>2</td>
<td>$1.03^{2}=1.0609</td>
<td>6.0900%</td>
</tr>
<tr>
<td>4</td>
<td>$1.015^{4}=1.061364</td>
<td>6.1364%</td>
</tr>
<tr>
<td>12</td>
<td>$1.005^{12}=1.061678</td>
<td>6.1678%</td>
</tr>
<tr>
<td>365</td>
<td>$1.0001644^{365}=1.061831</td>
<td>6.1831%</td>
</tr>
<tr>
<td>∞</td>
<td>$e^{0.06}=1.061837</td>
<td>6.1837%</td>
</tr>
</tbody>
</table>
\[ B = \sum_{t} C_t \cdot e^{-rt} \]

- Macauley Duration = \[ \sum_{t} t \cdot \frac{C_t \cdot e^{-rt}}{B} \]

- Modified Duration = \[ -\frac{\frac{dB}{dr}}{\frac{B}{dr}} \]

\[ = \frac{1}{B} \left( - \sum_{t} C_t \cdot e^{-rt} \right) \]

\[ = \frac{1}{B} \left( - \sum_{t} C_t \cdot e^{-rt} \cdot (-t) \right) \]

\[ = \frac{1}{B} \left( \sum_{t} t \cdot C_t \cdot e^{-rt} \right) \]

\[ = \sum_{t} t \cdot \frac{C_t \cdot e^{-rt}}{B} \]

= Macauley Duration
Duration dollars = \(-\frac{dB}{dr}\) (\$/%)
• Effective Duration

\[ D^E = \frac{B(r - \Delta r) - B(r + \Delta r)}{(2\Delta r)B} \]

• Coupon Curve Duration

\[ D^{CC} = \frac{B(r; c + \Delta c) - B(r; c - \Delta c)}{(2\Delta c)B} \]

* This approach is useful for securities that are difficult to price under various yield scenarios because only the market prices of securities with different coupons are required
Forward Rate Agreements

\[
100 \cdot (1 + r_7)^7 = 100 \cdot (1 + r_2)^2 \cdot (1 + f_{2,7})^5
\]

\[\Rightarrow f_{2,7} = \left[ \frac{(1 + r_7)^7}{(1 + r_2)^2} \right]^{\frac{1}{5}} - 1\]
• **Equity**
  - Systemic risk $\rightarrow \beta$
  - Idiosyncratic risk $\rightarrow \varepsilon$ (個股風險)

\[
\Delta V = V \cdot r_s = V (\alpha + \beta \cdot r_m + \varepsilon)
\]

• **Foreign Exchange**
  - 外匯市場交易量最大，流動性也好
  - 包括外匯現貨與期貨、外國的有價證券等都有匯率風險
  - For example, consider a U.S. bank holding a bond issued by a Mexican Company. The bank could lose money if the company defaults, if the peso interest rates increase, or if the peso devalues compares with the US$. 
• Forwards

- an agreement to buy a security or commodity at a point in the future
- delivery price (contract price)
- delivery date

Contract value at \( t = \frac{N}{(1+r_f)^{T-t}}(D_t - D_0) \)

\[
\begin{align*}
0 & \quad \quad t & \quad \quad T \\
\text{delivery Price} & \quad D_0 & \quad D_t \\
\text{Contract Value} & \quad 0 & \quad \frac{N}{(1+r_f)^{T-t}} \cdot (D_t - D_0)
\end{align*}
\]

- A more complex example for a forward (interest and exchange rates parity)

* Futures: standardized amounts and delivery dates, daily settlement
**SWAP**

- Interest-Rates Swap \( \tilde{r} \leftrightarrow \bar{r} \)
- Currency Swap = FX spot + FX forward
- Basis Swap \( \tilde{r}_{US} \leftrightarrow \tilde{r}_{LIBOR} \)
- Equity Swap \( \text{equity index} \leftrightarrow \tilde{r}_{LIBOR} \)

**Options**

- Vanilla options
- Packages of vanilla options
- Exotic options
  - Puts
  - Calls
    - European-style
    - Bermudan-style
    - Asian-style
• P.68 ~ 72, BS formula
• P.73 ~ 75, Figure 4-6 ~ 4-10, Greek Letters
• P.76 ~ 78, Figure 4-12 ~ 4-13, Volatility Smile and Skew
• P.77 ~ 79, Binomial Tree and Monte Carlo Simulation
• P.81 ~ 83, Trade Strategies of Options
• P.84, Exotic Options
  ■ Forward Start Options
  ■ Binary Options
  ■ Lookback Options
  ■ Barrier Options
  ■ Asian Options
  ■ Chooser Options
• Risk Measurement for Options

\[
\Delta = \frac{\partial P}{\partial S} \quad \Gamma = \frac{\partial^2 P}{\partial S^2} \quad \nu = \frac{\partial P}{\partial \sigma} \quad \rho = \frac{\partial P}{\partial r} \quad \theta = \frac{\partial P}{\partial T}
\]

\[
\delta V = \Delta \times \delta S + \frac{1}{2} \Gamma \times (\delta S)^2 + \nu \times \delta \sigma + \rho \times \delta r + \theta \times \delta T
\]