Chapter 2
Risk Measurement at the Corporate Level: Economic Capital and RAROC
**Risk adjusted Return on Capital (RAROC)**

\[
RAROC = \frac{\text{Expected net risk-adjusted profit (ENP)}}{\text{Economic capital (EC)}}
\]

* RAROC was developed by Bankers Trust in the late 1970s, which is designed to solve the problem of evaluating the performance of traders with different risk profiles.
* RAROC has become the industry’s standard way of measuring risk-adjusted profitability.
• Economic Capital: the net value the bank must have at the beginning of the year to ensure that there is a small probability of defaulting within the year (Table 2-5 default prob.)

  - Net value = Asset – Liability (equity can be viewed as being a cushion against default)
  - EC for Credit Risk
  - EC for Market Risk
  - EC for Operating Risk

* Economic Capital is also called Risk Capital, which is basically a value-at-risk measure
1. 資本、風險與破產機率

- Examples

\[ ROE = \frac{E_1 - E_0}{E_0} \]

<table>
<thead>
<tr>
<th>( A_0 = 100 ), ( r_A = 6% )</th>
<th>( D_0 = 95 ), ( r_D = 5% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(預計 ( t=1 ) 賺回 106)</td>
<td>(( t=1 ) 預計要付 99.8)</td>
</tr>
<tr>
<td>( E_0 = 5 )</td>
<td></td>
</tr>
</tbody>
</table>
## Base Case

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0%</th>
<th>4%</th>
<th>8%</th>
<th>16% (extra risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>106</td>
<td>101.8</td>
<td>97.5</td>
<td>89</td>
</tr>
<tr>
<td>$D_1$</td>
<td>99.8</td>
<td>99.8</td>
<td>97.5</td>
<td>89</td>
</tr>
<tr>
<td>$E_1$</td>
<td>6.3</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ROE</td>
<td>25%</td>
<td>-60%</td>
<td>-100%</td>
<td>-100%</td>
</tr>
</tbody>
</table>
## Extra Capital

$A_0 = 100$, $r_A = 6\%$, $D_0 = 90$, $r_D = 5\%$

$E_0 = 10$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0%</th>
<th>4%</th>
<th>8%</th>
<th>16%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>106</td>
<td>101.8</td>
<td>97.5</td>
<td>89</td>
</tr>
<tr>
<td>$D_1$</td>
<td>94.5</td>
<td>94.5</td>
<td>94.5</td>
<td>89</td>
</tr>
<tr>
<td>$E_1$</td>
<td>11.5</td>
<td>7.3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>ROE</td>
<td>15%</td>
<td>-27%</td>
<td>-70%</td>
<td>-100%</td>
</tr>
</tbody>
</table>
The Relation between Capital, Risk, and Default Probability

* Table 2-4 and Figure 2-1 (Credit-Loss Scenarios)
* Figure 2-2, 2-3, and 2-4 (Prob. Dist. of the payoff of assets)
EC for Credit Risk

MPL : maximum probable loss
EL : expected loss
UL : unexpected loss
EC : economic capital

\[ \text{EL} = 106 \times (1 - 4\%) \]
• 證明 $EC \approx MPL - EL$

$$EC_0 = A_0 - D_0$$

- $A_0 = D_0 + EC_0$
- $D_1 = (1 + r_D)D_0$
- $A_1 = (1 + r_A)(1 - \lambda)A_0$
- $EC_1 = A_1 - D_1 = (1 + r_A)(1 - \lambda)A_0 - (1 + r_D)D_0$
- $\Rightarrow 0 = (1 + r_A)(1 - \lambda_p)A_0 - (1 + r_D)D_0$

其中 $\lambda_p$ 指能承受最大的破產風險 (worse case of the lose of principal)

- $\Rightarrow D_0 = A_0 \frac{(1 + r_A)(1 - \lambda_p)}{1 + r_D}$
\[
= A_0 \left( \frac{(r_D - r_A) + \lambda_p + \lambda_p r_A}{1 + r_D} \right)
\]

\[
= A_0 \lambda_p \frac{1 + r_A}{1 + r_D} - A_0 \frac{r_A - r_D}{1 + r_D}
\]

\[
= MPL \frac{1}{1 + r_D} - A_0 \frac{EL / A_0}{1 + r_D}
\]

\approx MPL - EL

其中 MPL = A_0 (1 + r_A) \lambda_p

\[
r_A = \frac{(A_0 - EC_0)r_D + OC + EL + H \times EC - F}{A_0}
\]

(A_0 - EC_0 \approx A_0, \ OC = F = 0, \ H = 0)

\[
\approx \frac{A_0 r_D + EL}{A_0} = r_D + \frac{EL}{A_0}
\]

(\frac{EL}{A_0} = \mu, \ 期望壞帳機率)
• EC for Market Risk (假設投資在無風險資產)

\[ EC = \frac{W_P}{(1 + r_f)} \]

- \( W_P \): maximum probable loss
- \( p \): income attributed to holding risky asset
- \( 0 \): profit

\( EC \): Expected Cost
• EC for Operating Risk

\[
EC = \frac{L_{\text{operating}}}{(1 + r_f)}
\]

income attributed to operations
2. RAROC

- 之前銀行業，都用較不精確的ROA，ROE 來做performance的比較，這些都沒考慮 risk，無論是credit risk，market risk，或是 operating risk，所以比較的結果較沒有意義

  - ROE之E可以是
    - book capital
    - Regulatory capital (政府規定的，目前與風險大小無關，但其實 應該與風險有關，所以在新的巴塞爾協定中，建議所需的資 本，應該與承擔的風險相關)
• 若用了EC之觀念來做調整，會好很多
  ■ Risk-adjusted Return on Capital (RAROC)
    ◆ For a loan asset

\[
RAROC = \frac{\text{ENP}}{\text{EC}} = \frac{A_0 r_A + F - D_0 r_D - \text{OC} - L}{\text{EC}}
\]

ENP: expected net risk-adjusted profit
F: fee
D_0 : A_0 - EC
OC : Operating Cost
L : Expected Loss
For a trading transaction

\[ \text{RAROC} = \frac{\text{ENP}}{\text{EC}} = \frac{\Delta V - \text{OC}}{\text{EC}} \]

\( \Delta V \) : Net Change of Portfolio Value
\( \text{OC} \) : Operating Cost

If RAROC is calculated on a prospective basis

\( L \rightarrow E[L] \)
\( \Delta V \rightarrow E[\Delta V] \)

The minimum required value for RAROC is called the hurdle rate, and the actual value chosen is around 12 to 20%
Shareholder value added (SVA)
SVA gives a dollar-based measure of performance
SVA = actual or expected profitability – required profitability to meet the hurdle rate

For a loan asset

$$SVA = (A_0 r_A + F - (A_0 - EC) r_D - OC - E[L]) - H \times EC$$

For a trading transaction

$$SVA = (E[\Delta V] - OC) - H \times EC$$