Homework 4

Price a lookback put with the binomial tree model. The payoff function of the lookback put is as follows.

\[ \text{Payoff}_t = \max(S_{\text{max},t} - S_t, 0), \text{ where } S_{\text{max},t} = \max S_u, \text{ for } u = 0, \Delta t, 2\Delta t, ..., t. \]

- Basic requirement (80 points):
  
  (i) Implement the binomial tree model to price both European and American lookback puts.
  
  (ii) Implement the Monte Carlo simulation to price European lookback puts.
  
  (Inputs: \( S_t, r, q, \sigma, t, T, S_{\text{max},t}, n, \) number of simulations, number of repetitions. Outputs: Option values for both methods and 95% confidence level for Monte Carlo simulation.)

- Bonus 1 (5 points):
  
  Based on the same binomial tree framework, devise and implement a quick way to determine the \( S_{\text{max}} \) list for each node.

- Bonus 2 (10 points):
  
  Implement the method in Cheuk and Vorst (1997) to price European and American lookback puts.

\[
\begin{align*}
\text{max}(S_{\text{max}} - S, 0) &= S_u \max\left(\frac{S_{\text{max}}}{S_t} - 1, 0\right) \\
p &= S_u \left[ E[e^{-rT} \max\left(\frac{S_{\text{max}}}{S_T} - 1, 0\right)] \right]
\end{align*}
\]

- Reference