Homework 1

Derive the closed-form formula for an option with the following payoff function:

\[
\text{Payoff at } T_0 = \begin{cases} 
K_1 - K_2 & S_T > K_4 \\
K_3 - K_4 & K_2 < S_T < K_3 \\
0 & \text{otherwise}
\end{cases}
\]

- Basic requirement (80 points):
  (i) Derive the closed-form formula using the martingale pricing method by hands.
  (ii) Based on the formula you derive, implement a program to price this option.
    (Inputs: \(S_0, r, q, \sigma, T, K_1, K_2, K_3, K_4\). Output: Option value.)

- Bonus (10 points):
  Employ the Monte Carlo simulation to price this option.
  Based on \(\ln S_T \sim N(\ln S_0 + (r - q - \sigma^2/2)T, \sigma^2 T)\), draw 10,000 random samples for \(S_T\) to compute an option price. Repeat the above step 20 times to obtain the 95% confidence interval for the option value:
  \[
  \text{[mean of 20 repetitions} - 2 \times (\text{s.d. of 20 repetitions}), \text{mean of 20 repetitions} + 2 \times (\text{s.d. of 20 repetitions})].
  \]