Homework 1

Derive the closed-form formula for an option with the following payoff function:

\[ \text{Payoff at } T_0 = \min(S_T - K_1, K_2 - K_1) \]

- Basic requirement (80 points):
  
  (i) Derive the closed-form formula using the martingale pricing method by hands.
  
  (ii) Based on the formula you derive, implement a program to price this option.
  
  (Inputs: \( S_0, r, q, \sigma, T, K_1, K_2, K_3, K_4 \). Output: Option value.)

- Bonus (10 points):

  Employ the Monte Carlo simulation to price this option.
  
  Based on \( \ln S_T \sim N(\ln S_0 + (r - q - \sigma^2/2)T, \sigma^2 T) \), draw 10,000 random samples for \( S_T \) to compute an option price. Repeat the above step 20 times to obtain the 95% confidence interval for the option value:
  
  \[ \text{mean of 20 repetitions} - 2 \times (\text{s.d. of 20 repetitions}), \text{mean of 20 repetitions} + 2 \times (\text{s.d. of 20 repetitions}) \].