Abstract—IEEE 802.22 is the first cognitive-radio-based wireless communication standard. To manage resource sharing, IEEE 802.22 designs an interbase station (inter-BS) coexistence mechanism enabling distributed competing BSs to coexist effectively. In this IEEE 802.22 mechanism, BSs could be considered as players optimizing their own performance in a self-organizing way. Therefore, it is suitable to apply game theory to model the convergence of self-organizing behaviors. An operating point where no BS can unilaterally increase its performance describes an outcome of the inter-BS coexistence mechanism; this steady-state operating point is the well-known Nash equilibrium in game theory. In this paper, an IEEE 802.22 inter-BS coexistence issue is investigated from a game-theoretical perspective. As the IEEE 802.22 standard defines the inter-BS coexistence mechanism, we design a resource-transaction algorithm to realize IEEE 802.22 dynamic resource renting and offering (DRRO) and adaptive on-demand channel contention (AODCC) operation. We also use game theory to model and analyze the proposed algorithm. In game analysis, we first study a two-player game through a graphical method to gain insights for the solution. Afterward, we investigate a general n-player game and derive the Nash equilibrium where the resource allocation is uniquely determined. The resource allocation is efficient, as the overall system performance is maximized (allocative efficiency), and no BS can further improve its performance without impairing others (Pareto optimality). The resource allocation also achieves max-min fairness and proportional fairness. Furthermore, the proposed algorithm ensures all resource acquirers’ participation without resource loss (individual rationality). It also guarantees revenue maximization of the resource provider.

Index Terms—Cognitive radio, credit token, game theory, IEEE 802.22, interbase station (inter-BS) coexistence, Nash equilibrium.

I. INTRODUCTION

IEEE 802.22 is the first cognitive-radio-based standard that operates over the 54- to 862-MHz licensed TV bands [1]. IEEE 802.22 devices, which are composed of base stations (BSs) and consumer premise equipment (CPE), are enabled to opportunistically access the licensed bands on the premise of not interfering with licensed users. A cognitive radio technique [2], [3] is applied to perform spectrum sensing [4]. With spectrum sensing, IEEE 802.22 systems can discover vacant channels over which to operate. However, an important issue arises under a common scenario that multiple IEEE 802.22 BSs operate in the vicinity and cause severe interference. This issue is called inter-BS coexistence (or self-coexistence) in the IEEE 802.22 standard. To address this issue, the IEEE 802.22 standard defines an inter-BS coexistence mechanism.

A. IEEE 802.22 Inter-BS Coexistence Mechanism

The IEEE 802.22 inter-BS coexistence mechanism [1] consists of four stages: spectrum etiquette, interference-free 54 scheduling, dynamic resource renting and offering (DRRO), and adaptive on-demand channel contention (AODCC), as illustrated in Fig. 1. Spectrum etiquette is the first stage where BSs try to locally find channels that their neighbor BSs cannot or do not use. If there is no available channel under this rule, BSs will conduct interference-free scheduling. In interference-free scheduling, BSs share the same channel by scheduling traffic in a noninterfering manner. This, however, only occurs on the premise that the channel owner agrees to share the channel with others. If the channel owner needs to operate exclusively, interference-free scheduling cannot take place, and the inter-BS coexistence mechanism must go on to the next 66 stage.
IEEE 802.22 uses credit tokens for DRRO and AODCC operations. The concept of credit tokens and their utilization in dynamic spectrum sharing are first introduced in [13] and [14]. In IEEE 802.22, credit tokens are similar to money, except that credit tokens can be frozen but cannot be paid and exchanged. IEEE 802.22 assumes that each BS has a given credit token budget. In DRRO, two entities are defined. An offeror is a BS who currently has unused resources (in time and frequency). A renter is a BS who currently has an additional resource requirement. In IEEE 802.22, an offeror can offer its unused resources by broadcasting offering information, which includes the available resources and the minimum number of credit tokens (MNCT) required. Renters who hear the offering information can send renting requests, which include the desired resources and the number of credit tokens that they are willing to pay. After receiving and comparing the renting requests, the offeror derives the best renters in terms of higher credit tokens. These renters are granted access to their requested resources.

AODCC is the final stage of the mechanism. It is triggered when BSs do not get enough resources through the previous three stages. AODCC is very similar to DRRO except that a channel owner, which is also called a contention destination, now passively receives contention requests. When a contention destination receives a contention request from another BS, which is called a contention source, it compares the number of credit tokens that the contention source is willing to pay with its MNCT required. If the former is larger, the contention destination shall release the requested resources; otherwise, it replies with a rejection.

B. Related Work

Recently, researchers have been dealing with IEEE 802.22 inter-BS coexistence in several manners. Sengupta et al. [15] proposed a centralized utility-graph-coloring channel-assignment algorithm to maximize the summation of BSs’ utilities. Grandblaise et al. [13] applied the concept of ascending auctions to devise a credit-token-based rental protocol among BSs. This protocol can be regarded as a predecessor of DRRO in the IEEE 802.22 standard. Some researchers have also been applying game theory to solve IEEE 802.22 inter-BS coexistence. Sengupta et al. [5] applied minority game theory to investigate whether a BS should stay at the present channel or switch to another channel. A mixed-strategy Nash equilibrium existed, and the mixed strategy space performed better than the pure strategy space in achieving optimal solution. Gao et al. [6] modeled DRRO as a progressive second price (PSP) auction. The utilization of the PSP auction had a major benefit that BSs would truthfully make their requests. Chen et al. [16] further extended [6] to design a rental protocol to decrease signaling overhead and bidding time. Niyato et al. [7] investigated an interstratum resource transaction rather than inter-BS coexistence. They formulated the transaction of spectrum bands between licensed users and BSs by a sealed-bid double auction. They also introduced a pricing mechanism to model the service between BSs and CPEs. A Nash equilibrium is found through a numerical method.

In this paper, we investigate an IEEE 802.22 inter-BS coexistence problem through a game-theoretical approach. We focus on resource transactions in IEEE 802.22 rather than interference-avoidance channel selection (i.e., spectrum etiquette), which was studied in [5] and [15]. As other previous work [6], [13], [16] only considered resource renting and offering, we investigate the complete IEEE 802.22 inter-BS coexistence mechanism. In our model, a contention mechanism is included in addition to the renting and offering mechanism. We also take the limited credit token budgets, which were not considered in [6] and [16], into account. To the best of our knowledge, this paper is the first that considers the complete IEEE 802.22 inter-BS coexistence mechanism integrating resource renting, offering, and contention. As the IEEE 802.22 standard only defines the inter-BS coexistence mechanism without specifying the algorithm to allocate resources (the algorithm is typically considered as an implementation issue during the standardization process), we have proposed an effective algorithm and analyzed it with a game-theoretical approach. We derive unique, efficient (allocative and Pareto optimality), and fair (max-min fair and proportionally fair) resource utilization at the Nash equilibrium. Moreover, the proposed scheme ensures all resource acquirers’ participation without resource loss (individual rationality). The proposed scheme also achieves revenue maximization of the resource provider.

II. SYSTEM OVERVIEW

A. System Model

We use the term “offeror” to describe a BS that is willing to offer unused resources. For a BS having an additional 154 resource requirement, we use the term “acquirer” to describe it. The system we consider consists of one offeror, which is assumed to be the same across all BSs, and n acquirers, which are denoted by $\text{BS}_i$, $i = 1, 2, \ldots, n$. Each BS has default radio resources $T$ and a traffic requirement $x_i$ additional to $T$. Furthermore, each BS has a credit token budget $B$, which is assumed to be the same for all BSs since BSs should have equal “power,” in general. The proposed scheme also achieves revenue maximization of the resource provider.

![Traffic requirements of a system with one offeror and two acquirers.](image)

1Here, we formulate a one-offeror model because IEEE 802.22 does not synchronize DRRO, and there is barely more than one offeror at any instant of time. Nonetheless, solutions for the case of multiple offerors will be discussed in Section IX.
165 B. Resource-Sharing Algorithm

Based on DRRO and AODCC in the IEEE 802.22 inter-BS coexistence mechanism, we propose a resource-sharing algorithm consisting of a renting-and-offering procedure and a contention procedure. We assume that each acquirer should use credit tokens for resource acquisition and protection.

In the renting-and-offering procedure, the offeror BS₀ initially broadcasts that it is willing to provide the unused resources \( O(x_0 = -O) \). After hearing this offering information, each acquirer BSᵢ, \( i = 1, \ldots, n \) will make an acquisition request \( y_i \), which are the resources it claims to acquire. As each BSᵢ makes its acquisition request, we assume that every unit of the resources BSᵢ wants to acquire \( y_i \), and the protection of \( T \) is equally important. Hence, the credit token budget \( B \) will be equally allocated over \( (T + y_i) \). The unit acquisition price and the unit protection price of BSᵢ are then both equal to \( p_i(y_i) = B / (T + y_i) \). \( p_i(y_i) \) shows the priority of BSᵢ for resource acquisition and protection. On the other hand, as BS₀ receives the acquisition requests from all acquirers, it assigns the unused resources \( O \) to the acquirers, in decreasing order of the unit acquisition price, for their requested amount until exhaustion of \( O \). When multiple acquirers have the same unit acquisition price, and there are not enough resources for them, we assume that the amount assigned to them is equal.

If some acquirers cannot get enough resources in the renting-and-offering procedure, the contention procedure starts. All acquirers, including those getting enough resources in the previous procedure, should participate in the contention procedure since they should protect their original resources. We assume that an agent, which may be BS₀, carries out this contention procedure. The agent first collects the original resources \( T \) from all acquirers. The collected \( T \)'s are sorted in increasing order of the unit protection price. Afterward, the agent assigns the sorted \( T \)'s to the acquirers for their inadequate resources in decreasing order of the unit acquisition price. The assignment ends if the unit protection price is greater than or equal to the unit acquisition price. Finally, the agent returns the unassigned resources to the original acquirers. When multiple acquirers have the same acquisition price, and there are not enough resources for them, we assume that the amount assigned to them is equal.

In Table II, we show that the resources BSᵢ acquire from BS₀, \( \min(y_i, r_i) \), and the resources BSᵢ acquire or lose from the contention, \( q_i(y) = \min([y_i - r_i]⁺, t_i) \). Noting the fact that \( x_i \leq y_i \Leftrightarrow p_i \geq p_j \), \( \forall i, j = 1, \ldots, n \), we replace the relationship among prices with that among acquisition requests. Also, \( x_i \) is the function \([·]⁺\) gives a nonnegative value. All other notations are summarized in Table II.

### Table I

**RESOURCE-SHARING ALGORITHM**

<table>
<thead>
<tr>
<th><strong>Renting-and-offering procedure</strong></th>
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<tr>
<td><strong>Step 1.</strong> Offeror broadcasts that it is willing to provide resources ( O ).</td>
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<tr>
<td><strong>Step 2.</strong> Acquirers make acquisition requests. Each acquirer’s credit token budget is equally allocated over the resources it wants to acquire and to protect.</td>
</tr>
<tr>
<td><strong>Step 3.</strong> Offeror assigns ( O ) to the acquirers, in decreasing order of the unit acquisition price, till exhaustion. If the acquirers cannot get enough resources, the contention procedure starts.</td>
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**Contention procedure**

**Step 1.** Agent collects the original resources \( T \) from all acquirers. 
**Step 2.** Agent sorts the collected \( T \)'s in increasing order of the unit protection price. 
**Step 3.** Agent assigns the sorted \( T \)'s to the acquirers in decreasing order of the unit acquisition price. The assignment ends if the unit protection price is greater than or equal to the unit acquisition price. 
**Step 4.** Agent returns the unassigned resources to the acquirers.

### Table II

**NOTATIONS**

<table>
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<th><strong>Table II</strong></th>
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<tr>
<td><strong>O</strong></td>
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<tr>
<td><strong>B</strong></td>
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<td><strong>yᵢ</strong></td>
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<tr>
<td><strong>pᵢ(yᵢ)</strong></td>
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<td><strong>qᵢ(yᵢ)</strong></td>
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<td><strong>tᵢ(yᵢ)</strong></td>
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</table>

### C. Problem Description

Given the proposed resource-sharing algorithm, the problem we want to investigate is as follows.

**Problem:** Assuming that the credit token budget \( B \) and the additional traffic requirement \( xᵢ \) of each BSᵢ, \( i = 1, \ldots, n \) are known by each other, if the acquisition request \( yᵢ \) is constrained by 0 and \( xᵢ \), i.e., \( 0 \leq yᵢ \leq xᵢ \), how does each BSᵢ distributedly make an acquisition request to increase the resources? We model this resource-sharing problem. The aim is to explore whether an operational steady state exists. In this steady state, no acquirer can benefit by changing its own request while the additional traffic requirement \( xᵢ \) of each BSᵢ is known by each other.
III. INTRODUCTION TO GAME THEORY

Game theory is a set of mathematical tools that are used to analyze interactions between players. The core of game theory is called a Nash equilibrium. A Nash equilibrium is a set of strategies where no player can improve their outcome by unilaterally changing their strategy, assuming all other players keep their strategies unchanged. Such a state is often referred to as a stable state.

IV. RESOURCE-SHARING GAME FORMULATION

To find a Nash equilibrium for the proposed IEEE 802.22 resource-sharing problem, we apply game theory to construct a model, which is denoted by $G$. Besides the three main components, the credit token budget and the traffic requirement of each acquirer should be taken into account as well.

1) Player set $N$: Each acquirer $BS_i$ is a player in the game.

$$N = \{1, 2, \ldots, n\}.$$
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Fig. 3. Best response functions and the Nash equilibrium for $x_1 \leq O/2$ and $x_2 \leq O - x_1$. (a) $u_1$ and $BR_1$. (b) $u_2$ and $BR_2$. (c) $BR_1, BR_2$, and NE.

Fig. 4. Best response functions and the Nash equilibrium for $x_1 \leq O/2$ and $x_2 > O - x_1$. (a) $u_1$ and $BR_1$. (b) $u_2$ and $BR_2$. (c) $BR_1, BR_2$, and NE.

since it always dominates (or results in higher utility than) all other strategies. A similar observation is obtained in Fig. 3(b) that player 2’s best response function is $x_2$. Therefore, player 2 plays the unique dominant strategy, i.e., $y_2 = x_2$. By drawing two best response functions together in Fig. 3(c), we find their intersection $(x_1, x_2)$ a unique Nash equilibrium. The corresponding utility profile is $(x_1, x_2)$ as well.

B. $x_1 \leq O/2$ and $x_2 > O - x_1$

We already know that player 1 plays the unique dominant strategy, i.e., $y_1 = x_1$, when $x_1 \leq O/2$. As depicted in Fig. 4(b), player 2’s best response function is $BR_2(y_1) = O - y_1 \sim x_2$. This implies that the strategy $y_2 = x_2$ is player 2’s unique dominant strategy. However, it is not meaningful to discuss the concept of dominant strategy for player 2 while player 2 likes to play a single-player game. We explain why player 2 likes to play a single-player game as follows: When $x_1 \leq O/2$, player 1 plays the unique dominant strategy, i.e., $y_1 = x_1$, and acquires $x_1$ from $O$. For player 2, it has $(O - x_1)$ remained to acquire without any other player. Therefore, player 2 likes to play a single-player game. It can always acquire $(O - x_1)$ by playing $y_2$ such that $O - x_1 \leq y_2 \leq x_2$. This fact obviously results in multiple Nash equilibria. This can also be shown by drawing the two best response functions together. The resulting intersection is a line segment between $(x_1, O - x_1)$ and $(x_1, x_2)$, which means that multiple Nash equilibria exist. Although multiple Nash equilibria exist, the corresponding utility profile is uniquely $(x_1, O - x_1)$.

C. $x_1 > O/2$ and $x_2 > O/2$

We derive the best response function of player 1 from Fig. 5(a)–(c), i.e.,

$$BR_1(y_2) = \begin{cases} \min(O - y_2, x_1) \sim x_1, & \text{if } y_2 \leq \frac{O}{2} \\ y_2, & \text{if } \frac{O}{2} < y_2 \leq x_1 \\ x_1, & \text{if } x_1 < y_2 \end{cases}$$

and the best response function of player 2 from Fig. 5(d) and 5(e), i.e.,

$$BR_2(y_1) = \begin{cases} \min(O - y_1, x_2) \sim x_2, & \text{if } y_1 \leq \frac{O}{2} \\ y_1, & \text{if } \frac{O}{2} < y_1 \leq y_2 \end{cases}$$

Neither player 1 nor player 2 has any dominant strategy. By drawing two best response functions together, we find the intersection $(O/2, O/2)$ a unique equal-strategy Nash equilibrium. The corresponding utility profile is $(O/2, O/2)$. 
According to the analysis, we summarize the observations.

1) Condition for unique dominant strategies: When \( x_1 \leq O/2 \), player 1 plays the unique dominant strategy, i.e., \( y_1 = x_1 \). When \( x_2 \leq O - x_1 \), player 2 plays the unique dominant strategy, i.e., \( y_2 = x_2 \).

2) Existence of a Nash equilibrium: A Nash equilibrium always exists, regardless of the traffic requirements.

3) Necessary condition for the existence of a Nash equilibrium: No contention occurs at the Nash equilibrium in all cases. In other words, the necessary condition for the existence of a Nash equilibrium is no contention.

4) Condition for a unique equal-strategy Nash equilibrium: First, both players have no dominant strategies when \( x_1 > O/2 \) and \( x_2 > O/2 \). Also, when \( x_1 > O/2 \) and \( x_2 > O/2 \), the unique equal-strategy Nash equilibrium \((O/2, O/2)\) is attained. These are sufficient and necessary conditions for each other.

5) Condition for multiple Nash equilibria: The only cases where multiple Nash equilibria exist are when \( x_1 \leq O/2 \) and \( x_2 > O - x_1 \). We have explained that because player 2 likes to play a single-player game with \((O - x_1)\) offered, it can always acquire \((O - x_1)\) by playing \( y_2 \) such that \( O - x_1 \leq y_2 \leq x_2 \). Multiple Nash equilibria, i.e., \( x_1, O - x_1 \approx x_2 \), hence, exist.

6) Uniqueness of the utility profile at the Nash equilibrium: Even in the case where multiple Nash equilibria exist, the utility profile at the Nash equilibria is 377 unique. Therefore, the uniqueness of the utility profile at 378 the Nash equilibrium is guaranteed, regardless of traffic 379 requirements.

These observations will help us analyze the game model for 381 \( n \) acquirers. In fact, a Nash equilibrium always exists. The 382 necessary condition for the existence of a Nash equilibrium 383 always holds. The uniqueness of the utility profile at the Nash 384 equilibrium is always guaranteed.

VI. GAME THEORETICAL ANALYSIS—\( n \) ACQUIRERS

Based on the observations in the two-acquirer game, we 386 now analyze the \( n \)-acquirer game, i.e., \( G \) in Table III. The 387 traffic requirements, in accordance with that \( \mathbf{X} \) is sorted in 388 increasing order, can be classified into \((n + 1)\) cases. The 390 \((k + 1)\)th case, \( k \in \{0, N\} \), is \( x_j \leq -\sum_{i=0}^{k-1} x_i/(n - j + 1) \) 391 for \( j = 1, \ldots, k \) and \( x_j > -\sum_{i=0}^{k-1} x_i/(n - k) \) for \( j = k + 392 1, \ldots, n \), where \( x_0 = -O \). In the following, we first derive the 393 condition for unique dominant strategies.

We should mention that throughout this paper, we will use 395 \( r_j, u_j, q_j \), and \( t_j \) to express \( r_j(y), u_j(y), q_j(y) \), and \( t_j(y) \), 396 respectively, at any given strategy profile \( y \) for short. If we 397 need to compare the results between two different strategy 398 profiles, for example, \((y, y_{-i})\) and \((y', y_{-i})\), we will distinguish 399 by using \( r_j', u_j', q_j' \), and \( t_j' \) to express \( r_j(y', y_{-i}), u_j(y', y_{-i}), 400 q_j(y', y_{-i}) \), and \( t_j(y', y_{-i}) \), respectively.
Theorem 1: For game \( G \) with \( x_j \leq - \sum_{j=0}^{l=1} x_j/(n-j+1) \) for \( j = 1, \ldots, k \), where \( k \in \mathbb{N} \), player \( i \), \( i = 1, \ldots, k \), plays the unique dominant strategy, i.e., \( y_i = x_i \).

Proof: We prove it by mathematical induction.

At \( k = 0 \), we have \( x_0 = 0 \) and \( [x_0 - r_0]^+ = 0 \). Assume that it is true for \( 0 < k \). Then, for \( k + 1 \), we have

\[
\begin{align*}
\sum_{j=0}^{l=1} x_j/(n-j+1) &< 0 \\
\sum_{j=0}^{l=1} x_j/(n-j+1) &< 0 \\
\sum_{j=0}^{l=1} x_j/(n-j+1) &< 0
\end{align*}
\]

Next, we prove the necessary condition for the existence of a Nash equilibrium, i.e., contention will never occur. This necessary condition will be frequently used in other proofs.

Theorem 2: For game \( G \), a Nash equilibrium exists only if \( t_i \geq 0 \), \( \forall i \in \mathbb{N} \).

Proof: Assuming that there exists some player \( i \) having \( t_i < 0 \), there must be some other player \( k \), with \( y_k < y_i \), having \( [y_k - r_k]^+ = y_k - r_k > 0 \), since the term \( \sum_{j=0}^{l=1} [y_j - r_j]^+ \) in \( t_i \) must be greater than zero. Consequently, we have \( r_k = 440 y_k^+ = 0 \), and

\[
\begin{align*}
r_i &= \sum_{j=0}^{l=1} [O - \sum_{j=0}^{l=1} y_j]^+ \\
&< \sum_{j=0}^{l=1} [O - \sum_{j=0}^{l=1} y_j]^+
\end{align*}
\]

Equation (6), along with \( t_i < 0 \), means that the utility of 442 player \( i \) is \( u_i = t_i < 0 \), which is less than the utility in the case 443 that player \( i \) plays \( y_i \); a bit greater than zero. When \( t_i < 0 \) for 444 some \( i \), a Nash equilibrium does not exist. A Nash equilibrium 445 exists only if \( t_i \geq 0 \), \( \forall i \in \mathbb{N} \).

Remark: From the wireless system’s perspective, \( t_i \geq 0 \) means that there is no contention loss and, conse- 447 quently, no occurrence of contention. In other words, Theorem 2 shows that the contention procedure is like a thread that never 448 occurs and causes signaling overhead at the Nash equilibrium. 451 As a result, it does not matter who is the agent that is carrying 452 out the contention procedure.

Recall that for the two-acquirer game with \( x_1 > O/2 \) and \( x_2 > O/2 \), it has the unique equal-strategy Nash equilibrium of \( O/2, O/2 \). Analogously, we guess that game \( G \) with \( x_j > O/n \) has the unique equal-strategy Nash equilibrium of \( O/n, \ldots, O/n \). To verify that this guess is correct, we 458 prove that all strategy profiles other than \( (O/n, \ldots, O/n) \) are 459 not Nash equilibria. The proof is taken into two parts. One 460 part is to prove that \( y_i < O/n \) for any \( i \in \mathbb{N} \) is not in any 461 way a Nash equilibrium. It is shown in Lemma 1. The other 462 part is to prove that \( y_i > O/n \) for any \( i \in \mathbb{N} \) is not in any 463
way a Nash equilibrium. Although it is somewhat complicated, we can still prove, in Lemma 2, step by step that the set of players having strategies greater than $O/n$ is an empty set at a Nash equilibrium. Consequently, the fact that $y_i > O/n$ for any $i \in N$ is not in any way a Nash equilibrium is confirmed in Lemma 3.

**Lemma 1:** For game $G$ with $x_j > O/n$, $\forall j \in N$, the strategy $y_i < O/n$ for any $i \in N$ is not in any way a Nash equilibrium.

**Proof:** Assume that the strategy profile $(y_1,\ldots,y_n)$ is a Nash equilibrium. If $y_1 \leq O/n$ for some player $i$, then

$$r_i = \frac{1}{\sum_{y_j = y_i} 1} \left[ O - \sum_{y_j < y_i} y_j \right] +$$

$$= \frac{1}{\sum_{y_j = y_i} 1} \left( O - \sum_{y_j < y_i} y_j \right) \geq \frac{O}{n} \geq y_i. \quad (7)$$

From (7), we know $\min(y_i, r_i) = y_i$ and $[y_i - r_i]^+ = 0$. Also, along with $t_i \geq 0$, which is implied in Theorem 2, $y_i - r_i^+ = 0$ means that $q_i = \min([y_i - r_i]^+; t_i) = 0$. The utility function of player $i$, therefore, is

$$u_i = \min(y_i, r_i) + q_i = y_i. \quad (8)$$

Equation (8) tells that if $y_i \leq O/n$, the largest value of $u_i$ is obtained at $y_i = O/n$. It means that by the definition of the Nash equilibrium, $y_i < O/n$ is not in any way a Nash equilibrium. \hfill \Box

**Lemma 2:** For game $G$ with $x_j > O/n$, $\forall j \in N$, if the strategy profile $(y_1,\ldots,y_n)$ is a Nash equilibrium, $N_{\text{equal}}$ is the set of players having strategies equal to $O/n$, $N_0$ is the set of players having strategies greater than $O/n$, $N_1$ is the set of players in $N_0$ having largest strategies (smallest prices), and $N_2$ is the set of players in $N_0$ having smallest strategies (largest prices), the following statements are always true.

1) $t_i = 0$, $\forall i \in N_1$, and $y_i \leq r_i$, $\forall i \in N - N_1$.

2) $u_i = r_i \leq O/n$, $\forall i \in N_1$.

3) $N_{\text{equal}} \neq \emptyset$.

4) $N_0 = \emptyset$.

**Proof:**

1) Starting from $t_i \geq 0$, $\forall i \in N$, i.e., the result in Theorem 2, we have $\forall i \in N_1$

$$t_i = -T + \frac{1}{\sum_{y_j = y_i} 1} \left[ \sum_{y_j \geq y_i} T - \sum_{y_j < y_i} [y_j - r_j]^+ \right] +$$

$$= -T + \frac{1}{\sum_{y_j = y_i} 1} \left[ \sum_{y_j \geq y_i} T - \sum_{y_j < y_i} [y_j - r_j]^+ \right] +$$

$$= -T + \left[ T - \frac{1}{\sum_{y_j = y_i} 1} \sum_{y_j < y_i} [y_j - r_j]^+ \right] + \geq 0. \quad (9)$$

Noting that $[T - (1/\sum_{j:y_j=y_i} 1)[\sum_{j:y_j<y_i} [y_j - 497 r_j]^+]^+] \leq T$, the only possible solution to (9) is $t_i = 0$, $498 \forall i \in N_1$, and $y_i = r_i$, $\forall i \in N - N_1$.

2) From $y_i \geq O/n$, $\forall i \in N_1$, which is implied in Lemma 1 500

$$r_i = \frac{1}{\sum_{y_j = y_i} 1} \left[ O - \sum_{y_j < y_i} y_j \right] +$$

$$\leq \frac{1}{\sum_{y_j = y_i} 1} \left[ O - \sum_{y_j < y_i} \frac{O}{n} \right] = \frac{O}{n}. \quad (10)$$

From the result in point 1 and (10)

$$u_i = \min(y_i, r_i) + q_i = r_i + \min([y_i - r_i]^+, t_i)$$

$$= r_i + \min([y_i - r_i]^+, 0) = r_i \leq \frac{O}{n}. \quad (11)$$

3) If $N_{\text{equal}} = \emptyset$, $N_0 \neq \emptyset$, and player $i$, $\forall i \in N_1$ can play 502 $y_i^1 = y_m^1$ for any $m \in N_2$ to get higher utility since

$$r_i^1 = \left[ O - \sum_{y_j < y_i^1} y_j \right] + = O$$

$$\geq -\frac{1}{\sum_{y_j = y_m} 1} O = r_m \geq y_m > y_i^1$$

$$t_i^1 = -T + \left[ \sum_{y_j \geq y_i^1} T - \sum_{y_j < y_i^1} [y_j - r_j]^+ \right] +$$

$$= -T + \sum_{y_j \geq y_m} T = \sum_{y_j \geq y_m} T > 0 \quad (13)$$

$$u_i^1 = \min(y_i^1, r_i^1) + q_i^1 = y_i^1 + \min(0, t_i^1)$$

$$= y_i^1 > \frac{O}{n} \geq u_i. \quad (14)$$

Equation (14) contradicts that $(y_1,\ldots,y_n)$ is a Nash equilibrium. Hence, $N_{\text{equal}} \neq \emptyset$. 505

4) Going after point 3, if $N_0 \neq \emptyset$, player $i$, $\forall i \in N_{\text{equal}}$ can play 506 $y_i^1 = y_m^1$ for any $m \in N_2$ to get higher utility since

$$r_i^1 = \left[ O - \sum_{j:j \neq y_j \neq y_i^1} y_j \right] + \geq \left[ O - \sum_{j:y_j < y_m} \frac{O}{n} \right] +$$

$$\geq \frac{1}{\sum_{y_j = y_m} 1} \left[ O - \sum_{y_j < y_m} \frac{O}{n} \right] = \frac{O}{n}. \quad (15)$$
\[ t_i' = -T + \left[ \sum_{jy_j \geq y_i'} T - \sum_{jy_j < y_i'} [y_j - r_j] \right]^+ \]

\[ = -T + \sum_{jy_j \geq y_i'} T - \sum_{jy_j < y_i'} T > 0 \quad (16) \]

\[ u_i' = \min (y_i', r_i') + q_i' = y_i' + \min (0, t_i') \]

\[ = y_i' > \frac{O}{n} = u_i. \quad (17) \]

Equation (17) contradicts that \((y_1, \ldots, y_n)\) is a Nash equilibrium. Therefore, \(N_0 = \emptyset\). \(\Box\)

**Lemma 3:** For game \(G\) with \(x_j > O/n, \forall j \in N\), the strategy \(y_i > O/n\) for any \(i \in N\) is not in any way a Nash equilibrium.

**Proof:** Assume that the strategy profile \((y_1, \ldots, y_n)\) is a Nash equilibrium. Since \(N_0 = \emptyset\), which is derived in Lemma 2, the strategy \(y_i > O/n\) for any \(i \in N\) is not in any way a Nash equilibrium.

**Theorem 3:** For game \(G\) with \(x_j > O/n, \forall j \in N\), it has a unique Nash equilibrium, namely \((y_1, \ldots, y_n)\).

**Proof:** From Lemmas 1 and 3, we have verified that any strategy profile other than \((O/n, \ldots, O/n)\) cannot be a Nash equilibrium, given \(x_j > O/n, \forall j \in N\). In other words, only \((O/n, \ldots, O/n)\) can be a Nash equilibrium. We check its property and find it to be a unique Nash equilibrium.

**Theorem 4:** For game \(G\) with \(x_j > O/n, \forall j \in N\), it has a unique Nash equilibrium, namely \((O/n, \ldots, O/n)\).

**Proof:** From Lemmas 1 and 3, we have verified that any strategy profile other than \((O/n, \ldots, O/n)\) cannot be a Nash equilibrium, given \(x_j > O/n, \forall j \in N\). In other words, only \((O/n, \ldots, O/n)\) can be a Nash equilibrium. We check its property and find it to be a unique Nash equilibrium.

**Theorem 5:** For game \(G\) with \(x_j \leq -\sum_{l=0}^{j-1} x_l/(n-j+1)\) for \(j = 1, \ldots, k\) and \(x_j > -\sum_{l=0}^{k} x_l/(n-k)\) for \(j = k+1, \ldots, n\), where \(k \in \{0, N\}\), it has a unique Nash equilibrium. Theorem 5, in other words, reveals that the proposed resource-sharing algorithm always achieves the unique traffic-dependent resource allocation at the Nash equilibrium.

**VII. PROPERTIES ATTAINED AT THE NASH EQUILIBRIUM**

After deriving the Nash equilibrium, we prove that several desirable properties, including allocative efficiency, Pareto optimality, max-min fairness, proportional fairness, individual rationality, and revenue maximization of the resource offeror, are attained at the Nash equilibrium in game \(G\). In the following, we briefly introduce these properties and give their mathematical definitions. To conform to the expressions in our game, we use \(y\) and \(Y\) instead of \(s\) and \(S\) to represent the strategy profile and the strategy space, respectively.

Efficiency is one of the key system design issues. Allocative efficiency [10] is an efficient resource allocation in the sense of maximizing total utilities over all players. It is regarded as the most optimal since no other allocation can achieve greater social welfare. Another efficiency, which is named Pareto optimality [10], is defined as an allocation upon which no player can be made happier (in utility) without making at least one other player less happy. It is always true that allocative efficiency implies Pareto optimality.

**Definition 4:** A resource allocation game is allocative-efficient if the Nash equilibrium is a solution to the optimization problem

\[ \max_{i=1}^{n} u_i(y) \quad \text{s.t. } y \in Y. \]

**Definition 5:** A resource-allocation game is Pareto optimal if the Nash equilibrium \(y^*\) satisfies

\[ \exists y' \neq y^*, u_i(y') > u_i(y^*) \quad \Rightarrow \quad \exists j \in N, u_j(y') < u_j(y^*). \]

Fairness is another key system design issue. Two kinds of definitions of fairness are considered here. First, we say an allocation satisfies max-min fairness [11] if it is not possible to increase one player’s utility without simultaneously decreasing another player’s utility, which is already smaller. We say that an allocation exhibits proportional fairness [11] if it maximizes the sum of logarithmic utilities of all players, or equivalently, it maximizes the product of all players’ utilities.

**Definition 6:** A resource allocation game is max-min fair if the Nash equilibrium is a solution to the optimization problem

\[ \max \min (u_1(y), \ldots, u_n(y)) \quad \text{s.t. } y \in Y. \]
Definition 7: A resource-allocation game is proportionally fair if the Nash equilibrium is a solution to the optimization problem
\[
\max \prod_{i=1}^{n} u_i(y) \quad \text{s.t. } y \in Y.
\]

Another important property in game theory is individual rationality [10]. While players can freely and rationally decide to participate or not, individual rationality means that all rational players are willing to participate in a game. To achieve individual rationality, a game designer must provide some incentives to ensure that at the Nash equilibrium, all players can always obtain as much utilities from participation as without participation.

Definition 8: Assuming that all players have zero utility while not participating in a game, a game is individually rational if \(u_i(y^*) \geq 0\), \(\forall i \in N\), where \(y^*\) is the Nash equilibrium.

The last property is revenue maximization of the resource owner, which accounts for the willingness of the resource owner to offer unused resources. Revenue maximization [12] is generally defined as follows: Among all possible pricing mechanisms, a game adopts the one generating the most revenue for an agent at the Nash equilibrium. Since we are not comparing the adopted pricing mechanism to all other pricing mechanisms, we turn to a different issue: Given a pricing mechanism, the revenue of an agent is maximized at the Nash equilibrium of a 2-player game.

Definition 9: Given a pricing mechanism, a game achieves revenue maximization of an agent if the Nash equilibrium is a solution to the optimization problem
\[
\max \sum_{i=1}^{n} P_i(y) \quad \text{s.t. } y \in Y
\]
where \(P_i(y)\) is the revenue that the agent earns from player \(i\).

In game \(G\), the agent of which revenue is evaluated is BS\(_0\) since it is the only offeror. Although credit tokens rather than money are utilized, we can still explain the revenue as the amount of credit tokens frozen by BS\(_0\). Then, the total revenue that BS\(_0\) earns is \(\sum_{i=1}^{n} (B/(T + y_i)) \min(y_i, r_i)\), where \((B/T + y_i)\) is the unit acquisition price of BS\(_0\), and \(\min(y_i, r_i)\) is the resources that BS\(_0\) acquires from BS\(_0\).

Before we start the derivations, there are two things worth mentioning. First, remember that we omit the constant term \(T\) when setting utility functions. Upon the later derivation of proportional fairness, we shall replace \(u_i\) with \((T + u_i)\), \(\forall i \in N\); otherwise, the objective function will not be correctly characterized. For other properties, using \(u_i\), \(\forall i \in N\), will cause no difference.

Second, according to Lemma 4, which shows the range of the utility functions, we can transform the above property-related optimization problems from a strategy domain into a utility domain. Consequently, we can prove the properties by verifying that the utility profile at the Nash equilibrium is a solution to the corresponding optimization problems.

Lemma 4: For game \(G\) and \(\forall y \in Y\), the following statements are always true.
1) \(-T \leq u_i(y) \leq x_i, \forall i \in N\).
2) \(\sum_{i=1}^{n} \min(y_i, r_i) \leq \min(\sum_{i=1}^{n} x_i, O)\).
3) \(\sum_{i=1}^{n} q_i(y) = 0\).
4) \(\sum_{i=1}^{n} u_i(y) \leq \min(\sum_{i=1}^{n} x_i, O)\).

Proof: See the Appendix.

A. Allocative Efficiency

Theorem 6: Game \(G\) is allocatively efficient. Equivalently, the utility profile at the Nash equilibrium is a solution to the optimization problem
\[
\max \sum_{i=1}^{n} u_i
\]
\[\text{s.t. } -T \leq u_i \leq x_i \quad \forall i \in N\]
\[\sum_{i=1}^{n} u_i \leq \min(\sum_{i=1}^{n} x_i, O)\].

Proof: Recall in Theorem 5 that for game \(G\) with \(x_j \leq -\sum_{i=0}^{j} x_i/(n-j+1)\) for \(j = 1, \ldots, k\) and \(x_j > -\sum_{i=0}^{k} x_i/(n-k)\) for \(j = k+1, \ldots, n\), where \(x_j\) is the revenue that BS\(_0\) earns is \(\sum_{i=1}^{n} (B/(T + y_i)) \min(y_i, r_i)\), where \((B/T + y_i)\) is the unit acquisition price of BS\(_0\), and \(\min(y_i, r_i)\) is the resources that BS\(_0\) acquires from BS\(_0\).

Before we start the derivations, there are two things worth mentioning. First, remember that we omit the constant term \(T\) when setting utility functions. Upon the later derivation of proportional fairness, we shall replace \(u_i\) with \((T + u_i)\), \(\forall i \in N\); otherwise, the objective function will not be correctly characterized. For other properties, using \(u_i\), \(\forall i \in N\), will cause no difference.

Second, according to Lemma 4, which shows the range of the utility functions, we can transform the above property-related optimization problems from a strategy domain into a utility domain. Consequently, we can prove the properties by verifying that the utility profile at the Nash equilibrium is a solution to the corresponding optimization problems.

Lemma 4: For game \(G\) and \(\forall y \in Y\), the following statements are always true.
1) \(-T \leq u_i(y) \leq x_i, \forall i \in N\).
2) \(\sum_{i=1}^{n} \min(y_i, r_i) \leq \min(\sum_{i=1}^{n} x_i, O)\).
3) \(\sum_{i=1}^{n} q_i(y) = 0\).
4) \(\sum_{i=1}^{n} u_i(y) \leq \min(\sum_{i=1}^{n} x_i, O)\).

Proof: See the Appendix.

B. Pareto Optimality

Theorem 7: Game \(G\) is Pareto optimal. Equivalently, the Nash equilibrium \(y^*\) satisfies
\[\exists j \in N, \quad u'_j > u_j(y^*)\]
\[\forall i \in N, \quad u'_i \neq u_i(y^*)\]

Proof: Since allocative efficiency implies Pareto optimal-ity and game \(G\) is allocatively efficient, game \(G\) is Pareto optimal as well.

C. Max-Min Fairness

Theorem 8: Game \(G\) is max-min fair. Equivalently, the utility profile at the Nash equilibrium is a solution to the optimization problem
\[
\max \min_{i=1}^{n} (u_1, \ldots, u_n)
\]
\[\text{s.t. } -T \leq u_i \leq x_i \quad \forall i \in N\]
\[\sum_{i=1}^{n} u_i \leq \min(\sum_{i=1}^{n} x_i, O)\].
D. Proportional Fairness

**Theorem 9:** Game $G$ is proportionally fair. Equivalently, the utility profile at the Nash equilibrium is a solution to the optimization problem

$$\max \prod_{i=1}^{n} (T + u_i)$$

subject to

$$-T \leq u_i \leq x_i \quad \forall i \in \mathbb{N}$$

$$\sum_{i=1}^{n} u_i \leq \min\left(\sum_{i=1}^{n} x_i, O\right).$$

**Proof:** When $x_i > O/n$, $\forall i \in \mathbb{N}$, we have, from A.M. $\geq$ G.M.

$$\prod_{i=1}^{n} (T + u_i) \leq \left(\frac{T + \frac{1}{n} \sum_{i=1}^{n} u_i}{n}\right)^n \leq \left(\frac{T + O}{n}\right)^n. \quad (21)$$

The equality holds iff $u_i = O/n$, $\forall i \in \mathbb{N}$, or equivalently, $U = U_{O}^{T}$.

When $x_i \leq -\sum_{j=0}^{j-1} x_j/(n - j + 1)$ for $j = 1, \ldots, k$ and $x_j > -\sum_{j=0}^{j-1} x_j/(n - k)$ for $j = k + 1, \ldots, n$, where $k \neq 0$,

$$\prod_{i=1}^{n} (T + u_i) \leq \left(\frac{T + \frac{1}{n} \sum_{i=1}^{n} u_i}{n}\right)^n \leq \left(\frac{T + O}{n}\right)^n. \quad (21')$$

The equality holds iff $u_j = x_j$, for $j = 1, \ldots, k$ and $u_k = -\sum_{j=0}^{k-1} x_j/(n - k) < x_j$ for $j = k + 1, \ldots, n$, or equivalently, $U = U_{k}^{T}$.

In summary, $\prod_{i=1}^{n} (T + u_i)$ is maximized by $U_{k}^{T}$, $\forall k \in \{0, N\}$. Game $G$ is proportionally fair. 

**Proof:** When $x_j > O/n$, $\forall j \in \mathbb{N}$, we substitute $U_{0}^{T}$ into the objective function and derive that

$$\min(u_1, \ldots, u_n) = \min\left(\frac{O}{n}, \ldots, \frac{O}{n}\right) = \frac{O}{n}. \quad (19)$$

Because $\sum_{i=1}^{n} x_i > O$, we should have $\sum_{i=1}^{n} u_i \leq \min(\sum_{i=1}^{n} x_i, O) = O$. This equation implies that if

$$u_i = O/n + \Delta_i \quad \text{for some player } i \quad \text{where } \Delta_i > 0,$$

must be some player $j$ having $u_j = O/n - \Delta_j$ where $\Delta_j > 0$.

Then

$$\min(u_1, \ldots, u_n) = \min\left(\ldots, \frac{O}{n} + \Delta_i, \ldots, \frac{O}{n} - \Delta_j, \ldots\right) \leq \frac{O}{n} - \Delta_j < \frac{O}{n}. \quad (20)$$

Equation (20) reveals that $\min(u_1, \ldots, u_n)$ is maximized by $U_{k}^{T}$.

When $x_j \leq -\sum_{i=0}^{j-1} x_i/(n - j + 1)$ for $j = 1, \ldots, k$ and $x_j > -\sum_{i=0}^{j-1} x_i/(n - k)$ for $j = k + 1, \ldots, n$ where $k \neq 0$,

we know that $x_1 \leq \cdots \leq x_k \leq -\sum_{i=0}^{k} x_i/(n - k)$. This inequality reveals that the maximum value of $\min(u_1, \ldots, u_n)$ is constrained by $x_1$ since $u_1$ cannot exceed $x_1$. Therefore,

$$\min(u_1, \ldots, u_n) \leq x_1, \quad \text{and the equality holds iff } u_1 = x_1$$

and $x_1 \leq u_j$ for $j = 2, \ldots, n$. This condition is obviously satisfied

(20) (but not uniquely) by $U_{k}^{T}$.

In summary, $\min(u_1, \ldots, u_n)$ is maximized by $U_{k}^{T}$, $\forall k \in \{0, N\}$. Game $G$ is max-min fair.

From (23), we know that $\frac{\partial H}{\partial u_k} \leq 0$. Therefore, a nondecreasing function of $u_k$ iff $u_k \leq (O - \sum_{i=1}^{k-1} u_i)/(n - k)$, and $u_k = (O - \sum_{i=1}^{k-1} u_i)/(n - k)$ for $k = 1, \ldots, N - 1$. When we continue differentiating $H$ by $u_k$, and rearranging with $u_k = (O - \sum_{i=1}^{k-1} u_i)/(n - k)$, we have

$$\prod_{i=1}^{n} (T + u_i) \leq \left(\frac{T + \frac{1}{n} \sum_{i=1}^{n} u_i}{n}\right)^n \leq \left(\frac{T + O}{n}\right)^n. \quad (21')$$

The equality holds iff $u_j = x_j$, for $j = 1, \ldots, k$ and

$$u_j = -\sum_{i=0}^{j-1} x_i/(n - j + 1) < x_j \quad \text{for } j = k + 1, \ldots, n$$

or equivalently, $U = U_{k}^{T}$.

In summary, $\prod_{i=1}^{n} (T + u_i)$ is maximized by $U_{k}^{T}$, $\forall k \in \{0, N\}$. Game $G$ is proportionally fair.
E. Individual Rationality

733 Theorem 10: Game $G$ is individually rational. Equivalently, $u_i(y^*) \geq 0, \forall i \in \mathbb{N}$, where $y^*$ is the Nash equilibrium.

735 Proof: When $x_j \leq -\sum_{i=0}^{j-1} x_i / (n - j + 1)$ for $j = 1, \ldots, k$ and $x_j > -\sum_{i=0}^{k} x_i / (n - k)$ for $j = k + 1, \ldots, n$, where $k \in \{0, \mathbb{N}\}$, we can derive

$$0 < x_k \leq \frac{-\sum_{i=0}^{k-1} x_i}{n - k + 1} \iff 0 < \sum_{i=0}^{k} x_i \leq \frac{n - k}{n - k + 1} \sum_{i=0}^{k-1} x_i$$

$$\iff \frac{-\sum_{i=0}^{k} x_i}{n - k} \geq \frac{n - k}{n - k + 1} > 0.$$  

(25)

738 Equation (25) reveals that $u_i(NE_k) = x_i > 0$ for $i = 1, \ldots, k$ and $u_i(NE_k) = -\sum_{i=0}^{k} x_i / (n - k) > 0$ for $i = k + 1, \ldots, n$. In other words, game $G$ is individually rational. □

741 F. Revenue Maximization of Resource Offeror BS0

742 Theorem 11: In game $G$, revenue maximization of BS0 is attained at the Nash equilibrium, except for the case where multiple Nash equilibria exist. Equivalently, except for the case where multiple Nash equilibria exist, the Nash equilibrium is a solution to the optimization problem

$$\max \sum_{i=1}^{n} \frac{B}{T+y_i} \min(y_i, r_i)$$

s.t. $0 \leq y_i \leq x_i, \quad 0 \leq \min(y_i, r_i) \leq x_i$

$$\sum_{i=1}^{n} \min(y_i, r_i) \leq \min \left( \sum_{i=1}^{n} x_i, O \right).$$

747 Proof: We use $c_i$ to denote $\min(y_i, r_i)$ for simplicity. First, note that $\sum_{i=1}^{n} (B/(T+y_i)) c_i \leq \sum_{i=1}^{n} (B/(T+y_i)) c_i$. The equality holds iff $y_i = c_i, \forall i \in \mathbb{N}$. Second, note that $\sum_{i=1}^{n} (B/(T+y_i)) c_i$ can be rearranged as $\sum_{i=1}^{n} B - T \sum_{i=1}^{n} (B/(T+y_i))$, which means that we can maximize $\sum_{i=1}^{n} (B/(T+y_i)) c_i$ by minimizing $\sum_{i=1}^{n} (B/(T+y_i))$. When $x_i > O/n, \forall i \in \mathbb{N}$, by applying the Cauchy–Schwarz inequality, we have

$$\left( \sum_{i=1}^{n} \frac{B}{T+y_i} \right) \left[ \sum_{i=1}^{n} (T+c_i) \right] \geq \left( \sum_{i=1}^{n} \sqrt{B} \right)^2 = n^2 B.$$  

(26)

755 Since $\sum_{i=1}^{n} (T+c_i) \leq nT + O$, we can divide both sides of (26) by $\sum_{i=1}^{n} (T+c_i)$, and (26) becomes

$$\sum_{i=1}^{n} \frac{B}{T+c_i} \geq \frac{n^2 B}{\sum_{i=1}^{n} (T+c_i)} \geq \frac{nB}{T + \frac{O}{n}}.$$  

(27)

757 The equality holds iff $c_i = O/n, \forall i \in \mathbb{N}$. The Nash equilibrium $NE_0$ implies that $y_i = c_i = O/n, \forall i \in \mathbb{N}$, which satisfies the conditions for equality of $\sum_{i=1}^{n} (B/(T+y_i)) c_i \leq \sum_{i=1}^{n} (B/(T+y_i))$. In other words, $NE_0$ minimizes $\sum_{i=1}^{n} (B/(T+y_i)) c_i$, maximizes $\sum_{i=1}^{n} (B/(T+y_i)) c_i$, and, consequently, maximizes $\sum_{i=1}^{n} (B/(T+y_i)) c_i$.  

762 When $x_i \leq -\sum_{j=0}^{i-1} x_j / (n - i + 1)$ for $i = 1, \ldots, k$ and $x_i > -\sum_{j=0}^{k} x_j / (n - k)$ for $i = k + 1, \ldots, n$

$$\sum_{i=1}^{n} \frac{B}{T+c_i} \geq \frac{\sum_{i=1}^{k} B}{T+c_i} + \frac{(n-k)B}{O-\sum_{i=1}^{k} c_i}.$$  

(28)

The equality holds iff $c_i = (O - \sum_{j=1}^{k} c_j)/(n-k)$ for $i = 765 k+1, \ldots, n$. We denote the right-hand-side term in (28) as $C(c_1, \ldots, c_k)$. Differentiating $C$ by $c_k$ and rearranging it, we have

$$\frac{\partial C}{\partial c_k} = \frac{n-k+1}{n-k} \left( \frac{O - \sum_{j=1}^{k} c_j}{n-k+1} \right)$$

$$\times \frac{B}{2T+c_k + \frac{O-\sum_{i=1}^{k} c_i}{n-k}}$$

$$\left( T + c_k \right)^2 \left( T + \frac{O-\sum_{i=1}^{k} c_i}{n-k} \right).$$  

(29)

768 Thus, $\left( \frac{\partial C}{\partial c_k} \right)_{c_k \leq (O - \sum_{j=1}^{k-1} c_j)/(n-k+1)} \geq 0$, and this reveals that $C$ is a nondecreasing function of $c_k$ iff $c_k \leq (O - \sum_{j=1}^{k-1} c_j)/(n-k+1)$. We can iteratively apply the same 771 differential procedure to $c_k-1, \ldots, c_1$ and derive that $C$ is a 772 nondecreasing function of $c_i$ for $i = 1, \ldots, k$ iff $c_i \leq (O - \sum_{j=1}^{k-1} c_j)/(n-k+1)$. Therefore, we can solve this 773 with the above analysis on $C$, becomes

$$\sum_{i=1}^{n} \frac{B}{T+c_i} \geq \frac{\sum_{i=1}^{k} B}{T+c_i} + \frac{(n-k)B}{O-\sum_{i=1}^{k} c_i}.$$  

(30)

775 Thus, the equality holds iff $c_i = x_i < (\sum_{j=0}^{i-1} x_j)/(n - i + 1)$ for $i = 1, \ldots, k$ and $c_i = (\sum_{j=0}^{k} x_j)/(n-k) \leq x_i$ for $i = k + 1, \ldots, n$. When $k \neq n-1$, $NE_0$ implies 777 that $y_i = c_i$ for $i = 1, \ldots, k$ and $y_i = c_i = 779 (-\sum_{j=0}^{k-1} x_j)/(n-k)$ for $i = k + 1, \ldots, n$. When $k = n - 1$, 780 only $x_1, \ldots, x_{n-1} - \sum_{i=0}^{n-1} x_i) \in NE_{n-1}$ satisfies 781 $y_i = c_i = x_i$ for $i = 1, \ldots, n - 1$ and $y_n = c_i = -\sum_{j=0}^{n-1} x_j$, 782 others do not. Therefore, except for the case that multiple Nash equilibria exist, the Nash equilibrium maximizes $\sum_{i=1}^{n} (B/(T+y_i)) c_i$, 784 which, in other words, means that the revenue of BS0 is 786 maximized. □
VIII. NUMERICAL RESULTS

Here, we show the performance evaluation for the proposed resource-sharing algorithm. We compare three different methods of acquirers making acquisition requests: the proposed method, the random method, and the greedy method. The proposed method is the one in which the acquirers make requests at the Nash equilibrium. The random method is the one in which the acquirers randomly make requests between zero and their max traffic. In this scheme, we average the results over 1000 runs. The greedy method is the one in which the acquirers greedily make requests at the max traffic requirements. Four performance metrics will be used for the numerical evaluation: the allocative efficiency metric, the max-min fairness metric, the proportional fairness metric, and the revenue of the offeror, which are the objective functions in Definitions 4, 6, 7, and 9, respectively. The allocative efficiency metric measures the overall system performance of the resource allocation. The max-min fairness and proportional fairness metrics measure the fairness degree of the resource allocation. The revenue of the offeror measures the willingness of the offeror to offer unused resources.

We carry out the evaluation in two scenarios. In the first scenario, we vary the number of the acquirers from 1 to 20. The max traffic of each acquirer is randomly generated in [0, 20]. The other parameters are set as follows: $O = 25$, $T = 50$, and $B = 1000$. In the second scenario, we vary the amount of $O$ from 0 to 50. The max traffic of the 10 acquirers is randomly generated in [0, 20], while the other parameters are the same.

In scenario 1, all subgraphs in Fig. 6 give clear evidence to the fact that the proposed method achieves the most efficient and fair resource allocation. Fig. 6(b) shows that the proposed method protects the acquirers from losing the original resources as the max-min fairness metric remains greater than zero in all cases. This also indicates another advantage that the proposed method indeed achieves individual rationality. The other two methods cannot provide such a resource protection, particularly when the system becomes more crowded. As depicted in Fig. 6(b), both methods’ max-min fairness metrics rapidly approach $-50$. In Fig. 6(c), the logarithmic proportional fairness metric of the greedy method even drops to negative infinity since at least one acquired loses all its resources. Furthermore, the offeror also prefers the proposed method because it generates the highest revenue, as shown in Fig. 6(d).

In scenario 2, the proposed method outperforms the others in all metrics, as illustrated in Fig. 7. In the situation of scarce resources, Fig. 7(b) and (c) shows a prominent edge of the proposed method in fairness. As the amount of resources increases, the proposed method also gains further advantage in efficient resource sharing and revenue maximization, as depicted in Fig. 7(a) and (d).

IX. DISCUSSION ON MULTIPLE OFFERORS

In Section II-A, we have explained the reason for formulating a one-offeror model. It is because IEEE 802.22 does not synchronize DRRO, and there is barely more than one offeror at any instant of time. Nevertheless, we can still propose two solutions for the case of multiple offerors at the same instant.

In the first solution, we let an Access Service Networks (ASN) Gateway collect the offered resources from all offerors. The ASN Gateway then runs the renting-and-offering and
847 contention procedures to allocate resources. In the second 848 solution, we assume that the offeror with the smallest ID first 849 runs the resource-sharing algorithm, the offeror with the second 850 smallest ID runs the algorithm, and so on. Both solutions will 851 achieve efficient and fair resource allocation and will ensure all 852 acquirers’ participation.

X. Conclusion

853 In the proposed IEEE 802.22 resource-sharing game, the 855 acquirers always reach a Nash equilibrium. The acquirers’ re- 856 sources are uniquely determined at the Nash equilibrium, given 857 their traffic requirements. The proposed resource-sharing algo- 858 rithm is desirable because it derives efficient and fair resource 859 utilization among all acquirers. The resource allocation is effi- 860 cient as it achieves allocative efficiency and Pareto optimality. 861 The resource allocation also meets both max-min fairness and 862 proportional fairness criteria. Moreover, the resource-sharing 863 algorithm ensures all acquirers’ participation without resource 864 loss and revenue maximization of the resource offeror.

APPENDIX

PROOF OF LEMMA 4

865 1) It is the result directly from the setting of utility functions. 866 2) First, note that \( \sum_{i=1}^{n} y_i \leq \sum_{i=1}^{n} x_i \). If \( \sum_{i=1}^{n} y_i \leq O \), it 869 must be \( \sum_{i=1}^{n} y_i \leq \min(\sum_{i=1}^{n} x_i, O), \forall i \in \mathbb{N} \)

\[
r_i = \frac{1}{O_{jy_j=y_i}} \left[ O - \sum_{jy_j<y_i} y_j \right]^+ \\
\geq \frac{1}{\sum_{jy_j=y_i}} \left[ \sum_{jy_j=y_i} y_j - \sum_{jy_j<y_i} y_j \right]^+ \\
\geq \frac{1}{\sum_{jy_j=y_i}} \left[ \sum_{jy_j=y_i} y_j \right]^+ = y_i. \quad (31)
\]

Therefore, \( \min(y_i, r_i) = y_i \), and

\[
\sum_{i=1}^{n} \min(y_i, r_i) = \sum_{i=1}^{n} y_i \leq \min\left(\sum_{i=1}^{n} x_i, O\right). \quad (32)
\]

If \( O < \sum_{i=1}^{n} y_i \leq \sum_{i=1}^{n} x_i \), for player \( k \) having \( r_k > 0 \) with the largest \( y_k \), we have \( y_k \geq r_k \); otherwise

\[
r_k = \frac{1}{O_{jy_j=y_k}} \left[ O - \sum_{jy_j<y_k} y_j \right]^+ > y_k \\
\iff \left[ O - \sum_{jy_j<y_k} y_j \right]^+ > \sum_{jy_j=y_k} y_j \\
\iff \left[ O - \sum_{jy_j\leq y_k} y_j \right]^+ > 0. \quad (33)
\]

Equation (33) implies that for player \( m \) having \( y_m > 873 \) \( y_k \) with smallest \( y_m \) (such a player must exist since 874 \( \sum_{i=1}^{n} y_i > O \), \( r_m > 0 \), which, however, contradicts 875 player \( k \) who is the one having \( r_k > 0 \) with the largest 876 \( y_k \). Therefore, \( y_k \geq r_k \) for player \( k \) having \( r_k > 0 \) with 877 the largest \( y_k \) and \( r_k = 0, \forall i \) such that \( y_i > y_k \). Also, we 878 should have \( r_i > y_i, \forall i \) such that \( y_i < y_k \). If not, there 879 must exist some player \( m \) having \( y_m < y_k \) and \( r_m \leq y_m \). 880 Then

\[
r_m = \frac{1}{\sum_{jy_j=y_m}} \left[ O - \sum_{jy_j<y_m} y_j \right]^+ \leq y_m \\
\iff \left[ O - \sum_{jy_j<y_m} y_j \right]^+ \leq \sum_{jy_j=y_m} y_j \\
\iff \left[ O - \sum_{jy_j\leq y_m} y_j \right]^+ = 0. \quad (34)
\]

Equation (34) implies that for player \( i \) such that \( y_i > 882 \) \( y_m \), and, thus, \( r_k = 0 \), which contradicts \( r_k > 0 \). 883 From \( y_k \geq r_k, r_k = 0, \forall i \) such that \( y_i > y_k \), and \( r_i > 884 \) \( y_i, \forall i \) such that \( y_i < y_k \), we have

\[
\sum_{i=1}^{n} \min(y_i, r_i) = \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} r_i \\
= \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} \frac{1}{O_{jy_j=y_k}} \left[ O - \sum_{jy_j<y_k} y_j \right] \\
= O < \sum_{i=1}^{n} x_i. \quad (35)
\]

Combining (32) and (35), we have \( \sum_{i=1}^{n} \min(y_i, r_i) \leq 886 \min(\sum_{i=1}^{n} x_i, O) \).

3) Assume that player \( k \) is the player having \( t_k = -T \) with 888 the largest \( y_k \). Then, \( t_k \) must have the form

\[
t_k = \begin{cases} 
\frac{1}{\sum_{jy_j=y_k}} \left( \sum_{jy_j>y_k} T \right) - \sum_{jy_j<y_k} \left[ y_j - r_j \right]^+, & \text{if } t_k \geq 0 \\
-\sum_{jy_j=y_k} \left[ y_j - r_j \right]^+, & \text{if } 0 > t_k > -T.
\end{cases}
\]

(36)
Furthermore, we have \( [y_k - r_k]^+ \geq t_k \); otherwise
\[
[y_k - r_k]^+ < t_k = \frac{1}{\sum_{j:y_j = y_k}} \left( \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ \right)
\]
\[
\Leftrightarrow 0 < \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ - \sum_{j:y_j = y_k} [y_j - r_j]^+
\]
\[
\Leftrightarrow 0 < \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ .
\]
Equation (37) cannot hold if player \( k \) is the one with the largest \( y_k \) among all players since \( \sum_{j:y_j \leq y_k} T - \sum_{j:y_j > y_k} [y_j - r_j]^+ \) must be less than or equal to zero. However, if there are other players having a strategy that is larger than \( y_k \), then for player \( m \), who has \( y_m > y_k \), with smallest \( y_m \), (37) becomes
\[
0 < \sum_{j:y_j \geq y_m} T - \sum_{j:y_j < y_m} [y_j - r_j]^+
\]
\[
\Leftrightarrow - \sum_{j:y_j = y_m} T < \sum_{j:y_j > y_m} T - \sum_{j:y_j < y_m} [y_j - r_j]^+
\]
\[
\Leftrightarrow - T < \frac{1}{\sum_{j:y_j = y_m}} \left( \sum_{j:y_j > y_m} T - \sum_{j:y_j < y_m} [y_j - r_j]^+ \right).
\]
Applying (38) to \( t_m \), we will derive \( t_m > - T \), which contradicts that player \( k \) is the one having \( t_k > - T \) with the largest \( y_k \). Therefore, \( [y_k - r_k]^+ \geq t_k \) for player \( k \) having \( t_k > - T \) with the largest \( y_k \) and \( t_i = - T \), \( \forall i \) such that \( y_i > y_k \). Furthermore, we can derive \( t_i > [y_i - r_i]^+ \), \( \forall i \) such that \( y_i < y_k \) because by summing up all \( t_j \) with \( y_j = y_k \), we have
\[
\sum_{j:y_j = y_k} t_j = \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ > - \sum_{j:y_j = y_k} T .
\]
Then, for all player \( i \) having \( y_i < y_k \), (39) implies
\[
\sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ > - \sum_{j:y_j = y_k} T
\]
\[
\Leftrightarrow \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+
\]
\[
> - \sum_{j:y_j = y_k} T + \sum_{j:y_j \geq y_k} T + \sum_{j:y_j \leq y_k} [y_j - r_j]^+
\]
\[
\Rightarrow \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ > \sum_{j:y_j = y_k} [y_j - r_j]^+
\]
\[
\Rightarrow \frac{1}{\sum_{j:y_j = y_k}} \left( \sum_{j:y_j > y_k} T - \sum_{j:y_j < y_k} [y_j - r_j]^+ \right) > [y_i - r_i]^+
\]
\[
\Rightarrow t_i > [y_i - r_i]^+.
\]
Finally, from \( [y_k - r_k]^+ \geq t_k, t_i = - T, \forall i \) such that \( y_i > y_k \), \( t_i > [y_i - r_i]^+ \), \( \forall i \) such that \( y_i < y_k \), and (39)
\[
\sum_{i=1}^{n} q_i = \sum_{i=1}^{n} \min ([y_i - r_i]^+, t_i)
\]
\[
= \sum_{i:y_i < y_k} [y_i - r_i]^+ + \sum_{i:y_i = y_k} t_i - \sum_{i:y_i > y_k} T
\]
\[
= 0.
\]
4) From points 2 and 3
\[
\sum_{i=1}^{n} q_i = \sum_{i=1}^{n} (\min(y_i, r_i) + q_i) \leq \min \left( \sum_{i=1}^{n} x_i, O \right). \tag{42}
\]
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