1. B’s current wealth is \( w \). He considers to buy a stock which rate of return \( r \) is uncertain. \( E(r) > 0 \) and \( \text{var}(r) > 0 \). B is risk averse. Let \( u(.) \) denote B’s expected utility function of wealth.

   (a) (1 point) Let \( x \) denote B’s investment amount. Please explain B’s objective is:
   \[
   \max_x E u(w + xr), \quad x \geq 0.
   \]

   (b) (3 points) Please give the FOC of B’s problem, and check whether the SOC is satisfied.

   (c) (2 points) Will B buy the stock?

2. (3 points) C’s expected utility function of wealth \( (w) \) is: \( u_c(w) = -e^{-3w} \), and D’s expected utility function of wealth is: \( u_d(w) = -e^{-4w} \). Can we say one is globally more risk averse than the other one?

3. (3 points) There are 2 lotteries. Lottery 1 gives $10 with probability 2/3 and $20 with probability 1/3. Lottery 2 gives $5 with probability 1/3, $15 with probability 5/9 and $30 with probability 1/9. A is risk averse. Which lottery does A prefer?

4. A consumes 2 goods, his utility function is:
   \[
   u = x_1 x_2^2,
   \]
   where \( x_i \) is the consumption of good \( i \). Let \( p_i \) denote the price of good \( i \) and let \( I \) denote A’s income. A’s optimal consumption bundle is: \( x_1 = I/3p_1, x_2 = 2I/3p_2 \).

   (a) (2 points) Prove that A’s expenditure function is: \( e = 3(up_1 p_2^2/4)^{4/3} \)

   (b) (9 points) Suppose that in the last month, \( I = $9, p_1 = p_2 = $1 \). This month, \( p_1 \) increases to 3 while \( I \) and \( p_2 \) stay the same. Please calculate the equivalent variation, the compensating variation and the change of the consumer’s surplus due to this price change.

5. There are 100 people in a society. The \( i \)-th person’s indirect utility function is:
   \[
   v_i(p, m_i) = (p_1^r + p_2^r)^{-1/r} m_i,
   \]
   where \( p_1 \) and \( p_2 \) are prices of good 1 and good 2, \( m_i \) is the \( i \)-th person’s income, and \( r \) is some positive constant.

(a) (2 points) Are these indirect utility functions in the Gorman form?
(b) (2 points) What is their representative consumer’s indirect utility function?
(c) (2 points) What is their representative consumer’s demand for good 1?

6. (5 points) Consider the \( i \)-th person indirect utility function in the previous problem. What is his utility function \( u_i(x_1, x_2) \)?

7. Consider the following utility function:
   \[
   u = x_1 x_2^2 z^3.
   \]

   (a) (2 points) Is this utility function weakly separable in \((x_1, x_2)\)?

   (b) In the following, we shall construct a quantity index \( X \) and a price index \( P \):
   \[
   X = f(x_1, x_2), \quad P = g(p_1, p_2),
   \]
   where \( p_i \) denote the price of good \( x_i \). We wish to re-write the utility maximization problems as follows:
   \[
   \max_{X, z} U(X, z)
   \]
   \[
   \text{subject to } PX + p_z z = m,
   \]
   where \( m \) is the income and \( p_z \) the price of good \( z \).

   i. (3 points) What is the appropriate quantity index function \( f(x_1, x_2) \)?

   ii. (2 points) What is \( U(X, z) \)?

   iii. (3 points) What is the price index function \( g(p_1, p_2) \)?
解答

1a \( w - x + x(1 + r) = w + xr \)

1b FOC: \( Eu'(w + xr)r = 0 \), SOC: \( Eu''(w + xr)r^2 < 0 \) since \( u'' < 0 \).

1c yes, if \( x = 0 \), then \( Eu'(w + xr)r = Eu'(w)r > 0 \).

2 yes, \( D \) is more risk averse, since his risk aversion measure is always larger than \( C \)'s (4 > 3)

3 consider lottery 2 as \( 2/3 \circ (1/2 \circ $5 \oplus 1/2 \circ $15) \oplus 1/3 \circ (2/3 \circ $15 \oplus 1/3 \circ $30) \). \( A \) prefers $10 to \( 1/2 \circ $5 \oplus 1/2 \circ $15 \) and $20 to \( 2/3 \circ $15 \oplus 1/3 \circ $30 \). so \( A \) prefers lottery 1.

4a \( A \)'s indirect utility function is: \( u = 4I^3/27p_1p_2^2 \). To take the inverse function, we get the expenditure function.

4b \( EV = 3 \star 9^{1/3} - 9, CV = 9 - 9 \star 3^{1/3}, DCS = -3 \ln 3 \)

5a yes

5b \( v_i(p, m) = (p_1^r + p_2^r)^{1/r}m, \) where \( m = \sum m_i \)

5c by Roy’s identity, \( x_i(p, m) = (p_1^r + p_2^r)^{-1}p_1^{r-1}m \)

6 solve the following problem: \( \min_p v_i(p, 1), \) s.t. \( px = 1 \). in the solution, \( p \) is in terms of \( x \). substitute it in the objective function, we then have: \( u_i = (x_1^\rho + x_2^\rho)^{1/\rho}, \) where \( \rho = r/(r - 1) \)

7a yes

7(b)i \( f(x_1, x_2) = (x_1x_2^2)^{1/3} \)

7(b)ii \( U(X, z) = X^{3/z^3} \)

7(b)iii \( g(p_1, p_2) = 3p_1^{1/3}p_2^{2/3}4^{-1/3} \)