Chapter 5

Nonlinear Optical Effects in Optical Fibers

Though the nonlinearity of silica-based fiber is quite small, several nonlinear optical effects manifest themselves conspicuously owing to the fact that (a) the power density is very high because light is confined into a small cross-sectional area, (b) the interaction length between the light wave and the fiber material is quite long due to the low-loss property of fibers, and (c) coherent interaction is possible since the modal field distribution and polarization are well prescribed and maintained over the long length [1]. Various nonlinear optical effects in fibers will be explained, such as optical solitons, stimulated Raman scattering, stimulated Brillouin scattering, and second-harmonic generation.

5.1 Figure of merit for nonlinear effects

When 1 W of optical power is coupled into single-mode optical fiber with the core diameter of 10 μm, the optical power density exceeds 1 MW/cm². Such high power density and very long interaction length are the features of nonlinear optical effects in optical fibers. The $I \cdot L$ product, which is important to evaluate the nonlinear interactions, is defined as the product of optical intensity $I$ (optical power/active area of the beam) and the interaction length $L$. In bulk optics, when the spot size (beam radius at which an electric field becomes 1/e) at the focal point is $w_0$, the beam radius $w(z)$ at the distance $z$ from the focal point is expressed by [2]

$$w(z) = w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi n w_0^2}\right)^2}, \quad (5.1)$$

where $\lambda$ and $n$ denote the wavelength of light and the refractive index of the medium, respectively. The optical intensity at $z$ is given by

$$I_p(z) = \frac{P}{\pi w(z)^2}, \quad (5.2)$$

Then the $I \cdot L$ product of the bulk lens system is obtained as

$$[I \cdot L]_b = \int_{-\infty}^{\infty} I_p(z) \, dz = \frac{nP}{\lambda}. \quad (5.3)$$

It is known that there is no other way than to increase the input power $P$ in order to enhance the nonlinear effect in the bulk optics system. In contrast, the optical intensity of the fiber at distance $z$ is given by

$$I_f(z) = \frac{P \exp(-\alpha z)}{\pi w_0^2}, \quad (5.4)$$

where $\alpha$ and $w_0$ denote the attenuation coefficient and spot size of the fiber, respectively. The $I \cdot L$ product of the fiber with length $L$ is given by

$$[I \cdot L]_f = \int_{-\infty}^{\infty} I_f(z) \, dz = \frac{P}{\pi w_0^2} L_{\text{eff}}, \quad (5.5)$$

where $L_{\text{eff}}$ denotes the effective interaction length, which is defined by

$$L_{\text{eff}} = \frac{[1 - \exp(-\alpha L)]}{\alpha}. \quad (5.6)$$

It is seen from Eq. (5.5) that the $I \cdot L$ product can be increased by using low-loss and small-core (high refractive-index difference) fibers. When fiber length $L$ is sufficiently long, effective length can be approximated by $L_{\text{eff}} = 1/\alpha$. Then the ratio of the $I \cdot L$ product between bulk optics and optical fibers is expressed as

$$\frac{[I \cdot L]_f}{[I \cdot L]_b} \approx 5 \times 10^7 \frac{\lambda}{(\pi w_0)^2 n}, \quad (5.7)$$

For example, we have $[I \cdot L]_f/[I \cdot L]_b \approx 5 \times 10^7$ for the fiber parameters of $\lambda = 1 \mu m$, $n = 1.5$, $\alpha = 2.3 \times 10^{-4} \text{ m}^{-1}$ (loss of 1 dB/km), and $w_0 = 2.4 \mu m$. It is known that 1 W of optical power is sufficient in optical fibers to generate the same level of nonlinear effects for which several megawatts was required.

5.2 Optical Kerr effect

The optical Kerr effect is the phenomenon in which the refractive index of the medium changes when the electron orbit is deformed by the strong electric field [3]. The refractive index under the Kerr effect is expressed as $n_0 + n_2 E^2$, where $n_0$ and $n_2$ denote the linear refractive index and the Kerr coefficient, respectively. The Kerr coefficient in the silica glass fiber is typically given by $n_2 = 1.22 \times 10^{-22} \text{ m}^2/\text{N}^2$ [4]. It is expressed in different unit systems as $n_2 = 3.18 \times 10^{-20} \text{ m}^2/\text{W}$ or $n_2 = 1.1 \times 10^{-19} \text{ esu}$ (or cm²/statvolt²). The interesting and important nonlinear effects in optical fibers utilizing the optical Kerr effect are (1) optical solitons, (2) optical pulse compression, and (3) modulational instabilities.
5.2.1 Self-phase modulation

When a high-intensity short pulse is coupled to optical fiber, the instantaneous phase of the optical pulse rapidly changes through the optical Kerr effect. If we express the envelope of the optical pulse as $E$ and the linear refractive index at the angular frequency $\omega_0$ as $n(\omega_0)$, the effective index of the optical fiber is given by

$$n(\omega_0, |E|^2) = \frac{c\beta}{\omega_0} = n(\omega_0) + n_2|E|^2. \quad (5.8)$$

Since we define the optical phase as $\Phi = \omega_0 t - \beta z$, the instantaneous angular frequency is given as the derivative of $\Phi$ with respect to time $t$:

$$\omega(t) = \frac{\partial \Phi}{\partial t} = \omega_0 - \frac{\omega_0 n_2}{c} z \frac{\partial |E|^2}{\partial t}. \quad (5.9)$$

Here we consider the variation of optical intensity in the medium at $z$ during the passage of the optical pulse. Therefore, time $t$ progresses from the right-hand side (preceding edge of the pulse) of Fig. 5.1 to the left-hand side (trailing edge). As shown in Fig. 5.1 we have $\partial |E|^2/\partial t < 0$ at the trailing edge. Then it is seen from Eq. (5.9) that the angular frequency decreases ($\omega < \omega_0$) at the preceding edge and increases ($\omega > \omega_0$) at the trailing edge of the pulse. This is schematically illustrated in Fig. 5.1(b). This phenomenon is called self-phase modulation (SPM), which causes the frequency chirping to the optical pulse.

As described in Section 3.6.4, wavelength dependencies of the group velocity for a 1.3-µm zero-dispersion fiber and a 1.55-µm zero-dispersion fiber (dispersion-shifted fiber, DSF) are shown in Fig. 5.2. The spectral region shorter than the zero-dispersion wavelength $\lambda_0$ is called a normal dispersion region, and that longer than $\lambda_0$ is called an anomalous dispersion region. In the anomalous dispersion region, the lower the frequency (the longer the wavelength), the smaller the group velocity. Then the group velocity at the preceding edge of the optical pulse having a lower frequency becomes small. In contrast, the group velocity at the trailing edge of the optical pulse having a higher frequency becomes large. This con-

![Figure 5.1: Self-phase modulation of an optical pulse. (a) Pulse waveform and (b) instantaneous angular frequency (or wavelength) change.](image1)

![Figure 5.2: Wavelength dependencies of the group velocity of optical fibers. A: 1.3-µm zero-dispersion fiber; B: 1.55-µm zero-dispersion fiber (dispersion-shifted fiber, DSF).](image2)
presses the optical pulse. If the compression of the optical pulse due to self-phase modulation balances with and cancels the pulse broadening caused by dispersion, the optical pulse propagates through the fiber while maintaining its original pulse shape. This is called an optical soliton (more precisely, a bright optical soliton) [5], [6]. The historically well-known solitary wave on the surface of shallow water and the nonlinear vibration of the one-dimensional lattice are solitons of the waves themselves. In the optical fiber, however, the envelope of the optical pulse becomes a solitary wave. Therefore a soliton in the fiber is called an envelope soliton.

In the normal dispersion region, the group velocity at the preceding edge of the optical pulse having a lower frequency becomes large and the group velocity at the trailing edge of the optical pulse having a higher frequency becomes small. Then the energy of the optical pulse is dispersed into the preceding and trailing edges of the pulse and the pulse temporal shape becomes square. When such a frequency-chirped and square-shaped pulse is passed through an anomalous medium such as a grating pair, the optical pulse is compressed. This will be described in Section 5.4.

In the normal dispersion region, a dark soliton exists. A dark soliton is also a solitary wave that is generated by cutting a portion of a continuous wave. Nonlinear chirping of the intensity dip in the continuous wave balances the group velocity dispersion of optical fiber in the normal region.

5.2.2 Nonlinear Schrödinger equation
The wave equation in the medium with nonlinear electric polarization \( \mathbf{P}_{NL} \) is expressed as

\[
\nabla^2 \mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2},
\]

\[
\mathbf{D} = \varepsilon \mathbf{E} + \mathbf{P}_{NL}.
\]  

The nonlinear polarization \( \mathbf{P}_{NL} \) is assumed to be small and is treated as the perturbation from the linear polarization. Next the optical field is assumed to maintain its polarization along the fiber length so that a scalar approach is valid. Thirdly, the spectral width \( \Delta \omega \) of the optical pulse is assumed to be sufficiently small compared to the center angular frequency \( \omega_0 \). Since \( \Delta \omega_0 \approx 10^{13} \text{s}^{-1} \), the last assumption is valid for pulses whose width is \( \tau_0 > 0.1 \text{ ps} (\Delta \omega < 2 \times 10^{13} \text{s}^{-1}) \). The third assumption is called a slowly varying envelope approximation (SVEA), in which the temporal variation of the pulse envelope is sufficiently slow compared to the optical cycle. In the slowly varying envelope approximation, the rapidly varying part of the electric field is separated from the slowly varying envelope as

\[
E(r, t) = \frac{i}{2} u_s [E(r, t) \exp(j\omega_0 t) + \text{c.c.}],
\]  

where \( E(r, t) \) is a slowly varying function of time relative to the optical period, c.c. denotes complex conjugate, and \( u_s \) is the polarization unit vector of the light, assumed to be linearly polarized along the \( x \)-axis. When we add the optical loss or gain to the refractive-index change caused by the Kerr effect, we obtain [1]

\[
\mathbf{D} = \varepsilon \mathbf{E} + \mathbf{P}_{NL} = \varepsilon \frac{n + n_s |E|^2}{K} \mathbf{E},
\]

where \( \alpha \) is an attenuation coefficient and \( k = \omega_0/\varepsilon_0. \) Assuming that the nonlinear polarization and the fiber attenuation are small, we can separate Eq. (5.13) into

\[
\varepsilon \mathbf{E} = \varepsilon \frac{n^2 - j2n_s \alpha}{K} \mathbf{E},
\]

\[
\mathbf{P}_{NL} = 2\varepsilon \alpha n_s |E|^2 \mathbf{E}.
\]

The envelope function of the electric field \( E(r, t) \) is expressed as the product of the transverse field distribution \( R(r, \theta) \) and the axial amplitude variation \( A(z, t) \exp(-j\beta_0 z) \):

\[
E(r, t) = R(r, \theta) A(z, t) \exp(-j\beta_0 z),
\]

where \( \beta_0 \) denotes the propagation constant in the absence of the Kerr effect. Substituting Eqs. (5.12) and (5.16) in Eq. (5.10), \( \nabla^2 \mathbf{E} \) is given by

\[
\nabla^2 E = u_s \nabla^2 e_s = u_s \left( \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) e_s,
\]

where

\[
e_s = \frac{i}{2} R(r, \theta) A(z, t) \exp[j(\omega_0 t - \beta_0 z)] + \text{c.c.}
\]

Since the variation of \( A(z, t) \) along the \( z \)-direction is much slower than that of \( \exp(-j\beta_0 z) \), we can assume \( \frac{\partial^2 A}{\partial z^2} \ll \beta_0^2 |A| \). Taking this into account, Eq. (5.17) is reduced to

\[
\nabla^2 E = \frac{1}{2} u_s \left[ A(z, t) \left( \frac{\partial^2 R}{\partial \theta^2} + \frac{1}{r} \frac{\partial R}{\partial r} + \frac{1}{r^2} \frac{\partial^2 R}{\partial \theta^2} \right)
+ R(r, \theta) \left( -j2\beta_0 \frac{\partial A}{\partial z} - \beta_0^2 |A| \right) \exp[j(\omega_0 t - \beta_0 z)] + \text{c.c.} \right].
\]

The right-hand side of Eq. (5.10) is calculated by substituting Eqs. (5.11)–(5.16) in it and using the slowly-varying envelope approximation:

\[
\mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{2} u_s k^2 \left( n^2 - j2n_s \alpha \right) \left[ R(r, \theta) A(z, t) \exp[j(\omega_0 t - \beta_0 z)] + \text{c.c.} \right],
\]

\[
\mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2} = -u_s k^2 n_s |E|^2 \left[ R(r, \theta) A(z, t) \exp[j(\omega_0 t - \beta_0 z)] + \text{c.c.} \right].
\]
From Eqs. (5.19)–(5.21), Eq. (5.10) reduces to
\[
\left[ \frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + \frac{1}{r^2} \frac{\partial^2 R}{\partial \theta^2} + (k^2n^2 - \beta_0)R \right] = 0.
\]

Strictly speaking, the transverse mode profile of the optical fiber under Kerr effect nonlinearity becomes different from the unperturbed profile \( R(r, \theta) \). Based on first-order perturbation theory, however, the electric field profile \( R(r, \theta) \) can be approximated to be the same as that of the linear state, although the propagation constant becomes different as \( \beta_0 \to \beta(\omega) \). Then the electric field \( R(r, \theta) \) satisfies the wave equation
\[
\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + \frac{1}{r^2} \frac{\partial^2 R}{\partial \theta^2} + (k^2n^2 - \beta^2)R = 0.
\]  
(5.22)

Substitution of Eq. (5.23) in Eq. (5.22) gives
\[
(\beta^2 - \beta_0^2)RA + \left( -j2\beta_0 \frac{\partial A}{\partial z} - 2j \alpha A + 2k^2n_2 |A|^2 A \right) = 0.
\]
(5.24)

Multiplying this last equation by \( R^* \) and integrating it over the cross-sectional area, we have
\[
(\beta^2 - \beta_0^2)A - j2\beta_0 \frac{\partial A}{\partial z} - 2j \alpha A + 2k^2n_2 |A|^2 A = 0.
\]  
(5.25)

where \( \eta \) is defined by
\[
\eta = \frac{\int_{0}^{\pi} \left[ \int_{0}^{1} R^* r \, dr \, d\theta \right] \, d\theta}{\int_{0}^{\pi} \left[ \int_{0}^{1} R^* r \, dr \, d\theta \right] ^2}.
\]
(5.26)

Figure 5.3 shows the variation of \( \eta \) with the normalized frequency \( \nu \) for the HE_{11} mode in the step-index fiber, where the transversal electric fields in the core and cladding are given by \( J_0(\nu r/a) \) and \( J_1(\nu r/a)K_0(\nu r/a) \), respectively [refer to Eqs. (3.94) and (3.95) or (4.109) and (4.110)]. It is seen from Fig. 5.3 that we can approximate as \( \eta \approx 1/2 \) under the normal operating condition of the optical fibers \( (\nu = 1.5-2.4) \). Based on the weakly guiding approximation, we approximate \( \beta \approx \beta_0 \approx k \nu m \) in Eq. (5.25). Dividing Eq. (5.25) by \( 2k \nu m \), we obtain
\[
(\beta - \beta_0)A - j\frac{\partial A}{\partial z} - j \alpha A + \frac{1}{2} k \nu m |A|^2 A = 0.
\]
(5.27)

\[\text{Figure 5.3: Dependency of } \eta \text{ on the normalized frequency } \nu \text{ for the HE}_{11} \text{ mode in the step-index fiber.}\]

Propagation constant \( \beta(\omega) \) is approximated by the Taylor series expansion:
\[
\beta(\omega) - \beta_0 = (\omega - \omega_0) \beta' + \frac{1}{2} (\omega - \omega_0)^2 \beta'' + \cdots.
\]
(5.28)

Substitution of this equation into Eq. (5.27) gives
\[
j \frac{\partial A}{\partial z} - j(\omega - \omega_0) \beta' A - \frac{1}{2} (\omega - \omega_0)^2 \beta'' A + j \alpha A - \frac{1}{2} k \nu m |A|^2 A = 0.
\]
(5.29)

We notice here the Fourier transformation formula for the differential equation:
\[
\int_{-\infty}^{\infty} \frac{\partial A}{\partial t} \exp[-j(\omega - \omega_0) \nu] \, dt = \left[ A(t, \nu) \exp(-j(\omega - \omega_0) \nu) \right]_{-\infty}^{\infty}
\]
\[
+ \int_{-\infty}^{\infty} j(\omega - \omega_0) A \exp[-j(\omega - \omega_0) \nu] \, dt = \int_{-\infty}^{\infty} j(\omega - \omega_0) A \exp[-j(\omega - \omega_0) \nu] \, dt.
\]

\[
\int_{-\infty}^{\infty} \frac{\partial^2 A}{\partial z^2} \exp[-j(\omega - \omega_0) \nu] \, dt = \int_{-\infty}^{\infty} -j(\omega - \omega_0)^2 A \exp[-j(\omega - \omega_0) \nu] \, dt.
\]
Comparing both terms in Eqs. (5.30a) and (5.30b), we see that the following substitution relations hold:

\[ j(\omega - \omega_0)A \rightarrow \frac{\partial A}{\partial \tau}, \]  
(5.31a)

\[ -(\omega - \omega_0)^2 A \rightarrow \frac{\partial^2 A}{\partial \tau^2}. \]  
(5.31b)

Then Eq. (5.29) is rewritten, by using Eq. (5.31), as

\[ j \left[ \frac{\partial A}{\partial \tau} + \beta \frac{\partial A}{\partial z} + \alpha A \right] = - \frac{1}{2} \beta'' \frac{\partial^2 A}{\partial \tau^2} + \frac{1}{2} k n_2 |A|^2 A. \]  
(5.32)

This is the nonlinear Schrödinger equation that governs the envelope function of the optical pulse \( A(z, t) \) under Kerr effect nonlinearity and loss (or gain) in the optical fibers.

### 5.3 Optical solitons

We first solve the nonlinear Schrödinger equation under the ideal case of lossless fiber and examine the optical soliton.

#### 5.3.1 Fundamental and higher-order solitons

When the loss of the optical fiber is zero, Eq. (5.32) is expressed as

\[ j \left( \frac{\partial A}{\partial \tau} + \frac{1}{v_g} \frac{\partial A}{\partial t} \right) = - \frac{1}{2} \beta'' \frac{\partial^2 A}{\partial \tau^2} + \frac{1}{2} k n_2 |A|^2 A, \]  
(5.33)

where \( v_g = 1/\beta' \) is a group velocity given by Eq. (3.108). In order to investigate the progress of optical pulse shape, we change the fixed coordinate system to the moving coordinate at the velocity \( v_g \):

\[ \begin{align*}
A(z, \tau) &= \phi(z, \tau) \\
\tau &= t - \frac{z}{v_g}
\end{align*} \]  
(5.34a)

(5.34b)

Partial derivatives of \( A \) with respect to \( z \) and \( t \) are then given by

\[ \begin{align*}
\frac{\partial A}{\partial z} &= \frac{\partial \phi}{\partial z} - \frac{1}{v_g} \frac{\partial \phi}{\partial \tau}, \\
\frac{\partial^2 A}{\partial z^2} &= \frac{\partial \phi}{\partial z^2}, \\
\frac{\partial A}{\partial t} &= \frac{\partial \phi}{\partial t}, \\
\frac{\partial^2 A}{\partial t^2} &= \frac{\partial \phi}{\partial t^2}. 
\end{align*} \]  
(5.35a)

(5.35b)

(5.35c)

Substituting Eq. (5.35) into Eq. (5.33), we obtain

\[ j \frac{\partial \phi}{\partial \tau} = - \frac{1}{2} \beta'' \frac{\partial^2 \phi}{\partial \tau^2} + \frac{1}{2} k n_2 |\phi|^2 \phi. \]  
(5.36)

The fundamental solution of Eq. (5.36) is given by [6]

\[ \phi(z, \tau) = \phi_p \exp \left( j \frac{\beta''}{2 k} z \right) \text{sech} \left( \frac{z}{t_0} \right), \]  
(5.37)

where \( t_0 \) is related to the FWHM (full width at the half maximum) width of the optical intensity \( \phi_p \) by

\[ t_0 = \frac{\phi_p}{2 \cosh^{-1} \sqrt{2}} \approx 0.567 t_c. \]  
(5.38)

As is confirmed by substituting Eq. (5.37) into Eq. (5.36), the peak amplitude of the electric field \( \phi_p \) should satisfy

\[ |\phi_p|^2 = \frac{-2 \beta''}{i k n_2}. \]  
(5.39)

Since \( \beta'' = \partial n_2 / \partial \omega \), an optical soliton is generated in the anomalous region \( (\partial n_2 / \partial \lambda_c) < 0 \), where the signal wavelength is longer than the zero-dispersion wavelength, as shown in Section 5.2.1. \( \beta'' \) represents the dispersion of the group velocity with respect to the wavelength and is called the group velocity dispersion (GVD).

We next express the soliton condition of Eq. (5.39) in terms of the optical power. The electromagnetic fields of the HE\(_{11}\) mode is expressed by (refer to Section 3.5)

\[ \begin{align*}
H_x &= n_1 \epsilon_0 c E_x \\
H_y &= H_z = 0
\end{align*} \]

where we assumed \( \beta = k n_1 \). The instantaneous peak power of the HE\(_{11}\) mode is then given by

\[ P = \int_0^{\infty} \int_0^{2\pi} (E_x H_y^* - E_y H_x^*) r \, dr \, d\theta = n_1 \epsilon_0 c \int_0^{2\pi} \int_0^{2\pi} |E_x|^2 r \, dr \, d\theta. \]  
(5.40)

Substituting \( E_x = R(\rho, \phi) \phi(\rho, 0) \) to Eq. (5.40), we obtain

\[ P = n_1 \epsilon_0 c |\phi_p|^2 \int_0^{2\pi} \int_0^{2\pi} |R|^2 r \, dr \, d\theta. \]  
(5.41)
Equation (5.41) is also obtained for the HE$_{11}$ mode. Here we introduce the new parameter defined by [7]

\[ A_{\text{eff}} = \frac{\left[ \int |r|^2 |R|^2 r \, dr \, d\theta \right]^2}{\int \frac{|\psi|^2}{|R|^2} r \, dr \, d\theta} \]  \hspace{1cm} (5.42)  

$A_{\text{eff}}$ represents the cross-sectional area occupied by the substantial part of the light field and is called the effective area. Using Eqs. (5.26) and (5.42), Eq. (5.41) is rewritten as

\[ P = \eta n_2 \epsilon_0 c |\phi|^2 A_{\text{eff}} = \eta n_1 \epsilon_0 c |\phi|^2 A_{\text{eff}}, \]  \hspace{1cm} (5.43)  

where $\eta$ is assumed to be 0.5. Figure 5.4 shows the dependence of $A_{\text{eff}}$/core area on the normalized frequency $v$ for the HE$_{11}$ mode of the step-index fiber. Substitution of Eq. (5.39) in Eq. (5.43) gives the power condition to generate the fundamental optical soliton. We should note here that the Kerr coefficient $n_2$ of Eq. (5.39) is in units of $m^2/V^2$. In order to obtain the power condition of the optical soliton, the Kerr constant in m$^2$/V$^2$ ($n_2 = 1.22 \times 10^{-22} m^2/V^2$) should be transformed into units of m$^2$/W ($n_2 = 3.18 \times 10^{-20} m^2/W$). The transformation relation is given by

\[ n_2 [m^2/V^2] = n_2 [m^2/W] \cdot n_1 \epsilon_0 c, \]  \hspace{1cm} (5.44)  

where $\epsilon_0 = 8.854 \times 10^{-12} F/m$ and $n_1 = 1.45$. Substituting Eqs. (5.38), (5.39), and (5.44) into Eq. (5.43) and using the chromatic dispersion relation of $\beta = -(\lambda^2/2n_0)c$ [Eq. (3.143)], the peak power to generate the fundamental soliton is obtained:

\[ P = \frac{0.7768\lambda^3 A_{\text{eff}}|\sigma|}{\pi^2 c n_1 d_0}. \]  \hspace{1cm} (5.45)  

In order to simplify this last equation, we express the chromatic dispersion $\sigma$ in units of ps/km·nm, the wavelength $\lambda$ in units of $\mu$m, and the core diameter as $d(\mu$m) and define $\xi = A_{\text{eff}}/(\pi d/2)^2$. The power condition is then given by

\[ P_{\xi} = 6.48 \times 10^{-2} d^2 \lambda^3 |\sigma| \]  \hspace{1cm} [W·ps$^2$]  \hspace{1cm} (5.46)  

Figure 5.5 shows the dependence of $P_{\xi}$ on the chromatic dispersion $\sigma$ at a 1.55-$\mu$m wavelength for the effective area parameter $\xi = 1.5 (\nu = 2.0)$. When, for
example, the chromatic dispersion of the fiber is \( \sigma = 2 \text{ ps km } \text{nm} \) and the core diameter \( d = 5 \text{ \( \mu \text{m} \) } \), we obtain \( \beta_2 = 1.81 \). Therefore, peak power is known to be \( P = 18.1 \text{ mW} \) in order to generate the optical soliton with the FWHM pulse width of \( \tau_0 = 10 \text{ ps} \). When the bitrate of the pulse (pulse repetition rate) is \( B \), the average power of the soliton is obtained by

\[
P_{av} = B \int_{-1/2B}^{1/2B} \left[ P_{1/2} \text{sech}\left( \frac{1}{\tau_0} \right) \right]^2 dt \approx BP \int_{-\infty}^{\infty} \text{sech}^2 \left( \frac{1}{\tau_0} \right) dt
\]

\[
= \tau_0 BP \left[ \tanh \left( \frac{1}{\tau_0} \right) \right]_{-\infty}^{\infty} = 2\tau_0 BP = \frac{\tau_0 BP}{\cosh^{-1}(\sqrt{2})} \approx 1.135\tau_0 BP. \tag{5.47}
\]

Then the average power of the \( B = 10 \text{-Gbit/s} \) optical pulse is about \( P_{av} = 2.1 \text{ mW} \). Generally, the parameter

\[
\nu = \frac{\phi q_{0} I_0^{1/2}}{\sqrt{-2\beta}} \tag{5.48}
\]

is used to evaluate the generation of the higher-order optical solitons. Even if Eq. (5.48) is not unity, it is known that the fundamental soliton is generated when the parameter satisfies \( 0.5 \leq \nu \leq 1.5 \) [6]. However, for the case of \( \nu \neq 1 \), the excitation efficiency of the soliton becomes low, since the nonsoliton wave exists. When an optical pulse having a peak power higher than that of the fundamental soliton is coupled into an optical fiber, an \( N = [\nu + 0.5] \)th \( (N \) is an integer) higher-order soliton is generated for \( \nu \geq 1.5 \). A higher-order optical soliton periodically changes its temporal pulse shape, as shown in Fig. 5.6 (\( \nu = 3 \)). The length of the period is given by

\[
z_0 = \frac{\pi \beta_0^2}{2|\beta|} = 0.322 \frac{\pi^2 \tau_0^2}{\lambda^2 |\beta|}. \tag{5.49}
\]

The pulse width of the higher-order soliton becomes compressed at \( z < z_0 \), since pulse narrowing due to nonlinear chirping is dominant in this region. This is called a soliton compression and can be utilized for optical pulse compression [8].

### 5.3.2 Fiber loss compensation by optical amplification

The optical soliton is an ideal signal waveform for ultrahigh-bitrate long-distance optical communications, since it does not suffer from signal waveform distortion by the chromatic dispersion of optical fiber. However, we have neglected fiber loss in the preceding discussions. When loss exists in the fiber, pulse peak power decreases and pulse width becomes broadened, since pulse narrowing caused by nonlinear chirping is weakened. Figure 5.7 shows the variation of temporal pulse waveform calculated by using the beam propagation method (BPM) when the optical pulse with the peak power \( P = 18 \text{ mW} \) and FWHM width \( \tau_0 = 9.4 \text{ ps} \) is coupled to optical fiber having an attenuation of \( 0.18 \text{ dB/km} \) at a \( 1.55 \text{-\( \mu \text{m} \) wavelength}. Numerical simulation by BPM will be described in Chapter 7.
When signal loss becomes larger than about 3 dB (propagation distance over about 15 km in Fig. 5.7), the optical soliton loses its narrowing effect and is governed by chromatic dispersion. In order to compensate for the optical loss, the optical soliton should be amplified directly in the optical stage. Stimulated Raman scattering in optical fiber [9] and an Er-doped optical amplifier [10] can be utilized for the direct optical amplification of optical solitons. Stimulated Raman scattering is a phenomenon observed when strong monochromatic light irradiates the material [11]. Coherent light is scattered with the specific wavelength shift through the interaction between the optical phonon of the material and the excitation light field. The wavenumber shift in silica glass is about 440 cm\(^{-1}\). For example, when we use light at \(\lambda_p = 1.45 \mu m (1.45 \times 10^{-4} \text{ cm})\) as a pumping light, we can amplify the signal light directly at \(\lambda_s = 1.55 \mu m (1.55 \times 10^{-4} \text{ cm})\) by \(\lambda_p = 1/\lambda_s = \frac{1}{440 \text{ cm}^{-1}}\) through the interaction with the optical phonon. An Er-doped fiber amplifier is a fiber in which Er\(^{3+}\) ion is doped in the core, at several hundred parts per million. When it is pumped by the light at a wavelength of 0.98 \(\mu m\) or 1.48 \(\mu m\), the optical signal in the wavelength region of 1.53–1.55 \(\mu m\) is amplified through the stimulated emission between the transition lines \(^{4}I_{13/2} \rightarrow ^{4}I_{15/2}\). In the following, amplification of optical soliton by using stimulated Raman scattering will be described.

It is shown by numerical simulation that an optical soliton with signal wavelength \(\lambda_s = 1.55 \mu m\), FWHM pulse width \(\tau_0 = 9.4 \text{ ps}\), and peak power \(P = 18 \text{ mW}\) can be transmitted at 10 Gbit/s over several thousands of kilometers of fiber with chromatic dispersion \(\sigma = 2 \text{ ps/km nm}\), effective area \(A_{\text{eff}} = 25 \mu m^2\) (core diameter \(\approx 5 \mu m\)), loss at the signal wavelength of 0.18 dB/km, and loss at the pump wavelength \(\lambda_p = 1.45 \mu m\) of 0.29 dB/km when it is amplified by a pumping light of 50-mW (CW) fiber coupled power with a period of 50 km [12]. Figure 5.8 shows fiber length dependence of fiber loss, Raman gain, and net loss or gain with the preceding parameters. Figure 5.9 shows the variation of pulse waveform along one optical amplifier span calculated by BPM. It is shown that the fundamental soliton can be transmitted over a long fiber by direct optical amplification.

In the experiment on soliton transmission by Raman amplification, the signal from the F-center laser at 1.60 \(\mu m\) (FWHM pulse width of 55 ps, peak power of 15 mW, and repetition rate of 100 MHz) was recirculated in the fiber loop of length 41.7 km (total length of 4000 km) by using F-center laser pump light at 1.50 \(\mu m\) [13]. As the transmission experiment using an Er-doped fiber amplifier, an MQW-DFB laser oscillating at 1.52 \(\mu m\) (FWHM pulse width of 20 ps, peak power of 4–6 mW, and signal bitrate of 20 Gbit/s) was transmitted over 200 km by amplifying the signal with a period of 25 km using a 1.48-\(\mu m\) semiconductor laser as the pump source [14].

5.3.3 Modulational instability

Modulational instability (MI) is a phenomenon observed in a nonlinear dispersive medium in which the side-band component of the amplitude-modulated light grows exponentially when a certain condition is satisfied [15]. The steady-state solution of the nonlinear Schrödinger equation (5.36) (continuous-wave solution

![Figure 5.8: Fiber length dependence of fiber loss, Raman gain, and net loss or gain in Raman amplification.](image)

![Figure 5.9: Variation of temporal pulse waveform along one optical amplifier span.](image)
under the condition of $\delta \phi / \delta t = 0$ is given by

$$\phi_{\text{es}} = \phi_0 \exp(-j \delta z), \quad (5.50a)$$

$$\delta = \frac{k n^2}{2} |\phi_0|^2, \quad (5.50b)$$

where $\phi_0$ is a constant. When a small perturbation is applied to the steady-state solution [Eq. (5.50a)] in the anomalous region, the electric field becomes unstable under a certain condition. We add a small perturbation $q(z, \tau)$ to the steady-state solution $\phi_{\text{es}}$ to investigate the modulational instability. The field with the perturbation is expressed as

$$\phi = [\phi_0 + q(z, \tau)] \exp(-j \delta z). \quad (5.51)$$

Substituting this into Eq. (5.36) under the assumption that the perturbation is sufficiently small ($|q| \ll |\phi_0|$), we obtain the differential equation for $q$ of the form

$$j \frac{\partial q}{\partial z} = -\frac{\beta^2 q^3}{2} \frac{\partial^2 q}{\partial \tau^2} + \delta(q + q^*). \quad (5.52)$$

We assume that the general solution for $q(z, \tau)$ of the form

$$q(z, \tau) = q_1 \cos(\Omega \tau - K z) + j q_2 \sin(\Omega \tau - K z), \quad (5.53)$$

where $\Omega$ and $K$ denote the angular frequency and wavenumber of the perturbations, respectively. The dispersion relation between $K$ and $\Omega$ is obtained by substituting Eq. (5.53) into Eq. (5.52):

$$K^2 = \left(\frac{\beta^2 \Omega^2}{2}\right)^2 \left[\Omega^2 + \frac{2k n^2}{\beta^2} |\phi_0|^2 \right]. \quad (5.54)$$

It is seen from Eq. (5.54) that $K$ becomes a pure imaginary number when the following conditions are satisfied:

$$\frac{n^2}{\beta^2} < 0 \quad (5.55)$$

$$\Omega < \Omega_c = \left[\frac{2k n^2}{-\beta^2} |\phi_0|^2 \right]^{1/2}. \quad (5.56)$$

Then $q$ grows exponentially. Of course, we have the exponentially decaying solution. But it is not important here. If the perturbation is the amplitude modulation, this phenomenon implies that the modulation depth grows exponentially. Therefore, such a phenomenon is called the modulational instability.

The angular frequency at which the growth rate of the perturbation becomes the maximum is given by $\Omega = \pm \Omega_c / \sqrt{2}$. Here the growth rate of $q$ is expressed by $\exp(k n^2 |\phi_0|^2 / 2)$. Even when there is no perturbation feeding, the seed of the perturbation may originate in the thermally activated phonon. Figure 5.10 shows the experimental observation of the growth of side-band components by the modulational instability in optical fiber [16]. The light source in the experiment was a mode-locked Nd:YAG laser with a 100-MHz repetition rate and a 100-ps pulse width. The reason why a 100-ps pulse was used instead of using CW light was to suppress the stimulated Brillouin scattering (refer to Section 5.7). The parameters of the fiber used in the experiment were chromatic dispersion $\sigma = 2.4$ ps/km·nm, length of fiber $L = 1$ km, effective core area $A_{eff} = 60$ $\mu$m$^2$, and fiber loss of 0.67 dB/km. Figure 5.10(a) shows the output spectrum for low input power. The output spectrum at this power level coincides with that of the input spectrum. When

![Figure 5.10](image-url)
input power is increased to (b) 5.5 W, (c) 6.1 W, and (d) 7.1 W, a side band is generated from the thermally excited phonon at angular frequency $\Omega_c/\sqrt{2}$ [Fig. 5.10(b)], and it grows in accordance with the increase of input power [Fig. 5.10(c)]. When input power is increased still further, a higher-order side band appears [Fig. 5.10(d)]. The wavelength separation between pump light and side band becomes wider as input power increases, since the angular frequency of the side band $\Omega_c/\sqrt{2}$ is proportional to the input $P_0$.

Modulational instability can be generated intentionally by injecting a sideband component having angular frequency in the vicinity of $\Omega_c = \Omega_c/\sqrt{2}$ together with the carrier wave. This is called an induced modulational instability. A high-repetition optical pulse train can be generated by using this induced modulational instability. The repetition period is the inverse of the given modulation frequency $\Omega_c/2\pi$. An ultrahigh-bitrate pulse train having about a 0.34-THz repetition with 0.5-ps FWHM pulse width was generated based on the induced modulational instability [17]. The light source of the experiment was a mode-locked Nd:YAG laser oscillating at 1.319 $\mu$m (pulse width 100 ps and peak power 3 W), and an InGaAsP semiconductor laser with external cavity (cw output 0.5 mW) having the oscillation frequency at $\Omega_c/2\pi \approx 0.34$ THz (1.8 nm) apart from that of the source light is used as a side-band injection light. Figure 5.11 shows the measured auto-correlation traces when the pumping Nd:YAG laser and the side-band InGaAsP semiconductor laser are coupled to single-mode fiber having the zero-dispersion wavelength at $\lambda_0 = 1.275 \mu$m (chromatic dispersion at 1.319 $\mu$m is $\sigma = 375$ ps/km$\cdot$nm). Since wavelengths of side-band InGaAsP semiconductor lasers differ in their upper and lower traces, the repetition angular frequencies are different. The FWHM pulse width of the generated pulse is about 0.5 ps. Induced modulational instability is one of the promising techniques to generate ultrahigh-bitrate short-pulse trains.

Figure 5.11: Auto-correlation traces of pulse trains generated by the induced modulational instability. Wavelengths of side-band InGaAsP semiconductor lasers differ in their upper and lower traces. (After Ref. [17]).

5.3.4 Dark solitons

A bright soliton cannot exist in the normal dispersion region in which $\beta^\prime > 0$. However, a dark soliton, which is an amplitude dip in the continuous wave, can exist in this normal dispersion region [18]. The solution of Eq. (5.36) under the condition of $\beta^\prime > 0$ is given by

$$\phi(z, \tau) = \phi_p \exp \left( -j \frac{\beta^\prime}{2\tau_0} z \right) \tanh \left( \frac{\tau}{\tau_0} \right), \quad (5.57)$$

where the definition of $\tau_0$ is the same as for Eq. (5.38). The condition for the peak electric field $\phi_p$ is given by

$$|\phi_p|^2 = \frac{2\beta^\prime}{\xi_0 kn_2}, \quad (5.58)$$

Figure 5.12 shows the pulse waveform and chirping property of a dark soliton. The phase of the dark soliton jumps at the minimum amplitude point. Experimental observation of the dark soliton is already reported [19].
5.4 Optical pulse compression

Ultrasound optical pulses are indispensable for research into ultrafast phenomena. An oscillating optical pulse width of 27 fs from a colliding pulse-mode-locked dye laser is reported by compensating the group velocity dispersion (GVD) in the laser cavity with prism chains [20]. However, from a practical standpoint, the technique of generating ultrashort light pulses from the laser itself is rather disadvantageous, since the wavelength region of laser oscillation is limited. The optical pulse compression technique [21] based on, for example, the combination of single-mode fiber (chirping medium) and grating pair (dispersive medium) is able to cover a wide spectral range, since pulse compression is done outside of the laser cavity.

An optical pulse of 50-fs width from the colliding pulse-mode-locked dye laser oscillating at 620 nm was compressed down to 6 fs by the pulse compression technique [22]. So far this is the shortest optical pulse. Figure 5.13 shows the schematics of optical pulse compression using fiber and grating pair. Transients of the optical pulse waveform propagating in single-mode fiber is described by Eq. (5.32). In the normal dispersion region, in which the longer-wavelength component at the preceding edge of the pulse travels faster than the shorter one in the trailing edge, the pulse waveform becomes square shaped and linearly chirped. This is most favorable for the compression of pulse using a grating pair. As is well known, optical pulse width Δt and spectral width Δω are related by the uncertainty principle of Δt·Δω ≥ 1. Therefore self-phase modulation based on the nonlinearity in optical fiber serves to broaden the spectral width Δω so as to generate a shorter pulse width Δt by pulse compression.

Figure 5.14 shows transients of pulse instantaneous angular frequency, optical pulse intensity, and spectral intensity during propagation in normal-dispersion fiber calculated by numerical simulation based on the nonlinear Schrödinger equation [23]. Parameters τ₀ and x₀ are given by Eqs. (5.38) and (5.49). A linearly chirped and square-shaped pulse, as shown in Fig. 5.14(c), is the best pulse to compress by using a grating pair [24]. The wavelength- (frequency-) dependent delay time of the grating pair, as shown in Fig. 5.15, is expressed by

\[ \tau = \tau_0 - \frac{\omega - \omega_0}{\mu} + O(\omega - \omega_0)^2, \]  

(5.59)

Figure 5.13: Schematic configuration of optical pulse compression using fiber and grating pair.

Figure 5.14: Numerical simulations of pulse instantaneous angular frequency (top row), optical pulse intensity (center row), and spectral intensity (bottom row) during propagation in normal-dispersion fiber. (After Ref. [23]).

where τ₀ and μ are given by

\[ \tau_0 = \left( \frac{b}{c} \right) (1 + \cos \theta), \]  

(5.60)

\[ \frac{1}{\mu} = \omega^2 d^2 \left[ 1 - \left( \frac{2\pi c}{\omega d} - \sin \gamma \right)^2 \right]. \]  

(5.61)

In these equations, c, d, and b denote light velocity, grating pitch, and distance between gratings A and B, respectively, in Fig. 5.15. γ is the incident angle of light.
5.5 Light scattering in isotropic media

Generally, when light in the visible region is injected into a transparent medium, some part of the light is scattered inelastically. When we observe the scattered light, there is Rayleigh-scattered light having the same wavelength as the input (excitation) light and also new light having a longer or shorter wavelength than that of the excitation light [26]. This effect is generally called the Raman effect. When scattered new light is generated through the interaction of excitation light with molecular vibration, optical phonons of the solid, and other elementary excitations, such as impurity, plasma, and polarization, the scattering is called Raman scattering. On the other hand, if scattered light is generated by the interaction of the excitation field with acoustical phonons of the liquid or solid, it is called Brillouin scattering.

Spontaneous Raman or Brillouin scattering is the phenomenon in which Stokes light with angular frequency \( \omega_S = \omega_L - \omega_P \) and anti-Stokes light with angular frequency \( \omega_A = \omega_L + \omega_P \) are emitted, where \( \omega_L \) and \( \omega_P \) denote angular frequencies of excitation light and optical or acoustical phonon, respectively. Stokes and anti-Stokes waves are generated through the modulation of electric polarization \( P \) by the polarizability \( \alpha \) as \( P = \alpha E \cos(\omega_L t) \), since polarizability \( \alpha \) varies with the specific angular frequency of the phonon as \( \alpha = \alpha_0 + (d\alpha/d\omega)k \cos(\omega_L t) \). When the excitation field is small, scattered light is incoherent, since each lattice vibration is random in phase. However, when the excitation field becomes strong, the lattice vibration is coherently excited at the angular frequency \( \omega_L - \omega_P \) by the nonlinear interaction between the incident light and the Stokes light. The coupling of the coherent lattice vibration with the input light field causes third-order nonlinear polarization and generates stimulated scattering of the Stokes light. This is the origin of stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS).

5.5.1 Vibration of a one-dimensional lattice

A phonon is a quantized state of the lattice vibration. Let us consider the vibration of a one-dimensional lattice consisting of two kinds of atoms so as to obtain the physical image of a phonon. We consider the linear lattice in which two different atoms having masses \( M \) and \( m \) are aligned alternately as shown in Fig. 5.16 and obtain the dispersion relation for the vibrational wave propagating along the linear direction. The displacements of the two atoms from the static positions are expressed as \( u_n \) and \( v_n \) for the \( n \)th atoms \( M \) and \( m \), where the positions of the two atoms are denoted by \( x_n = na \) and \( x_n = x_n + (1/2)a \) for the \( n \)th atoms \( M \) and \( m \). Here \( a \) is a periodicity of the atom (or unit cell). If we assume that each atom is forced only by the adjacent atoms and that it obeys Hooke's law, we obtain the following equation of motion for the pair of \( n \)th atoms:

\[
M \frac{d^2u_n}{dt^2} = -q(u_n - v_n) - q(u_n - v_{n-1}) = -q(2u_n - v_n - v_{n-1}), \tag{5.63a}
\]

\[
m \frac{d^2v_n}{dt^2} = -q(v_n - u_{n+1}) - q(v_n - u_n) = -q(2v_n - u_{n+1} - u_n), \tag{5.63b}
\]
where \( q \) denotes Hooke’s constant. Since we are interested in the traveling vibrational wave, we express \( u_n \) and \( v_n \) in the following forms:

\[
\begin{align*}
u_n &= A \exp(j(\omega t - kx_n)), \\
v_n &= B \exp(j(\omega t - k(x_n + \frac{1}{2}a))). \quad (5.64a, 5.64b)
\end{align*}
\]

Substitution of Eqs. (5.64) in Eqs. (5.63) gives

\[
\begin{align*}
(-M \omega^2 + 2q)u_n &= q(1 + e^{jk})u_n, \\
q(1 + e^{-jk})u_n &= (-\omega^2 + 2q)v_n. \quad (5.65a, 5.65b)
\end{align*}
\]

Solution of these simultaneous equations gives \( \omega^2 \) as

\[
\omega^2 = q \left( \frac{1}{M + m} \right) \pm \frac{1}{M + m} \sqrt{ \frac{1}{4} \left( \frac{1}{M + m} \right)^2 - \frac{4 \sin^2(ka/2)}{Mm}}. \quad (5.66)
\]

Among the four possible solutions of Eq. (5.66), two positive solutions \( \omega_+ \) and \( \omega_- \) give the traveling wave solutions. Dispersion relations of \( \omega_+ \) and \( \omega_- \) with respect to wavenumber \( k \) are shown in Fig. 5.17. We have

\[
\omega_+ = \sqrt{2q \left( \frac{1}{M + m} \right)}, \quad \omega_- = 0, \quad (5.67)
\]

for \( k = 0 \). For small \( k \), where we can approximate \( \sin(ka/2) \approx ka/2 \), we obtain

\[
\omega_+ = \omega_0 \left[ 1 - \frac{mM(ka)^2}{8(M + m)^2} \right], \quad (5.68a)
\]

\[
\omega_- = ka \sqrt{\frac{q}{2(M + m)}}, \quad (5.68b)
\]

where \( \omega_0 \) is defined by

\[
\omega_0 = \sqrt{2q \left( \frac{1}{M + m} \right)}. \quad (5.69)
\]

Let us next consider \( \omega_+ \) and \( \omega_- \) at \( k = \pi/a \), which corresponds to the edges of the first Brillouin zone \((-\pi/a \leq k \leq \pi/a)\). When we assume \( M > m \), we obtain

\[
\omega_+ = \frac{2q}{\sqrt{M}}, \quad \omega_- = \frac{2q}{\sqrt{M}}. \quad (5.70)
\]

The two vibrational states corresponding to the upper and lower curves of Fig. 5.17 are called optical and acoustical vibrations, respectively. The physical meanings of these terminologies will be described in the following.

The relative amplitude ratios of the vibrations between the adjacent light atom \( m \) and heavy atom \( M \) near \( k \neq 0 \) are given, from Eqs. (5.65) and (5.68), as

\[
\begin{align*}
\frac{v_n}{u_n} &\approx -\frac{M}{m} \quad \text{(optical vibration)} \quad (5.71a) \\
\frac{v_n}{u_n} &\approx 1 \quad \text{(acoustical vibration)} \quad (5.71b)
\end{align*}
\]

It is seen that in the optical vibrational mode atoms \( M \) and \( m \) move in opposite directions and their amplitudes of vibration are inversely proportional to their masses. For example, when two atoms are ions with positive and negative signs, such as \( \text{Na}^+ \) and \( \text{Cl}^- \), optical vibration corresponds to the vibration of electric...
polarization and strongly interacts with the light field. Therefore it is called optical vibrational mode. On the other hand, in the acoustical vibrational mode, the directions of vibration for the two atoms are the same and their amplitudes are also the same. Since such a vibration is observed when an acoustic wave propagates in the crystal, it is called acoustical vibrational mode.

Generally, there exist transverse and longitudinal modes for the waves. Then there are four possible vibrational modes for each wavenumber $k$, as shown in Fig. 5.18: transverse optic (TO), longitudinal optic (LO), transverse acoustic (TA), and longitudinal acoustic (LA) modes. Moreover, there are two orthogonal vibrations ($x$- and $y$-directions) for the transverse mode, and TA and TO vibrations are twofold degenerated.

### 5.5.2 Selection rules for light scattering by phonons

Figure 5.19 shows the dispersion relations of optical and acoustical phonons and a light (electromagnetic) wave $\omega = ck$. The intersection points between the phonons and light give the selection conditions for the interaction between phonon and light. Since the speed of light $c$ is very large compared to that of an optical phonon wave, the dispersion curve of light ($\omega = ck$) intersects with that of the optical branch close to $\omega_{\omega}$.

The energy of an optical phonon is about $h\omega_{\omega} \approx 10^{-2}$ eV, where $h = h/2\pi$ and Planck's constant $h = 6.626 \times 10^{-34}$ J·s. Then angular frequency $\omega_{\omega}$ is seen to be $\omega_{\omega} \approx 1.5 \times 10^{13}$ s$^{-1}$, which corresponds to the electromagnetic wave with a wavelength of 120 μm. The wavenumber $k$ at the intersection point between optical branch and light is given by $k = \omega_{\omega}/c = 1.5 \times 10^{13}/3 \times 10^{10} \approx 500$ cm$^{-1}$. This wavenumber is quite small when compared to the wavenumber $k_{\text{max}} = n/a \approx 10^8$ cm$^{-1}$ at the edge of the Brillouin zone [27], where the lattice constant is assumed to be $a = 3 \times 10^{-8}$ cm. Therefore, the

\begin{align*}
\omega_r &= \omega_p \pm \omega_r, \\
\tilde{k}_r &= k_p \pm k_r,
\end{align*}

Figure 5.19: Dispersion relations between optical and acoustical phonons and a light (electromagnetic) wave.
we obtain
\[
\frac{k_p c}{\omega_p} = n_p, \quad (5.74a)
\]
\[
\frac{k_s c}{\omega_s} = n_s, \quad (5.74b)
\]
where \(k_p = |k_p|\) and \(k_s = |k_s|\). Since the angular frequency of phonon \(\omega_s\) is quite small \([\omega_s \approx \omega_i (\approx 10^{15} \text{ s}^{-1}) \gg \omega_f (\approx 10^{13} \text{ s}^{-1})]\), we obtain, from Eqs. (5.74):
\[
k_s = \frac{n_s}{n_p} k_p. \quad (5.75)
\]
If the medium is isotropic and refractive-index dispersion is small \((n_s \approx n_p)\), we can assume \(k_s \approx k_p\). Figure 5.20(a) shows the relation of the wave vectors between pump and scattered light and phonon. When we consider the Stokes scattering \((k_s = k_p - k_f)\), the scattering diagram is represented by Fig. 5.20(b), where \(\theta\) denotes the scattering angle. The wavenumber of the phonon \(k_f = |k_f|\) is obtained, from Eqs. (5.72) and (5.74), as
\[
k_f^2 = k_p^2 + k_s^2 - 2 k_p k_s \cos \theta = \left( \frac{n_p}{c} \right)^2 \left[ (\omega_p - \omega_f)^2 + 4 \omega_p \omega_s \sin^2 \left( \frac{\theta}{2} \right) \right]
\]
\[
= \left( \frac{n_p}{c} \right)^2 \left[ \omega_f^2 + 4 \omega_p (\omega_p - \omega_f) \sin^2 \left( \frac{\theta}{2} \right) \right], \quad (5.76)
\]
where we assumed \(n_s \approx n_p\). The last equation gives the relation between the angle \(\theta\) of the scattered light and the amplitude of the phonon wave vector \(k_f\) contributing the scattering. When we assume \(n_s \approx n_p\) and \(\omega_p \gg \omega_f\) in Eq. (5.76)

---

Figure 5.20: Relation of the wave vectors between pump and scattered light and phonon.

---

we obtain the following relationships for \(\theta\) and \(k_f\):
\[
\begin{align*}
\theta = 0^\circ \text{ (forward scattering)} \quad k_f &= \frac{n_p \omega_f}{c} \\
\theta = 90^\circ \text{ (right-angle scattering)} \quad k_f &= \sqrt{2} \frac{n_p \omega_f}{c} = \sqrt{2} k_p \\
\theta = 180^\circ \text{ (backward scattering)} \quad k_f &= 2 \frac{n_p \omega_f}{c} = 2 k_p
\end{align*} \quad (5.77)
\]
The wavenumber of the pump light is of the order of \(k_p = 4 \times 10^4 - 10^5 \text{ cm}^{-1}\). This is quite small when compared with the wavenumber at the edge of the first Brillouin zone \(k_{\text{max}} = 10^8 \text{ cm}^{-1}\) by about \(10^{-4} - 10^{-3}\). Therefore, \(k_f\)'s in Eqs. (5.77) lie in the vicinity of the center of the Brillouin zone and we have, irrespective of the scattering angle \(\theta\), the following relation:
\[
k_f \approx \frac{\omega_f}{c}
\]
Especially, \(k_f\) for the forward scattering is extremely small, since \(\omega_f (\approx 10^{13} \text{ s}^{-1}) \gg \omega_p (\approx 10^{15} \text{ s}^{-1})\).

5.6 Stimulated Raman scattering

Raman scattering is a scattering caused by the interaction between pump light and optical phonon. The first peak of Stokes light in silica fiber is located at the wavelength 440 cm\(^{-1}\) shifted from the pump wavelength, as shown in Fig. 5.21. The Raman gain coefficient of the first Stokes light when pumped at \(\lambda = 1 \mu m\) is \(g_r = 1 \times 10^{-11} \text{ cm/W} \text{ and is inversely proportional to the wavelength [11].}

In stimulated Raman scattering, forward and backward scattering are observed almost equally. This is why the dispersion relation of the optical phonon in the vicinity of the Brillouin zone center is
\[
\omega_s (k_f) \approx \text{constant}
\]
as shown in Eq. (5.68a) and Fig. 5.17. In other words, although the phonon wavenumber \(k_f\) contributing to the forward or backward scattering is different in the momentum conservation rule [Eq. (5.73)], the energy conservation rule is satisfied for any scattering angle, since \(\omega_s (k_f)\) is almost constant.

The progress of pump light intensity \(I_p\) and Stokes light intensity \(I_s\) in stimulated Raman scattering is described by the following coupled equation:
\[
\frac{dI_p}{dz} = g_r I_p I_s - \alpha_p I_p, \quad (5.79a)
\]
\[
\frac{dI_s}{dz} = -\alpha_p I_s - \frac{\omega_f}{\omega_p} g_r I_p I_s - \alpha_s I_s, \quad (5.79b)
\]
where \(\alpha_p\) and \(\alpha_s\) denote fiber losses at the wavelengths of pump and Stokes light, respectively. When we ignore the pump depletion and assume \(I_p = I_0 \exp (-\alpha_p z)\)
strictly speaking we should take into account the wavelength difference between pump and Stokes light. Then strict expression of the effective area is given by
\[ A_{\text{eff}} = \frac{\int_0^L R_p^2 r \, dr \, db \, \int_0^L R_s^2 r \, dr \, db}{\int_0^L [R_p^2 r] \, dr \, db} \]

where \( R_p(r) \) and \( R_s(r) \) represent electric field distributions of pump and Stokes light, respectively. The critical power for the single-mode fiber with \( A_{\text{eff}} = 50 \, \mu \text{m}^2 \), \( \alpha_p = 0.2 \, \text{dB/km} \) \((4.6 \times 10^{-2} \, \text{m}^{-1})\), \( L \gg 1/\alpha_p \) \((\approx 22 \, \text{km})\), and \( g_R = 0.6 \times 10^{-11} \, \text{cm/W} \) \((\lambda = 1.55 \, \mu \text{m})\) is obtained as \( P_c = 600 \, \text{mW} \).

Next we obtain the gain coefficient by SRS when the Stokes seed \( I_s(0) \) (signal to be amplified by SRS) is injected in Eq. (5.81). When we assume \( \alpha_p = \alpha_s = 0 \) for simplicity, the gain coefficient is obtained as

\[ G = \exp(g_R I_o L) \]  

(5.85)

If the pump power \( P_o = I_o A_{\text{eff}} = 4 \, \text{W} \) in the fiber having \( g_R = 0.6 \times 10^{-11} \, \text{cm/W} \) and \( A_{\text{eff}} = 50 \, \mu \text{m}^2 \), then a 21-dB gain is obtained at the fiber length of 1 km. Figure 5.22 shows the bit error rate (BER) characteristics of the fiber Raman

Figure 5.22: Bit error rate characteristics of the optical amplification experiment using SRS. Filled circles represent no pump light, and open circles and open rectangles represent 3.9 dB and 5.8 dB gain, respectively. (After Ref. [29]).
amplification experiment (signal and pump propagate in the opposite direction) when a 1.57-μm-wavelength DFB laser is used as the signal and a 1.47-μm-wavelength F-center laser is used as pump [29]. Signal bit rate and fiber length are 1 Gbit/s and 45 km, respectively. The gain coefficient at 100 mW of pump power is about 3.5 dB, and the maximum gain is 5.8 dB. The dashed lines in the figure represent the theoretical values for the BERs. Any difference from the theoretical value is attributed to the fluctuation of the pump light and to its back scattering.

5.7 Stimulated Brillouin scattering

Stimulated Brillouin scattering is a scattering caused by the interaction between pump light and acoustical phonon. Since the angular frequency of an acoustical phonon satisfies the linear dispersion relation with the wavenumber in the center of the Brillouin zone [Eq. (5.86b)], only the backward scattering occurs strongly.

Stimulated Brillouin scattering can be described by classical mechanics as the parametric interaction between the pump, Stokes, and acoustical waves [30]. First, the pump light $E_p \exp (i \omega_p t + k_p \cdot r)$ is spontaneously scattered by the thermally excited acoustical phonon in the medium and generates the scattered light $E_s \exp (i \omega_s t + k_s \cdot r)$. Then the electric fields of the pump and scattered light create the density fluctuations through the electrostriction. The density fluctuations is a traveling wave permittivity grating, which is expressed by $\exp \pm j(Q \xi - q \cdot r)$. Here, $Q$ and $q$ are given by

$$Q = \omega_p - \omega_s, \quad \text{(5.86a)}$$

$$q = k_p - k_s. \quad \text{(5.86b)}$$

This permittivity grating scatters the pump light and generates the Stokes light, which results in the stimulated Brillouin scattering. The variance of the relative permittivity $\delta e_s$ for the Stokes light that is caused by the permittivity grating is given by

$$\delta e_s = \rho_0 \left( \frac{\delta e}{\delta \rho} \right)^2 \frac{|E_p|^2}{10\pi} \frac{q^2}{(\Omega^2 - \Omega^2 - 2\Omega \Gamma)\rho^2}. \quad \text{(5.87)}$$

Here, $\rho_0$ is the density of the medium, $\delta e/\delta \rho$ is the rate of change in the relative permittivity with respect to the unit density change, and $\Gamma$ is the FWHM spectral width of the permittivity given by $\Gamma = 1/\tau_d$, where $\tau_d$ is the decay relaxation time of the acoustical wave. $\Omega_b$ is the angular frequency of the acoustical phonon, which is given by

$$\Omega_b = qv_A = qx_{ac}. \quad \text{(5.88)}$$

where $v_A$ is the velocity of a longitudinal acoustic wave. In silica glass, we have $\rho_0 = 2.2 \text{ g/cm}^3$, $\tau_d = 20 \text{ ns}$ at $\lambda = 1.55 \mu$m, and $v_A = 5940 \text{ m/s}$ [31, 32]. From Eq. (5.87), the refractive index for the Stokes light $n_s$ is obtained as

$$n_s = (n_0 + \delta e_s)^{1/2} = n_0 + \frac{\delta e_s}{2n_0}. \quad \text{(5.89)}$$

where $n_0$ denotes the original refractive index. When we ignore the attenuation of the Stokes wave, its amplitude variation is expressed by

$$E_s(L) = E_s(0) \exp \left( -j k_s n_s L \right) = E_s(0) \exp \left( -j k_s n_0 L - \frac{j k_s \delta e_s L}{2n_0} \right). \quad \text{(5.90)}$$

It is seen from this last equation that the imaginary part of $\delta e_s$ gives the gain of the Stokes light. Equation (5.90) is rewritten into the intensity form as

$$I_s(L) = I_s(0) \exp \left( \frac{k_s}{n_0} \int \delta e_s r \right) \quad \text{(5.91)}$$

We should note here that the relation between optical intensity (=power/effective area) and electric field amplitude is expressed in MKS units as

$$I_s = \frac{n_s \varepsilon_0 c |E_s|^2}{c^2} \quad \text{[W/m}^2]. \quad \text{(5.92a)}$$

and in cgs units is expressed as

$$I_s = \frac{n_s c}{8\pi} |E_s|^2 \quad \text{[erg/s}^{-1}/\text{cm}^2]. \quad \text{(5.92b)}$$

When we consider the Stokes light near the resonance angular frequency $\Omega \approx \Omega_b (= \omega_b)$, Eq. (5.87) reduces to

$$\int \delta e_s = \frac{n_s \varepsilon_0 c}{4\pi} \frac{q^2}{(\Omega - \Omega_b)^2 + \Gamma^2} I_0. \quad \text{(5.93)}$$

where $I_0 = (n_s c/8\pi)|E_s|^2$ in cgs unit and the following relation has been used:

$$\frac{\delta e}{\delta \rho} = n_0 \rho_{12} \quad \text{(5.94)}$$

The photoelastic coefficient $p_{12}$ for silica glass is $p_{12} = 0.27$. The gain coefficient of SBS is then given, from Eqs. (5.91) and (5.93), as

$$g_s = \frac{n_0 \rho_{12}}{2c\rho_0 v_A} \frac{q^2}{(\Delta v/2)^2} \left[ (\nu - \nu_0)^2 + (\Delta v/2)^2 \right]. \quad \text{(5.95)}$$
Here \( v \) is the frequency of Stokes light \( (v = \Omega/2\pi) \), \( v_B \) is the frequency shift of the Stokes light, and \( \Delta v_B \) is the FWHM width of the Brillouin gain, which is obtained by
\[
\Delta v_B = \frac{\Delta v_p}{\pi} \approx \frac{1}{\pi \tau_2}.
\]
Since \( q = |k_p - k_s| \ll 1 \) for forward scattering [Eq. (5.77)], the Brillouin gain is almost zero for forward Brillouin scattering [33]. In contrast, \( r \) becomes the maximum \( q = |k_p - k_s| \approx 2|k_s| \approx 4m_0c/\lambda \) for backward scattering. Then the Brillouin gain also becomes the maximum
\[
g_{B0} = \frac{2m_0^2 \beta \tau_2^2}{c^2 \Delta v_B \Delta v_p}. \tag{5.96}
\]
for backward scattering. Since the FWHM width of the Brillouin gain depends on the wavelength \( (\Delta v_B \propto 1/\lambda^2) \), the Brillouin gain itself is almost wavelength independent. The frequency shift \( v_B \) of the Stokes light becomes the maximum for backward scattering and is given by
\[
v_B = \frac{\Omega_v}{2\pi} = \frac{v_{B0} \Delta v_p}{2 \pi} = \frac{2m_0 v_s}{\lambda}. \tag{5.97}
\]
The frequency shift \( v_B \) of the Stokes light at \( \lambda = 1.55 \) \( \mu \)m is \( v_B = 11.1 \) GHz. Since the relaxation time \( \tau_2 \) of the acoustical phonon at \( \lambda = 1.55 \) \( \mu \)m is about \( \tau_2 \approx 20 \) ns, the FWHM spectral width of the Brillouin gain is given by \( \Delta v_B = 1/\pi \tau_2 \approx 16 \) MHz. At the \( \lambda = 1.3 \) \( \mu \)m wavelength region, \( v_B = 13.3 \) GHz and \( \Delta v_B \approx 23 \) MHz.

The Brillouin gain \( g_{B0} \) at the \( \lambda = 1.55 \) \( \mu \)m wavelength region is obtained, from Eq. (5.96), as \( g_{B0} = 4.1 \times 10^{-9} \) cm/W, where we assumed \( n_0 = 1.45 \). The gain of the stimulated Brillouin scattering is about 400 times larger than that of the SRS \( (g_R = 0.6 \times 10^{-11} \) cm/W). The Brillouin gain spectrum has a Lorentz shape, as shown in Eq. (5.95). When the spectrum of the pump light also has a Lorentz shape with an FWHM of \( \Delta v_p \), the Brillouin gain coefficient is given by the convolution integral of the two spectral distributions [34]:
\[
g_B = \frac{\Delta v_B}{\Delta v_B + \Delta v_p} g_{B0}. \tag{5.98}
\]

When the pump light has a rather narrow spectral width compared to the Brillouin gain width \( (\Delta v_p \ll \Delta v_B) \), the SBS gain becomes \( g_B \approx g_{B0} \). However, if the pump light has a wide spectral width compared to the Brillouin gain width \( (\Delta v_p \gg \Delta v_B) \), the SBS gain becomes small and inversely proportional to \( \Delta v_p \) as \( g_B \propto (\Delta v_B/\Delta v_p) g_{B0} \). Therefore it is seen that a pump light source with a narrow spectral width is necessary to generate SBS efficiently.

Variation of Stokes light in the SBS process is obtained in a manner similar to that in SRS. When we consider that Stokes light travels in a direction opposite to that of the pump light, we obtain
\[
I_s(L) = I_s(0) \exp(g_{B0} L - \alpha L), \tag{5.99}
\]
where we assumed \( \alpha_p \approx \alpha_s \approx \alpha \). Stimulated Brillouin scattering builds up from the

thermally excited spontaneous Brillouin scattering. The critical input power \( P_c \) at which Stokes light power becomes equal to the pump power is obtained by [28]
\[
P_c = \frac{21 A_{eff}}{g_{B0} L_{eff}} = \frac{\Delta v_B + \Delta v_p}{\Delta v_p} 21 A_{eff} \frac{g_{B0}}{g_{B0} L_{eff}}. \tag{5.100}
\]

In single-mode optical fiber with \( A_{eff} = 50 \) \( \mu \)m \( ^2 \), \( \alpha = 0.2 \text{ dB/km} \ (4.6 \times 10^{-5} \text{ m}^{-1}) \) at 1.55-\( \mu \)m wavelength, \( L = 1/\alpha (\approx 22 \) km), and \( g_{B0} = 4.1 \times 10^{-9} \) cm/W, the critical power is \( P_c = 1.2 \) mW for a narrow line width \( (\Delta v_p = \Delta v_B) \) and is \( P_c = (\Delta v_B/\Delta v_p) \times 1.2 \) mW for a wider pump line width.

A single-longitudinal-mode Ar laser and YAG laser with narrow spectral width have been utilized for the generation of SBS [31]. However, after the development of narrow spectral line width (less than 10 MHz) and high-power single-mode semiconductor lasers, experimental observation of SBS [35] and signal amplification by SBS [36] using the semiconductor laser as pump light have been reported. Figure 5.23 shows the measurement of the SBS threshold power using a 1.3-\( \mu \)m-wavelength DFB laser having spectral width of 15 MHz.
The attenuation and the length of the single-mode fiber are 0.46 dB/km at 1.3 µm and 30 km, respectively. It is known that the threshold input power for SBS is about 10 mW. This means that light with more than about 10 mW of power cannot be transmitted in single-mode fiber, since it is reflected back even if we increase the input power level.

The gain bandwidth of SBS can be enlarged up to about several hundred megahertz by connecting different kinds of fibers, since the Brillouin shift frequency of doped-silica glass differs slightly depending on the dopant material. Figure 5.24 shows the result of a signal amplification experiment by SBS using single-mode fibers with a broadened gain spectrum [36]. The signal and pump lasers are semiconductor lasers with external cavity oscillating at 1.5 µm (spectral width of 15 kHz and tuning range of 1.42–1.52 µm). The average loss of fibers is 0.27 dB/km, and the total fiber length is 37.5 km. The amplification gain with respect to 1 mW of pump power is 4.3 dB, which is far more efficient than that of stimulated Raman scattering. However, since the gain bandwidth of SBS is at most several hundred megahertz, it cannot amplify a broadband (high-bit rate) signal or ultrashort optical pulses.

### Figure 5.24
![Optical amplification by SBS using fibers with a broadened gain spectrum. (After Ref. [36]).](image)

5.8 Second-harmonic generation

Generally, second-harmonic generation (SHG) is believed to be impossible in center-symmetric materials such as silica glass. However, SHG with relatively high efficiency in single-mode optical fibers has been reported. Figure 5.25 shows the growth of SHG power when mode-locked and Q-switched Nd:YAG laser pulses at 1.06-µm wavelength (pulse width of 100–130 ps and average power of 125 mW) are propagated through a 1-m-long single-mode fiber [37]. The second-harmonic power grows almost exponentially with time, and the maximum conversion efficiency of 3% was obtained.

The physical mechanisms to explain second-harmonic generation are not fully understood. However, it is recognized that the origin of high-efficiency SHG may be due to the formation of periodic color centers or defects [38]. First, small higher-order nonlinearities exist to generate weak second harmonics even in optical fibers; they are: nonlinearities at the core-cladding interface and nonlinearities resulting from quadrupole and magnetic-dipole moments [39]. When the pump wavelength is \( \lambda_p = 1.06 \mu m \), the period of the beat component generated by the pump light and the SHG light (\( \lambda_s = 0.53 \mu m \)) is about \( \lambda_b = 2\pi/\Delta k \approx 30–40 \mu m \), by taking into account the dispersion effect [40]. Here \( \Delta k = k_s - 2k_p \), where \( k_s \) and \( k_p \) denote the wavenumbers of pump and SHG light, respectively. In normal SHG, the variation of the SHG power is given by [2]

\[
P_s(z) \propto \frac{\sin^2(\Delta k z/2)}{(\Delta k)^2}.
\]  

### Figure 5.25
![Second-harmonic power generated in a silica fiber as a function of time. (After Ref. [37]).](image)
Figure 5.26: Schematics to explain SHG in optical fiber. (a) Beat wave created by pump and SHG light, (b) $\chi^{(2)}$ grating caused by photoinduced defects, and (c) efficient SHG at positions where pump and SHG light get in phase.

When the phase-matching condition ($\Delta k = 0$) is satisfied, the SHG power increases with increase in propagation distance $z$. However, the SHG power does not grow when the phase-matching condition is not satisfied ($\Delta k \neq 0$).

In contrast to normal SHG, the phase-matching condition is automatically satisfied when photoinduced defects are generated periodically in optical fiber. As shown in Fig. 5.26(a), the pump and SHG light add up or cancel each other at the positions where they are in phase or out of phase, respectively. Then photoinduced defects and also $\chi^{(2)}$ are created at positions where the beat intensity is strong. The resulting distribution of $\chi^{(2)}$ in the fiber becomes a grating with a period of $\Lambda$. Since second harmonics are generated only where the pump and SHG light are in phase, the SHG light grows [Fig. 5.26(c)] in proportion to the propagation distance. The back-coupling from the SHG light to the pump light, which is observed in normal SHG, is prohibited, since there is no $\chi^{(2)}$ where SHG and pump light are out of phase (negligibly small $\chi^{(2)}$ exists caused by higher-order nonlinearity such as quadrupole moment). SHG in single-mode fibers is quite interesting for the nonlinear optical effects, since the phase-matching condition is automatically satisfied and the transverse modes are also well preserved throughout the interaction.

5.9 Erbium-doped fiber amplifier

The erbium-doped fiber amplifier (EDFA) had a great impact on optical fiber communication systems \[10,41\]. EDFA has a broad gain bandwidth around the 1.55-µm-wavelength region that coincides with the low-loss transmission wavelength of silica glass fibers.

Erbium ions have the energy level diagram shown in Fig. 5.27. Three states, i.e., the ground state ($^{4}I_{15/2}$), the first excited state ($^{4}I_{13/2}$), and one of the other higher excited states, participate and act as a three-level laser in the amplification of 1.55-µm-wavelength light, except for the 1.48-µm pumping scheme. In the case of 1.48-µm pumping, the upper portion of the $^{4}I_{15/2}$ state, which has a certain bandwidth, as described later, is used as a pumping level, and the Er ions are operated as a quasi-three-level laser. The energy difference between the $^{4}I_{15/2}$ state and the $^{4}I_{13/2}$ state corresponds to the 1.55-µm-wavelength band. When an incident signal photon comes into an Er ion in the $^{4}I_{13/2}$ excited state, the Er ion emits a photon with a certain probability and returns to the ground state through the stimulated emission process. The photon generated by the stimulated emission has exactly the same frequency and phase as the incident photon, thus enabling signal amplification. Even if the incident photon does not exist, the $^{4}I_{13/2}$ level decays to the ground state, accompanied by a spontaneous emission. Since spontaneous emission light has no correlation with the incident light in terms of frequency and phase, the amplified spontaneous emission (ASE) light becomes a noise source of optical fiber amplifiers. EDFA basically consists of an Er-doped fiber, a pumping light source, an optical coupler that combines signal and pump light, and optical isolators to reject the unwanted back-scattered light, as shown in Fig. 5.28. A fused-taper fiber WDM coupler or a dichroic mirror with dielectric multilayer is used for multiplexing pump and signal lights. The pump propagation direction is not necessarily the same as the direction of the signal. A better noise figure (NF) can be obtained by the copropagating pump scheme as compared to that of counter-propagating pump. However, the counterpropagating scheme is suitable as a booster application, since a higher conversion efficiency is obtained with this scheme. A bandpass filter is added in the output path to eliminate the ASE light.

In order to realize high gain and a low pumping threshold in the EDFA, it is essential to reduce the mode field diameter of the Er-doped fiber at the pump wavelength. Er-doped fiber generally has a high refractive-index difference and a small core to reduce the mode field diameter. A high doping concentration of Er

![Figure 5.27: Energy level diagram of the triply ionized erbium ion.](image)
ion is desirable to shorten the fiber length and to reduce the amplifier size. However, there is a limitation on the available maximum doping concentration due to the cooperative up-conversion process. The maximum Er concentration may be less than 100 wt.ppm for the Ge/Er doped fibers, while a concentration of up to 1000 wt.ppm is allowed for the Ge/Er/Al-doped fibers. Aluminum codoping is effective for increasing the allowable Er concentration. The aluminum codoping also offers an advantage in broadening and smoothing the signal gain spectrum.

Figure 5.29 shows a comparison between the gain spectra of Er-doped fibers with and without aluminum codoping. The gain spectrum of Er-doped fiber with 30,000 wt.ppm of Al is considerably flat and broad compared with that of fiber without Al.

The EDFA amplifies the optical signal directly, and thus the transmission rate of the system can be changed even after construction of the system. The EDFA is rapidly penetrating long-haul terrestrial and submarine transmission systems.

References

Chapter 5


