Solutions to Homework Chap 6 for Introduction to Optoelectronics

6.2
The EHP photogeneration rate from the illuminated crystal surface follows

\[ G_{\rho} = G_e \exp(-\alpha x) \]

Total number of EHP generated per unit time in a small volume \( d\chi \) is \( G_{\rho} d\chi \). Thus,

Total number EHP generated per unit time in \( l_{\alpha} + W + L_e = A \int_{l_{\alpha}} G_e \exp(-\alpha x) dx \)

or

\[ \frac{dN_{\text{ext}}}{dt} = \frac{G_e A}{\alpha} \left[ 1 - \exp(-\alpha (l_{\alpha} + W + L_e)) \right] \]

Since the photogenerated electrons flow through the external circuit, the photocurrent \( I_{\rho} \) is then

\[ I_{\rho} = \frac{eG_e A}{\alpha} \left[ 1 - \exp(-\alpha (l_{\alpha} + W + L_e)) \right] \]

For long wavelengths, \( \alpha \) will be small. Expanding the exponential we find,

\[ I_{\rho} = eG_e A (l_{\alpha} + W + L_e) \]

which applies under nearly uniform photogeneration conditions.

If \( d\rho \) is the change in the photon flux over an interval \( d\chi \), then

Rate of EHP generation in \( A d\chi = A d\rho = A G_e \alpha d\chi \)

\[ G_{\rho}(x) = \text{Photogeneration rate at } x = \frac{A G_e \alpha}{A d\chi} = \alpha d\rho(x) \]

Now the intensity is defined by

\[ I = \text{Intensity} = \text{Photon flux \times Energy per photon} = \Gamma_{\rho} h \gamma \]

Thus,

\[ G_{\rho}(x) = \frac{I(x)}{h \gamma} \quad \text{and} \quad G_{\rho}(0) = \frac{I(0)}{h \gamma} \]

or

\[ G_s = \frac{I}{h \gamma} \]

6.3

The solar cell is used under an illumination of 1 kW m\(^{-2}\). The short circuit current has to be scaled up by 1000/600 = 1.67. Figure 6Q3-2 shows the solar cell characteristics scaled by a factor 1.67 along the current axis. The load line for \( R = 20 \Omega \) and its intersection with the solar cell \( I-V \) characteristics is at \( P \) which is the operating point. Thus,

\[ I = 22.5 \text{ mA and } V = 0.45 \text{ V} \]

The power delivered to the load is

\[ P_{\text{out}} = IV = (22.5 \times 10^{-3})(0.45) = 0.101 \text{ W}, \text{ or } 10.1 \text{ mW}. \]

This is not the maximum power available from the solar cell. The input sun-light power is

\[ P_s = (\text{Light Intensity})(\text{Surface Area}) \]

\[ = (1000 \text{ W m}^{-2})(4 \text{ cm}^2 \times 10^4 \text{ m}^2/\text{cm}^2) = 0.4 \text{ W} \]

The efficiency is

\[ \eta = 100 \frac{P_{\text{out}}}{P_s} = 100 \frac{0.101}{0.4} = 2.5\% \]

which is poor.
The open circuit voltage depends on the temperature whereas $I_{sc}$ has very little temperature dependence. Use

$$V_{oc} = V_{oc} \left(1 - \frac{T}{T_0}\right)$$

(1)

to calculate the $V_{oc}$ at different temperature given $V_{oc}$ at one temperature. Then calculate $V_{oc}$ using

$$V_{oc} = V_{oc} \left(1 + \frac{0.72}{V_{oc} + 2}\right)$$

(2)

then FF using

$$FF = \frac{V_{oc} - \ln(V_{oc} + 0.72)}{V_{oc} + 2}$$

(3)

and then $P$ using

$$P = FF \times V_{oc}$$

(4)

as summarized in Table 6Q9-1.

<table>
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<tr>
<th>$n$</th>
<th>$I_{sc}$ mA/cm²</th>
<th>$V_{oc}$; Eq. (1)</th>
<th>$V_{max}$; Eq. (2)</th>
<th>FF; Eq. (3)</th>
<th>$P$ mW/cm²; Eq. (4)</th>
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<td>0.580 V</td>
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<td>0.545 V</td>
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Conclusions: $n = 2$ case has a lower FF and also lower power delivery.

NOTE: The temperature dependence of the open circuit voltage $V_{oc}$ was derived in the text as

$$V_{oc} = V_{oc} \left(1 - \frac{T}{T_0}\right)$$

This expression is valid whether $n$ is 1 or 2. Recall that $n = 1$ represents diffusion in the neutral regions and $n = 2$ is recombination in the space charge layer. In the $n = 1$ case $I_{sc} \propto n_i$ and in the $n = 2$ case $I_{sc} \propto n_i^2$, thus in general $I_{sc} \propto n_i^{2n}$.

Consider the open circuit voltage,

$$V_{oc} = \frac{n_i k T}{e} \ln \left(\frac{K T}{I_{sc}}\right)$$

or

$$\frac{e V_{oc}}{n_i k T} = \ln \left(\frac{K T}{I_{sc}}\right)$$

At two different temperatures $T_1$ and $T_2$, but at the same illumination level, by subtraction,

$$\frac{e V_{oc1}}{n_i k T_1} - \frac{e V_{oc1}}{n_i k T_2} = \ln \left(\frac{I_{sc2}}{I_{sc1}}\right) = \ln \left(\frac{n_i^{2n_2}}{n_i^{2n_1}}\right)$$
where the subscripts 1 and 2 refer to the temperatures $T_1$ or $T_2$ respectively.

We can substitute $n_{23} = (N_2/N_1)^{x_{23}} \exp(-E_g/kT_2)$ and neglect the temperature dependences of $N_2$ and $N_1$ compared with the exponential part to obtain,

$$\frac{aV_{oc1}}{nkT_1} - \frac{aV_{oc2}}{nkT_2} = \frac{E_g}{nkT_2} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

Rearranging for $V_{oc2}$ in terms of other parameters we find,

$$V_{oc2} = V_{oc1} \left( \frac{T_1}{T_2} \right) + \frac{E_g}{nk} \left( \frac{1}{T_1} \right)$$

6.10

a. The definition of $\alpha$ is shown in Figure 6Q10-1(a) and (b). We can plot the light intensity vs. $\alpha$ as shown in Figure 6Q10-2 which shows the maximum is at $\alpha = 90^\circ$ as expected. Notice how broad the curve is, implying that there is still substantial intensity when $\alpha$ is a low angle.

The light intensity is maximum when $\alpha = 90^\circ$, thus

$$I_{max} = 1.353(0.7)^{a_{\max}} = 0.95 \text{ kW m}^2$$

and

$$\text{Maximum Power per unit area = (Efficiency)(Maximum Intensity) = 0.095 kW m}^2 = 95 \text{ W m}^2$$

so that at the best spot on Earth, the best one can do with this solar cell is 95 W m$^2$.

b. Under test conditions, we have $V_{oc} = 0.45 \text{ V}$, $I_{oc} = I_{sc} = 400 \text{ mA}$ at 1 kW m$^2$, at 27 $^\circ$C, that is $V_{oc} - 0.0259 \text{ V}$

$$V_{oc} = nV_T \ln \left( \frac{I_{sc}}{I_T} \right) \quad \therefore \quad 0.45 = 1(0.0259) \ln \left( \frac{400 \times 10^{-3}}{I_T} \right)$$

giving

$$I_T = 1.1 \times 10^{-8} \text{ A}$$

At Eskimo Point, $\alpha = 90^\circ - 63^\circ - 27^\circ$ (see Figure 6Q10-1) which gives a light intensity of

$$I_{sc} = 1.353(0.7)^{a_{max}} = 1.353(0.7)^{90^\circ - 63^\circ - 27^\circ} = 0.736 \text{ kW m}^2$$

which means that the Eskimo Point photocurrent $I_{sc}$ is determined by

$$I_{sc} = \frac{I_{sc} \text{ (Eskimo Point)}}{1 \text{ kW m}^2}$$

$$\therefore \quad I_{sc} = 294 \text{ mA}$$

The dark current depends on the temperature. At $-10^\circ$C, $V_T = k_T/\alpha = 0.0227 \text{ V}$.

The change in the dark current $I_T$ can be found as follows. Let $V_T$ = bandgap voltage $= E_g/\alpha = 1.1 \text{ V}$ for Si. Then, $n = 1$ means that

$$I_T \propto n^2 \propto \exp \left( -\frac{V_T}{V_T} \right)$$

so that

$$\frac{I_T \text{ (Eskimo Point)}}{I_T \text{ (Test)}} = \exp \left[ \frac{V_T}{V_T \text{ (Eskimo Point)}} - \frac{V_T}{V_T \text{ (Test)}} \right]$$

$$\therefore \quad \frac{I_T \text{ (Eskimo Point)}}{1.1 \times 10^{-8} \text{ A}} = \exp \left[ -\frac{1.1}{0.0227} + \frac{1.1}{0.0259} \right]$$

solving gives

$$I_T = 2.77 \times 10^{-15} \text{ A}$$

so that the Eskimo Point OC voltage is

$$V_{oc} = nV_T \ln \left( \frac{I_{sc}}{I_T} \right) = 1(0.0227) \ln \left( \frac{294 \times 10^{-3}}{2.77 \times 10^{-15}} \right) = 0.523 \text{ V}$$

Maximum power is

$$P_{max} = FFV_{oc} = (0.73)(244 \text{ mA})(0.523 \text{ V}) = 93 \text{ mW}$$