Solutions to Homework Chap 3 for Introduction to Optoelectronics

3.1

(a) The energy distribution of the electrons in the conduction band \( n_g(E) \) is given by the product of the density of state \( g(E) \), function and the probability of occupancy of a given state with energy \( E, n(E) \). Thus,

\[ n_g(E) = A(E - E_g)^{3/2} \exp \left( \frac{E - E_g}{k_B T} \right), \]

where \( A \) is a constant. Introducing the new variable \( x = \frac{E - E_g}{k_B T} \) we obtain

\[ n_g(x) = A(k_B T)^{3/2} x^{3/2} \exp(-x) \exp \left( \frac{E_g - E}{k_B T} \right) \]

(c) Possible Answers

(a) It is essential to control of refractive index profile, core radius, and minimize variations in the refractive index due to variations in doping.

(b) Minimize impurities. Reduce scattering by reducing density and hence refractive index fluctuations (may not be possible). Use a glass material with a lower glass transition temperature so that the frozen fluctuations are smaller.

\[ y = n_g(x) = C x^{3/2} \exp(-x), \]

where \( C = A(k_B T)^{3/2} \exp \left( \frac{E_g - E}{k_B T} \right) \) is independent of \( E \) constant.

(b) The plot of the energy distribution \( y \) vs. \( x \) of the electrons for \( C = 1 \) is presented below.

\[ n_g(x) \text{ vs. } x. \text{ Notice that the maximum energy concentration of electrons is at about } x = 1/2. \]

The maximum of the distribution \( x_m \) can be found by setting the first derivative equal to zero. Thus

\[ \frac{dn_g(x)}{dx} = \frac{1}{2} C x^{-1/2} \exp(-x) - C x^{1/2} \exp(-x) = 0 \]

and

\[ x_m = \frac{1}{2} \quad \text{and} \quad n_{max} = 0.42888 \]

The FWHM is simply the distance between the two roots of the following equation

\[ x^{1/2} \exp(-x) = \frac{n_{max}}{2} \]

The two roots are \( x_1 = 0.0509 \) and \( x_2 = 1.846 \) and \( x_2 - x_1 = 1.795 \).

(c) The average electron energy is

\[ E_n = \frac{\int_{x_1}^{x_2} E n_g(E) dE}{\int_{x_1}^{x_2} n_g(E) dE} = \frac{\int_{x_1}^{x_2} E \exp(-x) \exp \left( \frac{E_g - E}{k_B T} \right) dx}{\int_{x_1}^{x_2} \exp(-x) \exp \left( \frac{E_g - E}{k_B T} \right) dx} = \frac{1}{2} k_B T \frac{\Gamma(5/2)}{\Gamma(3/2)} \]

We can obtain the exact value for \( E_n \) using the definition and one of the basic properties of the Euler gamma function: \( \Gamma(z+1) = z \Gamma(z) \). Since \( n_g(x) \) very rapidly approaches zero as \( x \) increases we can take the upper limits of the integration to be infinity and then

\[ E_n = k_B T \frac{\Gamma(5/2)}{\Gamma(3/2)} \]

(d) The thermal velocity of the electrons can be calculated from the relation

\[ \frac{1}{2} m_e v_e^2 = \frac{3}{2} k_B T \]

and the result is

\[ v_e = \sqrt{\frac{3 k_B T}{m_e}} \approx \sqrt{\frac{3(300 \text{ K})(1.3803 \times 10^{-23} \text{ J K}^{-1})}{0.067(9.109 \times 10^{-31} \text{ kg})}} = 4.5 \times 10^4 \text{ m s}^{-1}. \]

The mean free time is

\[ \tau_e = \frac{m_e |\mu|}{\pi} \approx \frac{0.067 \times 9.109 \times 10^{-31} \text{ kg} \times 8500 \times 10^{-24} \text{ m}^2 \text{V}^{-1} \text{s}^{-1}}{(1.602 \times 10^{-19} \text{ C})} = 3.2 \times 10^{-16} \text{ s}, \]

and the electron drift velocity is

\[ v_d = \mu_e E \approx (8500 \times 10^{-24} \text{ m}^2 \text{V}^{-1} \text{s}^{-1})(10^4 \text{ V m}^{-1}) = 8.5 \times 10^{10} \text{ m s}^{-1}. \]
The theory predicts that $\Delta \lambda / \lambda$ vs $T$ should be a straight line because:

$$\Delta \lambda = \lambda \cdot \frac{\Delta E_{ph}}{h c}$$

so that

$$\frac{\Delta \lambda}{\lambda} = \frac{m \Delta E_{ph}}{h c}$$

where $\Delta E_{ph} = m \Delta E_{ph}$ and $m$ is a numerical constant that represents the ratio $\Delta E(0)/\Delta E(T)$ and is determined from the slope of the $\Delta \lambda / \lambda$ vs. $T$ plot.

The three points plotted in the figure seem to follow this behavior. The best line forced through zero has a slope that indicates $m = 2.8$.

d) The bandgap decreases with temperature.

e) There are two factors to consider. (1) Spectral intensity means intensity per unit wavelength, thus it decreases. The intensity of the spectral curve gives the total intensity: the total number of photons emitted per unit area per unit time. As the spectrum broadens with temperature, we would naturally expect the peak to decrease with temperature. (ii) Higher the temperature, the stronger are the lattice vibrations (more phonons). Indeed, the radiation transitions, those that do not absorb phonons, require phonons (lattice vibration) which encourage inverse transitions. Thus increasing the temperature increases indirect transitions at the expense of direct transitions and the light intensity decreases. If (ii) was totally correct then the area under the curves for all the three spectra would be identical.

f) Use the peak emission wavelength to find $E_g$ as follows:

At $-40 ^{\circ}C$ (233 K), $\lambda_{peak} = 804$ nm.

At $25 ^{\circ}C$ (298 K), $\lambda_{peak} = 820$ nm.

At $85 ^{\circ}C$ (358 K), $\lambda_{peak} = 817$ nm.

We first note that we need the required bandgap $E_g$ at the wavelength of interest. The photon energy at peak emission is $h c / \lambda_{peak}$.

$$E_g = \frac{h c}{\lambda_{peak}} - \frac{k T}{e}$$

and at $\lambda_{peak} = 820 \times 10^{-9}$ m, taking $T = 25 + 273 K$,

$$E_g = (6.63 \times 10^{-34} \times 10^{-14}) - 0.0251 \text{ eV} = 1.4868 \text{ eV}$$

The bandgap $E_g$ of the ternary allows AlGaAs. As follows the empirical expression:

$$E_g(\text{eV}) = 1.424 + 1.266 \times 10^{-4} T + 0.266 T^2$$

Solving for $T$ we find $T = 0.05$.

g) From the definition of efficiency $\eta$,

$$\eta = \frac{P_{out}}{P_{in}}$$

where $P_{out}$ is the output electrical power and $P_{in}$ is the input electrical power.

$$\eta = \frac{90 \times 10^{-6}}{(25 \times 10^{-10} \text{ A})(2.2 \text{ V})} = 0.006417 = 0.0417%$$
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\[ D_e = kT \mu_e / e = (0.0259 \text{ V})(7000 \times 10^{-7} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}) = 0.01813 \text{ m}^2 \text{ s}^{-1} \]

and

\[ D_h = kT \mu_h / e = (0.0259 \text{ V})(310 \times 10^{-7} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}) = 0.0000803 \text{ m}^2 \text{ s}^{-1} \]

Recombination time of electrons diffusing in the p-region is

\[ \tau_e = \frac{1}{D_e N_e} = \frac{1}{(7.21 \times 10^{-14} \text{ m}^2 \text{ s}^{-1})(1 \times 10^{15} \times 10^{9} \text{ m}^{-3})} = 138.7 \text{ ns} \]

Recombination time of holes diffusing in the n-region is

\[ \tau_h = \frac{1}{D_h N_h} = \frac{1}{(7.21 \times 10^{-14} \text{ m}^2 \text{ s}^{-1})(1 \times 10^{15} \times 10^{9} \text{ m}^{-3})} = 138.7 \text{ ns} \]

The diffusion lengths are then:

\[ L_e = \sqrt{D_e \tau_e} = \sqrt{0.01813 \text{ m}^2 \text{ s}^{-1} \times 138.7 \times 10^9 \text{ s}^{-1}} = 5.02 \times 10^{-5} \text{ m}, \text{ or 50.2 \mu m} \]

and

\[ L_h = \sqrt{D_h \tau_h} = \sqrt{0.0000803 \text{ m}^2 \text{ s}^{-1} \times 138.7 \times 10^9 \text{ s}^{-1}} = 1.056 \times 10^{-5} \text{ m}, \text{ or 10.56 \mu m} \]

The diffusion component of the current is

\[ I = I_{diff} = I_n \left( \exp(\alpha V/kT) - 1 \right) = I_n \exp(\alpha V/kT) \text{ for } V \gg kT/e = (0.02586 \text{ V}) \]

where

\[ I_n = A \exp \left[ \frac{(D_e/L_e)(N_e/N_i) + (D_h/L_h)(N_h/N_i))}{(0.1 \times 10^{-3} \text{ m}^2 \text{ s}^{-1})(1.62 \times 10^{18} \text{ m}^{-3})(1 \times 10^{12} \text{ m}^{-3})} \right] \]

Thus

\[ I_n = \frac{(0.1 \times 10^{-3} \text{ m}^2)(1.62 \times 10^{18} \text{ m}^{-3})(1 \times 10^{12} \text{ m}^{-3})}{(0.06 \times 10^{-6} \text{ m})(1 \times 10^{12} \text{ m}^{-3})} \]

\[ \therefore I_n = 2.27 \times 10^{-31} \text{ A.} \]

The forward current due to diffusion is

\[ I_{diff} = I_n \exp[\alpha V/kT] = (2.27 \times 10^{-31} \text{ A}) \exp[(1 \text{ V})(0.0259 \text{ V})] \]

\[ \therefore I_{diff} = 0.00013 \text{ A or 0.13 mA} \]

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\[ n_{abs} = \frac{P_e}{2V} = \frac{2.5 \times 10^{-3}}{(30 \times 10^{-3} \text{ A})(1.6 \text{ V})} = 0.03125 = 3.125 \% \]

3.8

The linewidth \[ \Delta \lambda_{1/2} = \left[ \frac{6.57 \times 10^{-4} \text{ nm}}{\lambda^2} \right] \]

The slope is \[ m = 5.67 \times 10^{-4} \text{ nm}^{-1} \].

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Table 3Q8-3 shows the calculated spectral widths \[ \Delta \lambda_{1/4} \] using \[ m = 3 \]. The actual observed widths are substantially larger than the expected \[ \Delta \lambda_{1/4} \] using \[ m = 3 \], as we found above for the direct bandgap materials. If the recombination center were a discrete level, we would expect a spread in the photon energy that is controlled by the energy distribution of holes (electrons) in the valence (conduction) band. That is about \[ -1.5kT \text{ or } m = 1.5 \]. The observed spread is much more than \[ m = 1.5 \] and hence the energy level cannot be discrete. (It is possible to give a semiquantitative plausible explanation for this.) A captured electron has a wavefunction that is localized and hence a smaller uncertainty in its position than in the band; i.e., \( \Delta x \) will be small. That means the uncertainty \( \Delta p \) in its momentum will be higher and hence the uncertainty in its energy will also be higher. We would expect that the spread of photon energies will be much more than that in band to band recombinations.

Table 3Q8-3

<table>
<thead>
<tr>
<th>Peak wavelength of emission (( \lambda )) nm</th>
<th>468</th>
<th>545</th>
<th>585</th>
<th>600</th>
<th>635</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \lambda_{1/2} ) nm</td>
<td>66</td>
<td>28</td>
<td>36</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>Expected ( \Delta \lambda_{1/4} ) nm using ( m = 3 )</td>
<td>13.7</td>
<td>20.0</td>
<td>21.3</td>
<td>22.5</td>
<td>25.5</td>
</tr>
<tr>
<td>Color</td>
<td>Blue</td>
<td>Green</td>
<td>Yellow</td>
<td>Orange</td>
<td>Red</td>
</tr>
<tr>
<td>Material</td>
<td>SiC (Al)</td>
<td>GaP (N)</td>
<td>GaAsP (N)</td>
<td>GaAsP (N)</td>
<td>GaAsP</td>
</tr>
</tbody>
</table>
3.11

Consider

\[
\eta_{\text{int}} = \frac{1}{\tau_i} \left( \frac{1}{\tau_i} + \frac{1}{\tau_{\text{ns}}} \right) = \frac{1}{50 \text{ ns} + 100 \text{ ns}} = 0.667 = 66.7 \%
\]

From

\[
\eta_{\text{in}} = \frac{P_{\text{emis}}/h\nu}{J/\omega}
\]

\[
P_{\text{emis}} = \eta_{\text{in}} \left( \frac{f}{e} \right) h\nu = \eta_{\text{in}} \frac{hc}{e\lambda} = (0.667) \frac{(100 \times 10^{-5})(6.626 \times 10^{-34})(3 \times 10^5)}{(1.6 \times 10^{-19}) (850 \times 10^{-6})}
\]

\[= 0.097 \text{ W or } 97 \text{ mW} \]