1.3

Silica: $n$ vs $\lambda$ (\text{nm}), thick curve; $N_2$ vs $\lambda$ (\text{nm}), thin bold curve. $N_2$ vs $\lambda$ (\text{nm}), thick bold curve. $N_2$ vs $\lambda$ (\text{nm}) Minimum is around 1.3 \text{ nm}.

SIO$_2$-13.5 mol.\% GeO$_2$: $n$ vs $\lambda$ (\text{nm}), thin curve; $N_2$ vs $\lambda$ (\text{nm}), thick bold curve. $N_2$ vs $\lambda$ (\text{nm}) Minimum is around 1.4 \text{ nm}.

1.5

a. What should be the minimum incidence angle for TIR?

The critical angle for TIR is,

$$\theta_c = \arcsin(n_2/n_1) = \arcsin(1.430/1.460) = 78.4^\circ.$$ 

b. What is the phase change in the reflected wave when the angle of incidence $\theta = 85^\circ$ and when $\theta = 90^\circ$?

$$\frac{n_1}{n_2} = \sin^2 \theta - \frac{n_1^2 - n_2^2}{n_1^2 \cos \theta} = \frac{\sin^2(85^\circ) - (1.430^2 - 1.460^2)}{1.460^2 \cos(85^\circ)} = -2.0868 + \tan^2(128.79^\circ).$$

Thus, $\phi_1 = 128.8^\circ$.

For the $E_{x}$ component, the phase change is

$$\tan^2(\phi_x + 4\pi) = (n_1/n_2)^2 \tan^2(\phi_x) - \frac{1}{n_1^2} \tan^2(\phi_x)$$

so that

$$\tan^2(128.8^\circ + 4\pi) = (n_1/n_2)^2 \tan^2(128.8^\circ) - \frac{1}{n_1^2} \tan^2(128.8^\circ)$$

which gives $\phi_2 = -49.4^\circ$.

When $\theta = 90^\circ$ we have $\phi_x = 180^\circ$ and $\phi_2 = 0^\circ$.

d. What is the penetration depth of the evanescent wave into medium 2 when $\theta_1 = 85^\circ$ and when $\theta = 90^\circ$?

When $\theta_1 = 85^\circ$,

$$a_1 = \frac{2\pi \cdot 1.330}{(85^\circ \cdot 10^{-7})} \left[ \frac{1.460^2}{1.430^2} \sin^2(85^\circ) - 1 \right] = 1.96 \times 10^6 \text{ m}.$$ 

The penetration depth is $d = 1/a_1 = 5.09 \times 10^{-7} \text{ m}.$

When $\theta = 90^\circ$,

$$a_2 = \frac{2\pi \cdot 1.330}{(90^\circ \cdot 10^{-7})} \left[ \frac{1.460^2}{1.430^2} \sin^2(90^\circ) - 1 \right] = 2.18 \times 10^6 \text{ m}.$$ 

The penetration depth is $d = 1/a_2 = 4.95 \times 10^{-7} \text{ m}.$

e. What is the reflection coefficient and reflectance at normal incidence ($\theta = 0^\circ$) when the light beam traveling in the silica medium ($n = 1.455$) is incident on a silica/air interface? Thus $n_1 = 1.460$ and $n_2 = 1.$ Then,

$$R = \frac{r_0}{r_1} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.455 - 1}{1.455 + 1} = 0.185$$

and

$$R = \frac{r_1}{r_1 + r_2} = (0.185)^2 = 0.034 \text{ (3.4%)}. $$

1.8

Assume that $n_1 < n_2 < n_3$ and that the thickness of the coating is $d$. For simplicity, we will assume normal incidence. The phase change in traversing the coating thickness $d$ is $\phi = (2\pi/d)\varphi$ where $\varphi$ is the free space wavelength. The wave has to be multiplied by $\exp(-j\phi)$ to account for this phase difference.

The coefficients are given by,

$$t_1 = t_2 = \frac{n_1 - n_3}{n_1 + n_3}, \quad t_3 = \frac{n_2}{n_1 + n_3}, \quad l_1 = \frac{2n_2}{n_1 + n_3}, \quad l_3 = \frac{2n_3}{n_2 + n_3}, $$
Consider the transmission coefficient obtained in Question 1.8,
\[ t = \frac{r_1 e^{i \phi_0}}{1 + r_1 e^{i \phi_0}} \]

To maximize \( t \) we need \( r_1 \) \( \exp(-2\phi_0) = -1 \). However, \( r_1 \) and \( r_2 \) are positive numbers, which means \( \exp(-2\phi_0) = \cos(-2\phi_0) + \sin(-2\phi_0) = -1 \). This will be so when \( 2\phi = m\pi \) where \( m \) is an odd integer, or when
\[ \phi = \frac{2m\pi d}{A} = m\pi \]

leading to
\[ d = \frac{m\lambda}{4n_2} \]

In addition we need \( r_2 = 1 \). Consider choosing \( n_2 = (n_1 n_3)^{\frac{1}{2}} \). For light traveling in medium 1 incident on the 1-2 interface at normal incidence,
\[ r_1 = r_2 = \frac{n_1 - n_3}{n_1 + n_3} = \frac{n_1 - \sqrt{n_1 n_3}}{n_1 + \sqrt{n_1 n_3}} = \frac{1 - \frac{\sqrt{n_1 n_3}}{n_1}}{1 + \frac{\sqrt{n_1 n_3}}{n_1}} \]

For light traveling in medium 2 incident on the 2-3 interface at normal incidence,
\[ r_3 = \frac{n_3 - n_2}{n_3 + n_2} = \frac{n_3 - \sqrt{n_3 n_2}}{n_3 + \sqrt{n_3 n_2}} = \frac{1 - \frac{\sqrt{n_3 n_2}}{n_3}}{1 + \frac{\sqrt{n_3 n_2}}{n_3}} \]

thus,
\[ r_3 = r_1 \]

which confirms that we need \( n_2 = (n_1 n_3)^{\frac{1}{2}} \).

The reflection coefficient from Question 1.8 is
\[ t = \frac{r_1 + r_2 e^{i \phi_0}}{1 + r_1 e^{i \phi_0}} \]

This is zero (no reflected energy) when the numerator is zero, that is,
\[ r_1 = -r_2 \exp(-2\phi_0) \]

The magnitude of \( \exp(-2\phi_0) \) is unity and since \( r_1 \) and \( r_2 \) are positive quantities, we must have two conditions to obtain zero in the numerator:

Condition 1:
\[ r_1 = r_2 \]

This requires \( n_2 = (n_1 n_3)^{\frac{1}{2}} \) as derived above.

Condition 2:
\[ \exp(-2\phi_0) = \cos(2\phi_0) + \sin(2\phi_0) = -1 \]

which will be so when \( 2\phi = m\pi \) where \( m \) is an odd integer, or when
\[ \phi = \frac{2m\pi d}{A} = m\pi \]

leading to
\[ d = \frac{m\lambda}{4n_2} \]
Given \( \lambda = 632.8 \text{ nm}, \nu = 1.5 \text{ GHz} \). Thus,
\[
\nu = \frac{c}{\lambda} - \left( 3 \times 10^8 \text{ m/s} \right) \sin(632.8 \times 10^{-9} \text{ m}) = 4.738 \times 10^{14} \text{ s}^{-1} \text{ or Hz}.
\]
Thus,
\[
\lambda = \frac{c}{\nu} = \left( 1.5 \times 10^4 \text{ Hz} \right) \left( \frac{632.8 \times 10^{-9} \text{ m}}{4.738 \times 10^{14} \text{ Hz}} \right) = 2.00 \times 10^{-3} \text{ m} \text{ or } 2.00 \mu \text{m}.
\]

1.17
Consider the Bragg diffraction grating equation,
\[
\sin(\theta) = \sin(\theta) = m \lambda \quad m = 0, \pm 1, \pm 2, \ldots
\]
Take \( m = 0 \) to find the zero-order diffraction, which is the normal reflected beam. The result is \( \theta_n = 89^\circ \) as shown in Fig 1Q17; this identifies the meaning of the positive angle in the reflected beam with respect to the normal.

Substituting \( d = 1 \mu \text{m}, \lambda = 1.3 \mu \text{m}, \theta = 89^\circ, m = -1 \),

\[
(1 \mu \text{m}) \sin(\theta) = (1 \mu \text{m})(89^\circ) = (1)(1.3 \mu \text{m})
\]

Solving,

\[
\theta = 17.5^\circ
\]