Chapter 4
Stimulated Emission Devices:
LASERS

4.1 Simulated Emission and Photon Amplification

Stimulated emission:
- Two photons are in phase, same direction
- same polarization, same energy
- Incoming e' couples to the e' in E2

Basis for photon amplification
Re-absorbed? => population inversion

not in two level systems
(in steady state: \( R_{12} = R_{21} \))

Absorption, spontaneous (random photon) emission and stimulated emission.

3499 S.O. Kasap, *Optoelectronics* (Prentice Hall)

Three level system

LASER: Light Amplification by Stimulated Emission of Radiation

Pumping (optical)
e.g. ruby laser: chromium ions \( \text{Cr}^{3+} \) in \( \text{Al}_2\text{O}_3 \) crystal

Feedback: silvered mirror and partially silvered mirror

(Note: usu., more efficient in four level systems)

The principle of the LASER:
- (a) Atoms in the ground state are pumped up to the energy level \( E_2 \) by incoming photons of energy \( h\nu_1 = E_2 - E_1 \).
- (b) Atoms at \( E_2 \) rapidly decay to the metastable state at energy level \( E_2 \) by emitting photons or emitting lattice vibrations \( h\nu_2 = E_2 - E_1 \).
- (c) As the states of \( E_2 \) are long-lived, they quickly become populated and there is a population inversion between \( E_2 \) and \( E_1 \).
- (d) A random photon (from a spontaneous decay) of energy \( h\nu_3 = E_2 - E_1 \) can initiate stimulated emission. Photons from this stimulated emission can then stimulate further stimulated emissions leading to an avalanche of stimulated emissions and coherent photons being emitted.

3499 S.O. Kasap, *Optoelectronics* (Prentice Hall)
• Theodore Harold Maiman was born in 1927 in Los Angeles, son of an electrical engineer. He studied engineering physics at Colorado University, while repairing electrical appliances to pay for college, and then obtained a Ph.D. from Stanford. Theodore Maiman constructed this first laser in 1960 while working at Hughes Research Laboratories (T.H. Maiman, "Stimulated optical radiation in ruby lasers", Nature, 187, 493, 1960). There is a vertical chromium ion doped ruby rod in the center of a helical xenon flash tube. The ruby rod has mirrored ends. The xenon flash provides optical pumping of the chromium ions in the ruby rod. The output is a pulse of red laser light. (Courtesy of HRL Laboratories, LLC, Malibu, California.)

4.2 Stimulated Emission Rate and Einstein Coefficients

A useful LASER medium must have a higher efficiency of stimulated emission compared with spontaneous emission and absorption.

Upward transition rate

\[ R_{12} = B_{12}N_1\rho (h\nu) \]

\[ B_{12}, A_{21} : \text{Einstein Coefficients} \]

\[ \rho (h\nu) : \text{photon energy density / freq.} \]

\[ # \text{photons / volume at } h\nu \]

Downward transition rate

\[ R_{21} = A_{21}N_2 + B_{21}N_2\rho (h\nu) \]

\text{spontaneous} \quad \text{stimulated}

Thermal equilibrium: no change with time in the populations at E1 and E2

\[ R_{12} = R_{21} \]

Boltzmann statistics

\[ \frac{N_2}{N_1} = \exp \left[ \frac{(E_2 - E_1)}{k_B T} \right] \]

(only) in thermal equilibrium, by Plank's black body radiation distribution law

\[ \rho_{eq} (h\nu) = \frac{8\pi h\nu^3}{c^3 \exp \left[ \frac{h\nu}{k_B T} \right] - 1} \]

\( \rho \) is much larger in laser operation

Population inversion \( \Rightarrow \) depart from thermal equilibrium (negative abs. temp.)

The laser principle is based on non-thermal equilibrium.
4.3 Optical Fiber Amplifiers

Long haul communication suffers attenuation.

⇒ Regenerate signal by Optical-Electrical-Optical
⇒ Amplify optical signal directly ⇒ optical amplifier

EDFA: erbium (Er^{3+}) doped fiber amplifier
Other dopants: e.g. Nd^{3+}
Splicing: fibers are fused together

Optically pumped (usu. by LD)

\[ E = \frac{hv}{\lambda} = \frac{1.24 \text{ eV} \cdot \text{nm}}{\lambda \text{ (nm)}} \]

Energy of the Er^{3+} ion in the glass fiber

Optical gain:

\[ G_{op} = K(N_2 - N_1) \]

K: dep. on pumping intensity

\[ E_1 \text{; long-lived } \sim 10 \text{ms} \]

E_t: long-lived \sim 10ms

Fig. 4.3

Energy diagram for the Er^{3+} ion in the glass fiber medium and light amplification by stimulated emission from E_2 to E_1. Dashed arrows indicate radiationless transitions (energy emission by lattice vibrations)

\[ ?1999 \text{ S.O. Kassar, Optoelectronics (Prentice Hall)} \]

• Ali Javan and his associates William Bennett Jr. and Donald Herriott at Bell Labs were first to successfully demonstrate a continuous wave (cw) helium-neon laser operation (1960-1962).
4.4 Gas LASERS: The He-Ne LASER

He-Ne laser @632.8nm (red) from Ne atoms, He used to excite Ne

Ne: 1s²2s²2p⁶ or 2p⁶  He: 1s³

excited Ne: 2p⁵5s¹  Excited He: 1s¹2s¹

Fig. 4.5  Current regulated HV power supply
A schematic illustration of the He-Ne laser

Using dc or RF high voltage:
He atoms to become excited by collisions with drifting electrons
He + e⁻ → He⁺ + e⁻

Excited He⁺: 1s¹2s¹ parallel spin, metastable, not allowed to simply decay back to ground state,
Excited He collides with a Ne atom
He⁺ + Ne → He + Ne⁺  => population inversion of Ne

Various lasing transitions in the He-Ne laser

?1999 SO. Kasap, Optoelectronics (Prentice Hall)
Ne(2p^5s^1): 4 closely spaced energy levels
Ne(2p^53p^1): 10 closely spaced energy levels
Also energy level -- Ne(2p^54p^1)
IR: 3.39 µm
prevented by freq. selective mirrors

Ne(2p^53s^1) is a metastable level,
to ground state by colliding with the walls of laser tube
=> narrow tube is better

Typ. He-Ne laser
He: Ne = 5:1 several torrs
99.9% flat reflecting mirror, and 99% concave reflecting mirror, convergent lens
diameter: 0.5-1 mm divergence: 1 mrad few watts
polarized light by Brewster angle

\[ q \Delta q \wedge w q p = () \]

Gaussian beam
\[ 20 = \frac{4\lambda}{\pi (2w_o^2)} \]

\[ \Delta r \]
Laser tube
Gaussian beam
Laser tube

\[ v = v_0 \left( 1 - \frac{v_x}{c} \right) \]
\[ v = v_0 \left( 1 + \frac{v_x}{c} \right) \]

\[ \Delta v = v_1 - v_2 \approx 2v_0 \sqrt{\frac{2k_B T \ln(2)}{M c^2}} \]

\[ \Delta r \]

4.5 The Output Spectrum of a Gas Laser

Average kinetic energy of molecules: \( (3/2)k_B T \)

Doppler effect:
moving away \[ v_1 = v_0 \left( 1 - \frac{v_x}{c} \right) \]
moving toward \[ v_2 = v_0 \left( 1 + \frac{v_x}{c} \right) \]

Doppler broadened linewidth: \( \Delta v \approx v_2 - v_1 \)
Optical gain lineshape around \( \lambda_0 = c\nu_0 \)
\( \Delta v = 2.5GHz \) for many gas lasers, He-Ne laser \( \sim 0.02 \AA \)

Full width at half maximum linewidth (FWHM) \( \Delta v_{1/2} = 2v_0 \sqrt{\frac{2k_B T \ln(2)}{M c^2}} \)
Finite width due to NL of cavities acoustic and thermal fluctuations of L nonideal end mirror (R<100%)

Typ. ~1MHz for He-Ne Stabilized gas laser as low as 1kHz

Fig. 4.8

(a) Optical gain vs. wavelength characteristics (called the optical gain curve) of the laser medium, (b) Allowed modes and their wavelengths due to stationary EM waves within the optical cavity, (c) The output spectrum (relative intensity vs. wavelength) is determined by satisfying (a) and (b) simultaneously, assuming no cavity loss:

Optical Gain

Relative intensity

Doppler broadening

Optical Gain

Relative intensity

Doppler broadening

Optical Gain

Relative intensity

Doppler broadening

Optical Gain

Relative intensity

Doppler broadening

(a)

Optical gain vs. wavelength characteristics (called the optical gain curve) of the laser medium, (b) Allowed modes and their wavelengths due to stationary EM waves within the optical cavity, (c) The output spectrum (relative intensity vs. wavelength) is determined by satisfying (a) and (b) simultaneously, assuming no cavity loss: 

Typ. freq. width of an individual spike in He-Ne laser is ~1MHz ( low ~1kHz)

Typ. freq. width of an individual spike in He-Ne laser is ~1MHz ( low ~1kHz)

Typ. freq. width of an individual spike in He-Ne laser is ~1MHz ( low ~1kHz)

Typ. freq. width of an individual spike in He-Ne laser is ~1MHz ( low ~1kHz)

Typ. freq. width of an individual spike in He-Ne laser is ~1MHz ( low ~1kHz)
4.6 LASER Oscillation Conditions

A. Optical Gain Coefficient \( g \)

exp(-\( a \times x \)) \( a \): absorption coefficient

\[ g \] \( g \): optical gain coefficient (Fig. 4.10)

\[ g = \frac{\delta N_{ph}}{P \delta x} = \frac{n \delta N_{ph}}{N_{ph} \delta x} (\text{optical gain coefficient}) \]

\[ \frac{dN_{ph}}{dt} = \text{Net rate of stimulated photon emission} \]

\[ = N_2 B_{21} \rho (hv) - N_1 B_{21} \rho (hv) \] (stimulated – absorption)

\[ = (N_2 - N_1) B_{21} \rho (hv) \] (considering directional wave

\( \Rightarrow \) spon. emission is neglected)

\[ \Delta \nu_{1/2} \approx \Delta \nu \approx (1.51 \times 10^8 \text{ Hz}) (632.8 \times 10^{-9} \text{ m}) / (4.74 \times 10^4 \text{ s}^{-1}) \]

\[ \Delta \nu = 2.02 \times 10^{-12} \text{ m or } 0.0020 \text{ nm} \]

B. Optical Threshold Gain \( g_{th} \)

Steady state conditions, no optical power loss in the round trip

\( \Rightarrow \) net round-trip optical gain \( G_{op} = 1. \)

\[ G_{op} = \frac{P_f}{P_i} = 1 \] Losses: R1, R2,

absorption (e.g. by impurities, free carriers)

scattering (defects and inhomogeneities)

\( g(v_0) = (N_2 - N_1) \frac{B_{12} n \nu_0}{c \Delta \nu} \) (optical gain coefficient)

\[ P_f = P_i R_1 R_2 \exp \left[ g (2L) \right] \exp \left[ -\gamma (2L) \right] \] (considering directional wave

\( \Rightarrow \) spon. emission is neglected)
Courses Code: 901 37500-01  Subject Name: Optoelectronics  Instructor: Huang Ding Wei

\[ P_f = P_R R_2 \exp \left[ g(2L) \right] \exp \left[ -\gamma(2L) \right] \]

Threshold optical gain
\[ g_{th} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

Threshold population inversion
\[ (N_2 - N_1)_{th} = g_{th} \frac{c \Delta V}{B_2 n h v_0} \]

Threshold pump rate and \( P_0 \)
\[ N_2 - N_1 \approx \text{pumping} \]

\( (N_2 - N_1)_{th} \approx \text{pumping} \]

Fig. 4.12
Simplified description of a laser oscillator. \((N_2 - N_1)\) and coherent output power \(P_0\) vs. pump rate under continuous wave steady state operation.

Threshold pump rate
Pump rate

C. Phase condition and Laser modes

\[ E_f = E_i \text{ in Fig. 4.11} \]

Phase condition for laser oscillations
\[ \Delta \phi_{\text{round-trip}} = m(2\pi) \]

Neglect \( \phi \) changes at mirrors
\[ nk_m (2L) \equiv m \left( 2\pi \right) \]

Approx. laser cavity modes
\[ m \left( \frac{\lambda_m}{2n} \right) = L \quad \text{longitudinal (axial) modes} \]

Ideally, infinitely wide mirrors plane waves are assumed
Practically, finite size mirrors Gaussian beams are the solutions

Off-axis modes can exist and replicate themselves
\( \Rightarrow \) transverse modes or transverse electric and magnetic (TEM) modes

Each transverse mode with a given \( p,q \) has a set of longitudinal modes.
Usu. \( m \) is very large \( \sim 10^6 \) in gas lasers

Transverse modes depend on: optical cavity dimensions, reflector sizes, Cartesian (rectangular) or polar (circular) symmetry about the cavity axis (Brewster angle)

highly desirable TEM00:
lowest mode, radially symmetric, lowest divergence

Fig. 4.13
Laser Modes (a) An off-axis transverse mode is able to self-replicate after one round trip. (b) Wavefronts in a self-replicating wave (c) Four low order transverse cavity modes and their fields. (d) Intensity patterns in the modes of (c).

\[ \Delta N_a = (N_2 - N_1)_{th} \]

\[ \Delta N_a = g_{th} \frac{8\pi m^2 n^2 L \tau_m \Delta \nu}{c^2} \]

\[(12)\]

其中 \( n_k \) = 发射频率峰值（输出频率峰值）, \( n = \) 相对折射係數, \( \tau_m = 1/A_21 = \) 自發性轉換之平均時間, \( \Delta \nu = \) 光強增益頻寬（光強增益線形之頻率線寬）。

考慮 He–Ne 氣體雷射操作波長為 632.8 nm，雷射管長度 \( L = 50 cm \), 直徑為 1.5 mm，兩端鏡面反射率為 100% 及 90%，鏡面 \( \Delta \nu = 1.5 GHz \), 捐耗係數 \( \gamma = 0.05 m^{-1} \), 自發性衰減時間常數 \( \tau_m = 1/A_21 = 300 ns \), \( n = 1 \) 則閾值電流反轉為何？

例 4.6.1 He–Ne 雷射之閾值電流反轉
4.7 Principle of the Laser Diode

The energy band diagram of a degenerately doped $p-n$ with no bias. (b) Band diagram with a sufficiently large forward bias to cause population inversion and hence stimulated emission.

See Fig. 4.14(a) degenerate doping

No applied voltage, $E_{Fp} = E_{Fn}$

applied voltage $V$, $\Delta E_p = eV$

Fig. 4.14(b) $\Delta E_p = eV > E_g$

Diminishes the built-in potential barrier

SCL is no longer depleted

In SCL, more electrons in the conduction band at energies near $E_c$ than electrons in the valence band near $E_v$

Fig. 4.15(a) Population inversion between energies

near $E_c$ and those near $E_v$ around the junction

Called inversion layer or active layer

More stimulated emission than absorption

$E_g < \hbar \nu < E_{Fn} - E_{Fp}$ Stimulated emission

$\hbar \nu > E_{Fn} - E_{Fp}$ Absorption

Injection pumping: pumping by the forward diode current

Fig. 4.15

(a) The density of states and energy distribution of electrons and holes in the conduction and valence bands respectively at $T = 0$ in the SCL under forward bias such that $E_{Fn} - E_{Fp} > E_g$. Holes in the VB are empty states.

(b) Gain vs. photon energy.
4.8 Heterostructure Laser Diodes

Reduction of the threshold current by improving
A. Rate of stimulated emission
   => carrier confinement
B. Efficiency of the optical cavity
   => photon confinement by waveguide

Both can be achieved by heterostructured devices
LEDs vs LDs: stimulated emission > spontaneous emission

Fig. 4.18
p-GaAs and p-AlGaAs are degenerately doped

(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs).
(b) Simplified energy band diagram under a large forward bias. Lasing recombination takes place in the p+ GaAs layer, the active layer
(c) Higher bandgap materials have a lower refractive index
(d) AlGaAs layers provide lateral optical confinement

Typical output optical power vs. diode current (I) characteristics and the corresponding output spectrum of a laser diode.

Problems of homojunction LD:
threshold current density \( J_m \) is too high for practical use
\( \sim 500 \text{ A/mm}^2 \) for GaAs, can only be operated at very low temperature
\( J_m \) can be reduced by orders of magnitude by using heterojunction LD

\[ m \left( \frac{\lambda_m}{2n} \right) = L \]

Fig. 4.16
A schematic illustration of a GaAs homojunction laser diode. The cleaved surfaces act as reflecting mirrors.
p-GaAs: 870-900 nm
Stripe contact: define current density

$\Rightarrow$ reduce $I_s$, better coupling to fiber
$W$: few µm, $I_{th}$: tens mA

![Schematic illustration of the the structure of a double heterojunction stripe contact laser diode](image)

71999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

**Buried double heterostructure**
Index guided: better lateral optical confinement (can be single mode)

![Schematic illustration of the cross sectional structure of a buried heterostructure laser diode](image)

71999 S.O. Kasap, *Optoelectronics* (Prentice Hall)

**Example 4.8.1** 
**Ray and Optical Resonance Length of the Modulator**

Consider a modulator in which length is 200 µm. AlGaAs, GaAs, and GaAsP are characterized. The bandgap is 0.78 eV. The refractive index of AlGaAs is 3.54, GaAs of 3.46, and GaAsP of 3.49. The index of refraction of AlGaAs is 3.54, GaAs of 3.46, and GaAsP of 3.49. The index of refraction of AlGaAs is 3.54, GaAs of 3.46, and GaAsP of 3.49.

<table>
<thead>
<tr>
<th>Table 4.1</th>
<th>Model Material</th>
<th>λ (nm)</th>
<th>$\eta_{external}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>GaAs</td>
<td>870 - 900</td>
<td>10</td>
</tr>
<tr>
<td>Al$<em>{x}$Ga$</em>{1-x}$As$_{y}$</td>
<td>GaAs</td>
<td>640 - 870</td>
<td>5 - 20</td>
</tr>
<tr>
<td>In$<em>{x}$Ga$</em>{1-x}$As$_{y}$</td>
<td>InP</td>
<td>1 - 1.6 µm</td>
<td>&gt;10</td>
</tr>
<tr>
<td>In$<em>{x}$Ga$</em>{1-x}$N$_{y}$ alloys</td>
<td>GaN or SiC</td>
<td>430 - 460</td>
<td>2</td>
</tr>
<tr>
<td>Saphire</td>
<td>Saphire</td>
<td>500 - 530</td>
<td>3</td>
</tr>
<tr>
<td>SiC</td>
<td>Si : SiC</td>
<td>460 - 470</td>
<td>0.02</td>
</tr>
<tr>
<td>GaAs$<em>{x}$P$</em>{y}$</td>
<td>GaP</td>
<td>560 - 700</td>
<td>&lt;1</td>
</tr>
<tr>
<td>GaAs$<em>{x}$P$</em>{y}$</td>
<td>GaP</td>
<td>650 - 750</td>
<td>&lt;1</td>
</tr>
<tr>
<td>GaP (Zn-O)</td>
<td>GaP</td>
<td>700</td>
<td>2 - 3</td>
</tr>
<tr>
<td>GaP (N)</td>
<td>GaP</td>
<td>565</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>

Fig. 4.19

Schematic illustration of the the structure of a double heterojunction stripe contact laser diode

71999 S.O. Kasap, *Optoelectronics* (Prentice Hall)
4.9 Elementary Laser Diode Characteristics

Output Spectrum dep. on

A. Optical Resonator
- L: longitudinal mode separation
- W, H: lateral modes   small W, H => single mode, TEM₀₀
- divergence: smaller aperture => larger diffraction (e.g. H in Fig. 4.21)

B. Optical Gain Curve

Fig. 4.21

The laser cavity definitions and the output laser beam characteristics.

?1999 S.C. Kuo, Optoelectronics (Prentice Hall)

Red shift: temperature-induced gain shifting due to heating

Output spectrum of lasing emission from an index guided LD. At sufficiently high diode cur rents corresponding to high optical power, the operation becomes single mode. (Note: Relative power scale applies to each spectrum individually and not between spectra)

?1999 S.C. Kuo, Optoelectronics (Prentice Hall)
Slope efficiency

\[ \eta_{\text{slope}} = \frac{P_0}{I-I_{\text{th}}} \]

Conversion efficiency may be as high as 30-40%.

**Fig. 4.23**

Output optical power vs. diode current as three different temperatures. The threshold current shifts to higher temperatures.


---

Peak wavelength vs. case temperature characteristics. (a) Mode hops in the output spectrum of a single mode LD. (b) Restricted mode hops and none over the temperature range of interest (20 - 40 °C). (c) Output spectrum from a multimode LD.


---

4.10 Steady State Semiconductor Rate Equation

Rate of electron inject by current

\[ \frac{I}{edLW} = \frac{n}{\tau_{\text{sp}}} + CnN_{ph} \]

\[ C: \text{constant dep. on } B_{ij} \]

Steady state:

Rate of the coherent photon loss = Rate of stimulated emission

\[ \frac{N_{ph}}{\tau_{\text{ph}}} = CnN_{ph} \]

\[ \tau_{\text{ph}}: \text{due to trans. thru end-faces, scattering, absorption} \]

At threshold \( n_{\text{th}} \)

\[ n_{\text{th}} = \frac{1}{C\tau_{\text{ph}}} \]

Stimulated emission just balanced by all loss

\[ I_{\text{th}} = \frac{n_{\text{th}}edLW}{\tau_{\text{sp}}} \]

Smaller \( I_{\text{th}} \): heterostructure and stripe geometry.

---

例題 4.9.1 雷射輸出波長之變動

若 GaAs 之相對折射率隨溫度之變化為 \( dn/dT = 1.5 \times 10^{-4} \text{ K}^{-1} \) ，試計算在發光波長 870 nm 下，模態跳動隨溫度之變動情形。

第 8 節考慮一個特定波長 \( \lambda_n \)

\[ m \left( \frac{\lambda_n}{2n} \right) = L \]

則

\[ \frac{d\lambda_n}{dT} = \frac{dn}{dT} \left( \frac{2m}{n} \right) \]

\[ = \frac{2L}{m} \frac{dn}{dT} \]

\[ \frac{d\lambda_n}{dT} = \frac{870 \text{nm}}{(3.7)} \left( 1.5 \times 10^{-4} \text{ K}^{-1} \right) = 0.035 \text{ nm K}^{-1} \]

注意：一般使用到的 \( n \) 與共振振有關，而上面我們使用的 \( n \) 爲主動區共振振之有效相對折射率，亦同時與介質之光增益有關，因此，由於溫度的相依性使得較我們使用之 \( dn/dT \) 為高。
4.11 Light Emitters for Optical Fiber Communications

Short haul: LED    larger $\Delta \lambda$
  simpler to drive, economic, longer lifetime
  usual with multimode graded index fibers

Long haul: LD
  narrow linewidth and high output power

Comparison, see Fig. 4.26 and Table 4.1

A laser diode pigtailed to a fiber. Two of the leads are for a back-facet photodetector to allow the monitoring of the laser output power. (Courtesy of Alcatel)

Above threshold
$$\frac{P}{P_{th}} = CN_{ph}/N_{th}$$

O/P optical power
$$P_0 = \left( \frac{1}{2} N_{ph} \right) \left( \text{Cavity Volume} \right) \left( \text{Photon energy} \right) \left( 1 - R \right)$$

=> Laser diode equation
$$P_0 = \left[ \frac{hc}{2 \pi \lambda} \left( 1 - R \right) \right] \left( J - J_{th} \right)$$

Fig. 4.25

Simplified and idealized description of a semiconductor laser diode based on rate equations. Injected electron concentration $n$ and coherent radiation output power $P_0$ vs. diode current $I$.

Typical optical power output vs. forward current for a LED and a laser diode.

Laser diode
One mode by suppressing unwanted mode thru design, $\Delta \lambda \sim 0.01$ to $0.1$ nm

$\Delta \lambda < 0.01$ nm in single frequency LD

<table>
<thead>
<tr>
<th></th>
<th>Double Junction</th>
<th>InGaAsP on InP</th>
</tr>
</thead>
<tbody>
<tr>
<td>材料</td>
<td></td>
<td></td>
</tr>
<tr>
<td>輸出幅射</td>
<td>非同調（自發性放射）</td>
<td>同調（受激發射）</td>
</tr>
<tr>
<td>當型光譜線寬, $\Delta \lambda$</td>
<td>100 nm</td>
<td>2 - 4 nm (多模雷射)</td>
</tr>
<tr>
<td>上升時間</td>
<td>5 - 20 ns</td>
<td>&lt; 1 ns</td>
</tr>
</tbody>
</table>
4.12 Single Frequency Solid State Lasers

Distributed Bragg reflector (DBR) laser

\[ \frac{\lambda_B}{\Lambda} = \frac{2}{q+1} \]

\( \lambda_B \): Bragg wavelength
\( q \): diffraction order

Right and left traveling waves are coupled

(a) Distributed Bragg reflection (DBR) laser principle. (b) Partially reflected waves at the corrugations can only constitute a reflected wave when the wavelength satisfies the Bragg condition. Reflected waves \( A \) and \( B \) interfere constructively when \( q(\frac{\lambda_B}{2}\Lambda) = \Lambda \).

Cleaved-coupled-cavity (C³) laser

Two lasers are pumped by different currents

Only those waves that can exist as modes in both cavities are allowed

Wide separation between the modes \( \Rightarrow \) single mode operation

Fig. 4.29 Cleaved-coupled-cavity (C³) laser

Distributed feedback (DFB) laser

Right and left traveling waves are coupled

(a) Distributed feedback (DFB) laser structure, (b) Ideal lasing emission output, (c) Typical output spectrum from a DFB laser.

\( \Lambda \gg \lambda_B \)

Commercially available 1.55\( \mu \text{m} \) \( \Delta \lambda \approx 0.1 \text{nm} \)

例題 4.12.1 DFB 雷射

考慮 DFB 雷射之波長規則 \( \Lambda \) 為 0.22\( \mu \text{m} \)，光柵長度為 400\( \mu \text{m} \)，假設介質之有效折射率為 3.5。在考慮第一階光柵情況下，試計算布拉格波長，模態波長及模態間距。

\( \lambda_B \) 布拉格波長為

\[ \lambda_B = \frac{2\Delta n}{q} = \frac{2(0.22\mu \text{m})(3.5)}{1} = 1.540\mu \text{m} \]

相對於 \( \lambda_B \) 之對稱型模態波長

\[ \lambda_n = \lambda_B \pm \frac{\lambda_B^2}{2nL}(m+1) = 1.54 \pm \frac{(1.54\mu \text{m})^2}{2(3.5)(400\mu \text{m})}(0+1) \]

因此 \( m=0 \) 之模態波長為

\[ \lambda_0 = 1.539 \text{ 或 } 1.508\mu \text{m} \]

這兩者之間距為 0.0017\( \mu \text{m} \) 或 1.7nm，由於某些不對稱特性，因此其輸出僅有單一模態，且就許多實際之目的而言，模態波長通常是 \( \lambda_B \)。
4.13 Quantum Well Devices

Fig. 4.30
Ultra thin, typ. < 50nm, narrow bandgap semiconductor devices
Lattice match is required
d << \(D_x, D_y, D_z\)
One dimensional PE well in \(x\)-direction and as if free in \(yz\)-plane
\[ E = E_c + \frac{\hbar^2 n_x^2}{8m^*_x d^2} + \frac{\hbar^2 n_y^2}{8m^*_y D_y} + \frac{\hbar^2 n_z^2}{8m^*_z D_z} \]
Step density of states
a large concentration of electrons (holes) can easily occur at \(E_1 (E'_1)\)
population inversion occurs quickly without a large current
Advantages
\(I_p\) is markedly reduced (eg. 0.5-1mA in SQW, 10-50mA in DH laser)
linewidth is substantially narrower because \(\epsilon^*(\hbar^*)\) near \(E_1 (E'_1)\)

Fig. 4.31
In single quantum well (SQW) lasers electrons are injected by the forward current into the thin GaAs layer which serves as the active layer. Population inversion between \(E_1\) and \(E'_1\) is reached even with a small forward current which results in stimulated emissions.

Multiple quantum well (MQW) laser

Fig. 4.32
A multiple quantum well (MQW) structure. Electrons are injected by the forward current into active layers which are quantum wells.

\(\epsilon^*(\hbar^*)\) near \(E_1 (E'_1)\)
4.14 Vertical Cavity Surface Emitting Lasers (VCSELs)

See Fig. 4.33

Dielectric mirrors

\[ n_1 d_1 + n_2 d_2 = \frac{1}{2} \lambda \]

(DBR structure)

High reflectance end mirrors are needed due to short cavity length

20-30 or so layers to obtain the required reflectance (99%) Active layer: very thin <0.1µm, likely MQW e.g. 980 nm InGaAs in GaAs substrate

Circular cross-section

Height: several microns ⇒ single mode longitudinally (maybe not laterally)

Spectral width 0.5nm

---

Fig. 4.33

A simplified schematic illustration of a vertical cavity surface emitting laser (VCSEL).

1999 S. K. Kaner, Optical Electronics (Prentice Hall)
4.15 Optical Laser Amplifiers

Amplified by induced stimulated emission
Pumped to achieve optical gain
(i.e. population inversion)
Random spontaneous emission
=> noise => filtered out
Typ. ~20dB, dep. on the AR coating
Pump current

Under threshold current
Amplified by stim. Emission
Multiple reflection
Higher gain, less stable

Signal in
Active region
Signal out
AR = Antireflection coating
AR
Partial mirror
Partial mirror
(a) Traveling wave amplifier
(b) Fabry-Perot amplifier

Fig. 4.34 Simplified schematic illustrations of two types of laser amplifiers

4.16 Holography

A technique of reproducing three dimensional (3D) optical image of an object by using a highly coherent radiation from a laser source

Fig. 4.35

$E_{\text{ref}}$; both amplitude and phase variations representing the cat’s surface
Reflected $E_{\text{ref}}$ interferes with $E_{\text{cat}}$ at photographic plate
Interference pattern depends on the magnitude and phase variations in $E_{\text{cat}}$

Hologram: recorded interference pattern in the photographic film

Diffracted beam: virtual image
(Bragg condition $d \sin \theta = m \lambda$)
Thru beam: real image

Fig. 4.35

A highly simplified illustration of holography. (a) A laser beam is made to interfere with the diffracted beam from the subject to produce a hologram. (b) Shining the laser beam through the hologram generates a real and a virtual image.
\[ E_{\text{ref}}(x, y) = U_r(x, y)e^{j\omega} \quad \text{and} \quad E_{\text{cat}}(x, y) = U(x, y)e^{j\omega} \]

\[ I(x, y) = |E_{\text{ref}} + E_{\text{cat}}|^2 = |U_r + U|^2 = (U_r + U)(U_r^* + U^*) \]

Pattern on the hologram
\[ I(x, y) = UU^* + U,U_r^* + U^*U + U_rU^* \]

Illuminating the hologram with reference beam
\[ U_r \propto U_r I(x, y) = U_r(UU^* + U_r^*U + U_r^*U + U_r^*U^*) \]

i.e. \[ U_r \propto U_r I(x, y) = U_r(UU^* + U_r^*U) + (U_r^*U)U + U_r^*U^* \]

\[ U_r \propto a + bU(x, y) + cU^*(x, y) \]

\[ a = U_r(UU^* + U_r^*U) \text{ constant: through beam} \]

\[ bU \text{: scaled version of } U \text{: virtual image} \]

\[ cU^* \text{: complex conjugate of } U \text{: real image, conjugate image} \]

Eye can not detect phase shift. Observer always sees the positive image.

**Holography is a method of wavefront reconstruction.**